LARDNER'S
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OPTICS.


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CHRISTO ET ECCLESIAE
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Geo. [illegible] Esq.

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March 30, 64 Emerson

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Necrosis in the upper
Jaw, of a woman
Mass. Gen. Hospital

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A

HAND-BOOK

OF

OPTICS.

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IN UNIVERSITY COLLEGE, LONDON.

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LIGHT.

CHAPTER I.

LUMINOUS AND NON-LUMINOUS BODIES. — TRANSPARENCY. — OPACITY.

896. *Physical nature of light.* — Light is the physical agent by which the external world is rendered manifest to the sense of sight.

Opinion has long been divided as to its nature; one party has regarded it as a specific fluid, another as the effect of undulation.

The former consider that the eye is affected by light as the sense of smell is affected by the odoriferous effluvia; the latter maintain that light is to the eye what sound is to the ear. Before these theories, however, can be understood, or their claims to adoption be appreciated, it will be necessary that the chief properties of light, and the phenomena consequent upon them, be explained.

897. *Bodies luminous and non-luminous.* — In relation to the production of light, bodies are considered as luminous and non-luminous.

Luminous bodies, or luminaries, are those which are original sources of light, such, for example, as the sun, the flame of a lamp or candle, metal rendered red-hot, the electric spark, lightning, and so forth.

Luminaries are necessarily always visible when present, provided the light they emit be strong enough to excite the eye.

Non-luminous bodies are those which themselves produce no light, but which may be rendered temporarily luminous when placed in the presence of luminous bodies. These cease, however, to be luminous, and therefore visible, the moment the luminary from which they borrow their light is removed. Thus the sun, placed in the midst of the planets, satellites, and comets, renders these bodies luminous and visible; but when any of them is removed from the solar influence by the interposition of any object not pervious to light, they cease to be visible, as is manifest in the case of lunar eclipses, when the globe of the earth is interposed between the sun and moon, and the latter object is therefore deprived of light. A candle or lamp placed in the

room renders the walls, furniture, and surrounding objects temporarily luminous, and therefore visible; but if the candle be screened by any object not pervious to light, those parts of the room from which light is intercepted would become invisible, did they not receive some light from the other parts of the room still illuminated. If, however, the candle or lamp be completely covered, all the objects in the room become invisible.

898. *Transparency and opacity.*—In relation to the propagation of light, bodies are considered as transparent and opaque. Bodies through which light passes freely are called transparent, because the eye placed behind them will see such light through them. Bodies, on the contrary, which do not admit light to pass through them, are called opaque; and such bodies consequently render a luminary invisible if interposed between it and the eye.

Transparency and opacity exist in various bodies in different degrees. Glass, air, and water are examples of very transparent bodies. The metals, stone, earth, wood, &c. are examples of opaque bodies.

Correctly speaking, no body is perfectly transparent or perfectly opaque.

899. *No body perfectly transparent.*—There is no substance, however transparent, which does not intercept some portion of light, however small. The light is thus intercepted in two ways; first, when the light falls upon the surface of any body or medium, a portion of it is arrested, and either absorbed upon the surface, or reflected back from it; the remainder passes through the body or medium, but in so passing more or less of it is absorbed, and this increases according to the extent of the medium through which the light passes. Analogy, therefore, justifies the conclusion that there is no transparent medium which, if sufficiently extensive, would not absorb all the light which passes into it.

A very thin plate of glass is almost perfectly transparent, a thicker is less so, and according as the thickness is increased the transparency will be diminished. The distinctness with which objects are seen through the air diminishes as their distance increases, because more or less of the light transmitted from them is absorbed in its progress through the atmosphere. This is the case with the sun, moon, and other celestial objects, which when seen near the horizon are more dim, however clear the atmosphere may be, than when seen in the zenith. In the former case, the light transmitted from them passes through a greater mass of atmosphere, and more of it is absorbed. According to Bouguer, sea-water at about the depth of 700 feet would lose all its transparency, and the atmosphere would be impervious to the sun's light if it had a depth of 700 miles.

900. *Various degrees of transparency.*—The transparency of the same substance varies according to the density of its structure, the transparency generally increasing with the density. Thus, charcoal

is opaque; but if the same charcoal be converted into a diamond, which it may be, without any change of the matter of which it is composed, it will become transparent.

Bodies are said to be imperfectly transparent, or semi-transparent, when light passes through them so imperfectly, that the forms and colours of the objects behind them cannot be distinguished. Ground glass, paper, and thin tissues in general, foggy air, the clouds, horn and various species of shell, such as tortoise-shell, are examples ot this.

The degrees of this imperfect transparency are infinitely various; some substances, such as horn, being so nearly transparent as to render the form of a luminous object behind it indistinctly visible. Porous bodies, which are imperfectly transparent, usually have their transparency increased by filling their pores with some transparent liquid. Thus paper, which is imperfectly transparent, is rendered much more transparent by saturating it with oil, or by wetting it with any liquid. The variety of opal called hydrophane is white and opaque when dry, but when saturated with water it becomes transparent. Ground glass is rendered more transparent by pouring oil upon it. Two plates of ground glass placed one upon the other are very imperfectly transparent; but if the space between them be filled with oil, and their external surfaces be rubbed with the same liquid, they will be rendered nearly transparent.

901. *Opaque bodies become transparent when sufficiently attenuated.* — Bodies, however opaque, lose their perfect opacity when reduced to the form of extremely attenuated laminæ. Gold, one of the most dense of metals, is, in a state of ordinary thickness, perfectly opaque; but if it be reduced to the form of leaf-gold by the process of the gold-beater, and attached to a plate of glass, light will pass partially through it, and to an eye placed behind it it will appear of a greenish colour. Other metals, when equally attenuated, show the same imperfect opacity.

CHAP. II.

RECTILINEAR PROPAGATION OF LIGHT. — RADIATION. — SHADOWS AND PENUMBRÆ — PHOTOMETRY.

902. *Light transmitted in straight lines.* — One of the first properties recognised in light by universal observation and experience is, that when transmitted through a uniform medium, it maintains a rectilinear course.

A luminous point is a centre from which light issues in every direc-

tion through the surrounding space in straight lines. This effect of rectilinear propagation in all directions from a common centre is called *radiation.*

Any straight line along which light is transmitted is called a *ray of light.*

Any point from which rays of light radiate through the surrounding space, is called a *luminous point.*

The rectilinear propagation of light is established by numerous examples, and by a vast variety of effects, of which it affords the explanation. If any opaque object be interposed in a right line between the eye and a luminous point, the luminous point will cease to be visible; but if the opaque object be removed in the slightest degree from the direct line between the eye and the luminous point, the latter will become immediately visible.

This law, in its strictest sense, may be verified by the following experiment. Let three disks be pierced, each with a small hole, and let them be attached to a straight rod, in such a manner that the three holes shall be precisely in the same straight line, and consequently, at the same distance from the rod. If a light be placed behind one of the extreme disks, and the eye behind the others, the light will be visible. The ray, therefore, which renders it visible, must pass successively through the holes in the two extreme disks, and in the intermediate disk; but if the intermediate disk be slightly moved on either side, or upwards or downwards, or, in a word, have its position deranged in any manner, so that a thread stretched between the holes in the extreme disks would not pass through the hole in the intermediate disk, then the light will be no longer visible.

903. *Pencil of rays.* — Any collection of rays having a luminous point as their common origin, and included within the surface of a cone, or any other regular limit, is called a *pencil of rays.* The point from which such rays diverge, and which is the apex of the cone, is called the focus of the pencil.

When the surface of any object receives light from a luminous point, it is customary to consider each portion of such surface as the base of a pencil of rays, the focus of which is the luminous point, so that the illuminated surface of any body is considered as composed of the bases of a number of pencils of rays having the luminous point as their common focus.

When rays radiate from a luminous point in this manner, they are called *divergent.*

But cases will be shown hereafter, in which such rays may be so changed in their direction, that, instead of diverging from the same point, they will converge to a common point. In this case the rays are called *converging* rays, the pencils *converging* pencils, and the point towards which the rays converge, and at which they would meet, if not intercepted, is called the *focus of the pencil.*

904. *Shadow of a body.* — When light radiating from a luminous point through the surrounding space encounters an opaque body, it will be excluded from the space behind such body. The space from which it is thus excluded is called the *shadow* of the opaque body.

This term *shadow* is sometimes applied, not to the space from which the light is thus excluded, but to a section of such space formed upon the surface of some body placed behind the opaque body which intercepts the light. Thus, the floor or wall of a room intersecting the space from which light is excluded by an opaque body placed between such wall or floor and a luminary, will exhibit a dark figure, resembling more or less in outline the body which intercepts the light.

If a straight line be imagined to be drawn from the luminous point to the boundary of the opaque body, and to be continued beyond it indefinitely, such line being imagined to be moved round the opaque body following its limits and its form, that part of the line which is beyond the body will pass through a surface which will form the limits of the shadow of such body, or of the space from which it excludes the light. If such line, however, encounter a wall, screen, or other surface, it will trace upon such surface the limits of the shadow, in the common acceptation of that term.

If the opaque object be a sphere, or any other figure whose section taken at right angles to the direction of the luminous pencil is a circle, the shadow will be a truncated cone, as represented in *fig.* 267.

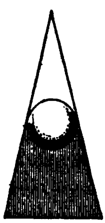

Fig. 267.

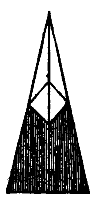

Fig. 268.

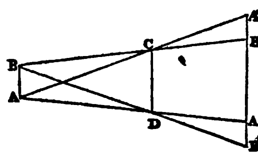

Fig. 269.

If the section be of a rectilinear figure, such, for example, as a square, the shadow will be what in geometry is called a truncated pyramid, as represented in *fig.* 268.

There is, however, no luminary which, strictly speaking, is a luminous point. All luminous objects have a certain definite surface of more or less extent, and consist therefore of an infinite number of luminous points. Now each luminous point of such a body is the focus of an independent pencil of luminous rays, and each such pencil encountering an opaque object will produce an independent shadow

905. *Cause of penumbræ.*—This gives rise to phenomena which it is necessary here more fully to explain.

Let C D, *fig.* 269, represent the section of an opaque object, and let B A represent the section of a luminary. B A will then consist of a line of luminous points, from each of which a pencil of rays will issue. The pencil which issues from the point B, will encounter the object C D, and the extreme rays of the pencil grazing the edge of the object, will proceed in the direction C B′ and D B″, being the continuation of the lines B C and B D. Now it is evident that the light proceeding from the point B will be excluded from the space included between the lines C B′ and D B″.

In like manner it may be shown that the light issuing from the point A will be excluded from the space included between the lines C A′ and D A″. It will also be easily perceived that the light proceeding from all the luminous points from A to B will be excluded from the space included between the lines C B′ and D A″; while more or less of such light, according to the position of the luminous points, will enter the space included between the lines C A′ and C B′, and the lines D A″ and D B″ respectively. The space, therefore, included between the lines C B′ and D A″, from which the entire light of the luminary A B is excluded, is called the umbra or absolute shadow; while the spaces included between C A′ and C B′ and between D A″ and D B″, from which the light of the luminary A B is only partially excluded, is called the *penumbra*, or imperfect shadow.

If a screen be fixed behind the body C D, the shadow and penumbra will be cast upon it, and will be perceptible. At B′ and A″, the boundaries between the shadow and the penumbra, the limit of shadow will be scarcely discernible, and the shadow will become gradually less dark, proceeding from such points to the points A′ and B″, which are the limits of the penumbra. The points A′ and B″ respectively receive light from all the points between A and B, but a point below A′ receives no light from the point A, or from the points immediately above it.

In like manner the points immediately above B″ receive no light from the point B, or the points immediately below it; and as we proceed onwards along the penumbra, the nearer we approach to the limits B′ and A″, the less will be the number of luminous points of the luminary A B from which light will be received. Hence it is, that the obscurity of the penumbra augments by degrees in proceeding from its outward limits to the limits of the umbra, where the obscurity becomes complete.

906. *Forms and dimensions of shadow.*—When an object is placed with its principal plane parallel to the plane of a screen, both being at right angles to the pencil of rays which proceeds from the luminary, the outline of the shadow will resemble the outline of the

object; but if the pencil fall obliquely on the object, or if the screen be not parallel to it, then the form and dimensions of the shadow will be distorted, the relative proportions and directions being different from those of the object.

When the sun is near the horizon, the shadow of an object standing vertically, which is cast upon a vertical wall, will present the form of the object with but little distortion, but the shadow which is cast upon the level ground will be disproportionally elongated in relation to its breadth.

907. *Light diminished in brightness by distance.—The intensity of light which issues from a luminous point diminishes in the same proportion as the square of the distance from such point increases.*

This is a common property of radiation, and has been already explained in the case of the radiation of sound. The intensity of the light at any point is in the direct proportion of the number of rays which fall upon a surface of given magnitude, or in the inverse proportion of the surface over which a given number of rays are diffused.

Now let us suppose a luminous point radiating in all directions round it to be the centre of a sphere. Let two spheres be imagined, having the luminous point as a common centre, and the radius of one being double the radius of the other. The surface of the greater sphere will be therefore twice as far from the luminous point as the surface of the lesser sphere; and since the surfaces of spheres are in the ratio of the squares of their radii, the surface of the greater sphere will be four times that of the lesser. Now since all the light issuing from the luminous point is diffused over the surface of such sphere, it is clear that its density on the surface of the lesser sphere will be greater than its density on the surface of the greater sphere, in the exact proportion of the magnitude of the surface of the greater sphere to the magnitude of the surface of the lesser sphere; that is, in the present example, as 4 to 1. In general it is evident, therefore, that the superficial space over which the rays issuing from a luminous point are diffused, is in the inverse proportion of the squares of the distances from the luminous point.

If, therefore, any opaque surface be presented at right angles to the rays proceeding from a luminous point, the intensity of the illumination which it receives will be increased in the same proportion as the square of its distance from its luminous point is diminished.

Since, then, the intensity of the light proceeding from each luminous point is inversely as the square of the distance from such point, it follows that the intensity of the light proceeding from any luminary will depend conjointly on, first, the number of luminous points upon the luminary, or, what is the same, the magnitude of the luminous surface; secondly, on the intensity of the light of each luminous point

composing such surface; and thirdly, upon the distance from the luminary at which the illuminated object is placed.

908. *Absolute brilliancy depends conjointly on absolute intensity and distance.* — The absolute brilliancy of each luminous point composing any luminous object is called the absolute intensity of its light. Let this be expressed by I. Let the number of luminous points composing it, or the magnitude of its luminous surface, be expressed by S, and let the distance of the illuminated object from the luminary be expressed by D. The brilliancy of the illumination will then be expressed by

$$B = \frac{I \times S}{D^2}.$$

In other words, the brilliancy of the illumination is proportional to the absolute intensity of the luminary multiplied by the magnitude of its illuminating surface, and divided by the square of the distance of the illuminated object from it.

909. *Effect of obliquity of the light.* — It is here supposed, however, that the illuminated surface is placed at right angles to the rays of light, as would be the case with the surface of a sphere surrounding a luminous centre; but as it seldom happens that the illuminated surface has exactly this position, it is necessary to inquire in what manner the brightness of the illumination will be affected by its obliquity to the rays of light falling upon it.

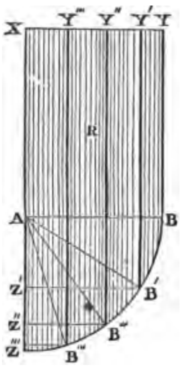

Fig. 270.

Let R, *fig.* 270., be a pencil of rays which we shall here suppose to be parallel; and let A B be a surface on which these rays fall. Let this surface be supposed to be capable of being turned upon the point A as a centre or hinge, so as to assume different obliquities in relation to the rays. If it were in the position A B, at right angles to the direction of the rays, it would receive upon it all the rays included between the lines A X and B Y. If it be in the position A B′, it will receive upon it only the rays included between the lines A X and B′ Y′. If it be in the position A B″, it will receive upon it only the rays included between the lines A X and B″ Y″. Again, if it be in the position A B‴, it will receive upon it only the rays which are included between the lines A X and B‴ Y‴.

Thus it is quite apparent that as the obliquity of the surface upon which the rays fall to the direction of the rays is increased, the number of rays incident upon such surface will be diminished, and that this diminution will be in the proportion of the distances B′ Z′, B″ Z″, B‴ Z‴, &c.

These lines are called in geometry the *sines* of the angles formed by the surfaces B′ A, B″ A, &c., with the direction of the rays.

It follows, therefore, that the intensity of the illumination produced upon a given surface by a given pencil of rays will diminish in the same proportion as the sine of the angle of obliquity of such surface to the direction of the rays is diminished.

It follows, therefore, evidently from this that the illumination is greatest when the surface is at right angles to the rays, and gradually diminishes until the surface is in the direction of the rays, when it ceases altogether to be illuminated.

910. *Methods of comparing the illuminating power of lights.*— If two luminaries, having equal luminous surfaces at equal distances from the same white opaque surface, placed at the same angle with the rays, shed lights of equal brightness on such surface, it follows that their absolute intensities must be equal.

In that case, the distances and the luminous surfaces being respectively equal, there is no other condition which can affect the illumination, except the intensity of the light proceeding from each luminous point; and since, therefore, the illuminations are equal, these intensities must be equal.

If, on the contrary, two such luminaries so placed produce different degrees of illumination on the same surface, their absolute intensities must be different, and must be in the proportion of the illuminations they produce. If in this case that luminary which produces the more feeble illumination be moved towards the illuminated object, until its proximity is increased, so that it produces an illumination equal to that of the other luminary, then the absolute intensity of the two luminaries will be as the squares of their distances. This may be demonstrated as follows:—

Let B express the brilliancy of the illumination produced by the two luminaries. Let S express the common magnitude of their luminous surfaces. Let I and I′ express their intensities, and let D and D′ express those distances which render their illuminations equal; we shall then have for the one

$$B=\frac{I\times S}{D^2},$$

and for the other,

$$B=\frac{I'\times S}{D'^2};$$

consequently, we shall have

$$\frac{I}{D^2}=\frac{I'}{D'^2};$$

and consequently,

$$I : I' :: D^2 : D'^2.$$

911. *Photometry.*—The art of measuring the intensity of light by observation is called *photometry*, and the instruments or expedients serving this purpose are called *photometers.*

The most simple form of photometer is that which may be called the method of shadows, and which is founded upon the principle which has just been demonstrated,—that with equal illumination the intensity of the light is directly as the square of the distance of the luminary.

912. *Photometer by shadows.*—This photometric apparatus, the invention of which is due to Count Rumford, consists of a white screen fixed in a vertical position, having a small opaque rod placed at a short distance from it, also in a vertical position. The screen, rod, and the two lights whose powers are to be compared, are so placed relatively to each other, that the two shadows of the rod formed by the two luminaries on the screen shall just touch without overlaying each other. Under these circumstances, it is evident that the space on the screen occupied by the shadow proceeding from each luminary will be illuminated by the other luminary. Thus, two spaces on the screen are exhibited in juxtaposition, each of which is illuminated by one of the luminaries independent of the other. It will at first be found that these two spaces will be unequally bright. The position of the luminaries, or of the screen or rod, must then, one or all, be changed until the two shadows, being still kept in juxtaposition, appear to be equally bright, so as to present a uniform shadow. Let the distance of the two luminaries from the shadows be then measured, and it will follow, according to the principle that has been already established, that the intensities of the two luminaries will be as the squares of these distances.

If in this case the two luminaries have equal luminous surfaces, their absolute intensities will be in the ratio of the squares of their distances; but if either luminous surface be unequal, the squares of the distances will represent the proportion, not of their absolute intensities, but of the products of their absolute intensity multiplied by their luminous surface.

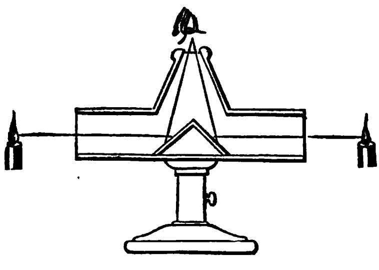

Fig. 271.

913. *Ritchie's photometer.*—Another photometer, on a simple and beautiful principle, proposed by the late Professor Ritchie, and represented in *fig.* 271., consists of a rectangular box about an inch and a half or two inches wide, and eight inches long, open at both ends, and blackened in the middle. In the centre of its length are two

surfaces placed at right angles with each other, and at an angle of 45° with the bottom of the box. Upon these surfaces, white paper is pasted. A round hole is made in the top of the box immediately over the line formed by the edges of the paper, so that an eye looking in at this hole may see equally the two surfaces of paper. To compare two lights, the instrument is placed in such a manner before them that each may illuminate one of the pieces of paper. The distance of the lights from the surfaces of the paper are then to be so adjusted by successive trials that the two surfaces of paper shall appear to the eye of uniform brightness. In that case, the illumination of the surfaces being the same, the illuminating powers of the luminaries will be in the same proportion as the squares of their distances from the paper, the principle of this being the same as that of the photometer of Count Rumford.

In this and all similar experiments, the colour of the light exercises a material influence on the results; and the comparative brilliancy cannot be ascertained with any precision, unless the two luminaries give light of nearly the same colour.

914. *Method of comparing the absolute intensity of light.*—When it is desired to ascertain the absolute intensities of the lights, it is, as has been stated, necessary to expose equal illuminating surfaces to the photometric apparatus; but as it is not always easy to produce luminaries having surfaces exactly equal, this object may be attained by the following expedient:—Let two opaque screens, having holes in them of exactly equal magnitude, be placed near and exactly opposite to the middle of each luminous surface. The rays of light which pass through the two apertures will in such case proceed from equal portions of the surfaces of the two luminaries, and the result of the experiment will therefore show the absolute intensities.

915. *Intensity of solar light.*—The sun produces the most intense illumination with which we are acquainted. This arises partly from the absolute intensity of that luminary, and partly from the vast extent of his luminous surface. The diameter of the sun is very near a million of miles, and consequently, being a sphere, the superficial extent of his surface is about three millions of square miles; but as one-half the surface only is presented to us at any one time, the magnitude of it will be a million and a half of square miles.

916. *Electric light.*—The most brilliant artificial light yet produced is inferior to the splendour of solar light in an incredible proportion. The brightest artificial lights are those produced by the contact of charcoal points, through which a galvanic current passes, and by lime submitted to the heating power of the oxyhydrogen blow-pipe. These lights, however, when projected on the disk of the sun, appear nevertheless as a blank spot.

CHAP. III.

REFLECTION OF LIGHT.

917. *Reflection varies according to the quality of the surface.*— When rays of light encounter the surface of an opaque body, they are arrested in their progress, such surfaces not being penetrable by them. A certain part of them, more or less according to the quality of the surface and the nature of the body, is absorbed, and the remaining part is driven back into the medium from which the rays proceed. This recoil of the rays from the surface on which they strike is called *reflection*, and the light thus returning into the same medium from which it had arrived, is said to be *reflected*.

The manner in which the light is reflected from such a surface varies according as the surface is polished or unpolished, and according to the degree to which it is polished.

We shall consider three cases: 1st, that of a surface absolutely unpolished; 2dly, that of a surface perfectly polished; and 3dly, that of a surface imperfectly polished.

CHAP. IV.

REFLECTION FROM UNPOLISHED SURFACES.

If light fall upon a uniformly rough surface of an opaque body, each point of such surface becomes the focus of a pencil of reflected light, the rays of such pencil diverging equally in all directions from such focus.

The pencils which thus radiate from the various points are those which render the surface visible. If the light were not thus reflected indifferently in all directions from each point of the surface, the surface would not be visible, as it is, from whatever point it may be viewed.

The light which is thus reflected from the various points upon the surface of any opaque body, has the colour which is commonly imputed to the body. The conditions, however, which determine the colour of bodies will be fully explained hereafter; for the present, it will be sufficient to establish the fact, that each point of the surface of an opaque body which is illuminated is an independent focus from which light radiates, having the colour proper to such point, by which light each such point is renuered visible.

918 *Irregular reflection.*—This mode of reflection, by which the forms and qualities of all external objects are rendered manifest to sight, has been generally denominated, though not as it should seem with strict propriety, the irregular reflection of light.

There is, nevertheless, nothing irregular in the character of the phenomena. The direction of the reflected rays is independent of each of the incident rays; but, nevertheless, such direction obeys the common law of radiation.

The existence of these radiant pencils proceeding from the surface of any illuminated object, and their independent propagation through the surrounding space, may be rendered still more manifest by the following experiment.

Let A B, *fig.* 272., be an illuminated object, placed before the window-shutter of a darkened room. Let *c* be a small hole made in the window-shutter, opposite the centre of the object. If a screen be held parallel to the window-shutter, and the object at some distance from the hole, an inverted picture of the object will be seen upon it, in which the form and colour of the object will be preserved; the magnitude, however, of such picture will vary according to the distance of the screen from the aperture. The less such distance, the less will be the magnitude of the picture.

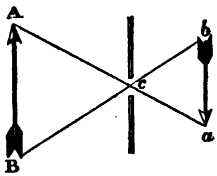

Fig. 272.

This effect is easily explained. According to what has been already stated, each point of the surface of the illuminated object A B is a focus of a pencil of rays of light having the colour peculiar to such point. Thus, each portion of the pencil of rays which radiates from the point B, and has for its base the area of the aperture *c*, will pass through the aperture, and will continue its rectilinear course until it arrives at the point *b* upon the screen, where it will produce an illuminated point corresponding in colour to the point B.

In the same manner, the pencil diverging from A, and passing through the aperture *c*, will produce an illuminated point on the screen at *a*, corresponding to the point A.

Each intermediate point of the object will produce a corresponding illuminated point on the screen. It is evident, therefore, that a series of illuminated points corresponding in arrangement and colour to those of the object will be formed upon the screen between *a* and *b*; their position, however, being inverted, the points which are highest in the object being lowest in the picture.

919. *Picture formed on wall by light admitted through small aperture.*—These effects may be witnessed in an interesting manner in any room which is exposed to a public thoroughfare frequented by moving objects. Let the window-shutters be closed and the interstices stopped so as to exclude all light except that which enters

through a small hole made for the purpose, and if no hole be found in the shutters sufficiently small, a piece of paper or card may be pasted over any convenient aperture, and a hole of the required magnitude pierced in it. Coloured inverted images of all the objects passing before the window will thus be depicted on a screen conveniently placed. They will be exhibited on the opposite wall of the room; but unless the wall be white, the colours will not be distinctly perceptible. The smaller the hole admitting the light is, the more distinct but the less bright the pictures will be. As the hole is enlarged the brightness increases, but the distinctness diminishes. The want of distinctness arises from the spots of light on the screen, produced by each point of the object overlaying each other, so as to produce a confused effect.

920. *Different reflecting powers of surfaces.*—Surfaces differ from each other in the proportion of light which they reflect and absorb. In general, the lighter the colour, other things being the same, the more light will be reflected and the less absorbed, and the darker the colour the less will be reflected and the more absorbed; but even the most intense black reflects some light. A surface of black velvet, or one blackened with lamp-black, are among the darkest brown, yet each of these reflects a certain quantity of rays. That they do so we perceive by the fact that they are visible. The eye recognizes such surfaces as differing from a dark aperture not occupied by any material surface, and it can only thus recognize the appearance of the material surface by the light which it reflects. The following experiment, however, will render this more evident.

921. *The deepest black reflects some light.*—Blacken the inside of a tube, and fasten upon the extremity remote from the eye a plate of glass. To the centre of this plate of glass attach a circular opaque disk, somewhat less in diameter than the tube, so that in looking through the tube a transparent ring will be visible, as represented in *fig.* 273. In the centre of this transparent ring will appear an intensely dark circular space, being that occupied by the disk attached to the glass.

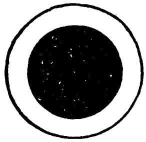

Fig. 273.

Now, let a piece of black velvet be held opposite the end of the tube, so as to be visible through the transparent ring. If the velvet reflected no light, then the transparent ring would become as dark as the disk in the centre; but that will not be the case. The velvet will appear by contrast with the disk, not black, but of a greyish colour, proving that a certain portion of light is reflected, which in this case is rendered perceptible by the removal of the brighter objects from the eye.

922. *Irregular reflection necessary to vision.*—Irregular reflection, as it has been so improperly called, is one of the properties of light which is most essential to the efficiency of vision.

Without irregular reflection, light must be either absorbed by the surfaces on which it falls, or it must be regularly reflected. If the light which proceeds from luminous objects, natural or artificial, were absorbed by the surfaces of objects not luminous, then the only visible objects in the universe would be the sun, the stars, and artificial lights such as flames.

These luminaries would, however, render nothing visible but themselves.

If the light radiating from luminous objects were only reflected regularly from the surface of non-luminous objects, these latter would still be invisible. They would have the effect of so many mirrors, in which the images of the luminous objects only could be seen. Thus, in the day-time, the image of the sun would be reflected from the surface of all objects around us, as if they were composed of looking-glass, but the objects themselves would be invisible. The moon would be as though it were a spherical mirror, in which the image of the sun only would be seen. A room in which artificial lights were placed would reflect these lights from the walls and other objects around as if they were specula, and all that would be visible would be the multiplied reflections of the artificial lights.

Irregular reflection, then, alone renders the forms and qualities of objects visible. It is not, however, merely by the first irregular reflection of light proceeding from luminaries by which this is effected. Objects illuminated and reflecting irregularly the light from their surfaces, become themselves, so to speak, secondary luminaries, by which other objects not within the direct influence of any luminary are enlightened, and these in their turn reflecting light irregularly from their surfaces, illuminate others, which again perform the same part to another series of objects. Thus light is reverberated from object to object through an infinite series of reflections, so as to render innumerable objects visible which are altogether removed from the direct influence of any natural or artificial source of light.

923. *Use of the atmosphere in diffusing light.*—The globe of the earth is surrounded with a mass of atmosphere extending forty or fifty miles above the surface.

The mass of air which thus envelopes the hemisphere of the earth presented towards the sun, is strongly illuminated by the solar light, and, like all other bodies, reflects irregularly this light. Each particle of air thus becomes a luminous centre, from which light radiates in every direction. In this manner, the atmosphere diffuses in all directions the light of the sun by irregular reflection. Were it not for this, the sun's light could only penetrate those spaces which are directly accessible to his rays. Thus, the sun shining upon the window of an apartment would illuminate just so much of that apartment as would be exposed to his direct rays, the remainder remaining in darkness. But we find, on the contrary, that although that part of

the room upon which the sun directly shines is more brilliantly illuminated than the surrounding parts, these latter are nevertheless strongly illuminated. All this light proceeds from the irregular reflection of the mass of atmosphere just mentioned.

924. *Diffusion of solar light by all opaque objects.* — But the solar light is further diffused by being again irregularly reflected from the surface of all the natural objects upon which it falls. The light thus irregularly reflected from the air falling upon all natural objects, is again reciprocally reflected from one to another of these through an indefinite series of multiplied reflections, so as to produce that diffused and general illumination which is necessary for the purposes of vision.

Light and shade are relative terms, signifying only different degrees of illumination. There is no shade so dark into which some light does not penetrate.

It is the same with artificial lights. A lamp placed in a room illuminates directly all those objects accessible to its rays. These objects reflect irregularly the light incident upon them, and illuminate thus more faintly others which are removed from the direct influence of the lamp, and thus, these again reflecting the light, illuminate a third series still more faintly; and so on.

925. *Effect of the irregular reflection of lamp-shades.* — When it is desired to diffuse uniformly by reflection the light which radiates from a luminary, the object is often more effectually attained by means of an unpolished opaque reflector than by a polished one. White paper or card answers this purpose very effectually. Shades formed into conical surfaces placed over lamps are thus found to diffuse by reflection the light in particular directions, as in the case of billiard-tables or dinner-tables, where a uniformly diffused light is required. A polished reflector, in a like case, is found to diffuse light much more unequally.

In case of white paper or card, each point becomes a centre of radiation, and a general and uniform illumination is the consequence. The light obtained by reflection in such cases is always augmented by rendering the reflector perfectly opaque; for if it be in any degree transparent, as is sometimes the case with paper shades put over lamps, the light which passes through them is necessarily subtracted from that which is reflected.

CHAP. V.

REFLECTION FROM PERFECTLY POLISHED SURFACES.

926. *Regular reflection.* — By what has been just explained, it appears that light reflected from rough and unpolished surfaces radi

ates from all the parts composing them, as from so many foci of divergent pencils. If, however, the surface were absolutely smooth and perfectly polished, then totally different phenomena would ensue, which have been denominated *regular reflection.*

927. ***Mirrors and specula.***—Surfaces which possess this reflecting power in the highest degree are called *mirrors* or *specula.*

The most perfect specula are those composed of the metals, the best being produced by various alloys of copper, silver, and zinc. If a glass plate be blackened on one side, the surface of the other will form for certain purposes a good reflector.

928. *Law of regular reflection.*—To explain the law of regular reflection, let C, *fig.* 274., be a point upon a reflecting surface A B, upon which a ray of light D C is incident. Draw the line C E perpendicular to the reflecting surface at C; the angle formed by this perpendicular and the incident ray D C, is called the *angle of incidence.*

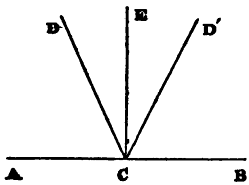

Fig. 274.

From the point C, draw a line C D′ in the plane of the angle of incidence D C E, and forming with the perpendicular C E an angle D′ C E equal to the angle of incidence, but lying on the other side of the perpendicular. This line C D′ will be the direction in which the ray will be reflected from the point C. The angle D′ C E is called the *angle of reflection.*

The plane of the angles of incidence and reflection which passes through the two rays C D and C D′, and through the perpendicular C E, and which is therefore at right angles to the reflecting surface, is called the *plane of reflection.*

This law of regular reflection from perfectly polished surfaces, which is of great importance in the theory of light and vision, is expressed as follows:—

When light is reflected from a perfectly polished surface, the angle of incidence is equal to the angle of reflection, in the same plane with it, and on the opposite side of the perpendicular to the reflecting surface.

From this law it follows, that if a ray of light fall perpendicularly on a reflecting surface, it will be reflected back perpendicularly, and will return upon its path; for in this case, the angle of incidence and the angle of reflection being both nothing, the reflected and incident rays must both coincide with the perpendicular. If the point C be upon a concave or convex surface, the same conditions will prevail; the line C E which is perpendicular to the surface, being then what is called in geometry, the normal.

929. *Experimental verification of this law.*—This law of reflection may be experimentally verified as follows:—

Let *c d c'*, *fig.* 275., be a graduated semicircle, placed with its diameter *c c'* horizontal. Let a plumb-line *b d* be suspended from its centre *b*, and let the graduated arc be so adjusted that the plumb-line shall intersect it at the zero point of the division, the divisions being numbered from that point in each direction towards *c* and *c'*. Let a small reflector (a piece of looking-glass will answer the purpose) be placed upon the horizontal diameter at the centre with its reflecting surface downwards, and let any convenient and well-defined object be placed upon the graduated arc at any point, such as *a*, between *d* and *c*. Now, if the point *a'* be taken upon the arc *d c'* at a distance from *d* equal to *d a*, the eye placed at *a'* and directed to *b* will perceive the object *a* as if it were placed in the direction *a' b*. It follows, therefore, that the light issuing from the point of the object *a* in the direction *a b*, is reflected to the eye in the direction *b a'*. In this case, the angle *a b d* is the angle of incidence, and the angle *d b a'* is the angle of reflection; and, whatever position may be given to the object *a*, it will be found that in order to see it in the reflector *b*, the eye must be placed upon the arc *d c'* at a distance from *d* equal to the distance at which the object is placed from *d* upon the arc *d c*.

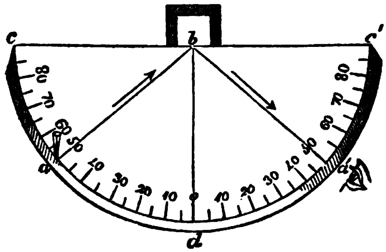

Fig. 275.

The same principle may also be experimentally illustrated as follows:—

If a ray of sun-light admitted into a dark room through a small hole in a window-shutter strike upon the surface of a mirror, it will be reflected from it, and both the incident and reflected rays will be rendered visible by the particles of dust floating in the room. By comparing the direction of these two visible rays with the direction of the plane of the mirror and the position of the point of incidence, it will be found that the law of reflection which has been announced is verified.

930. *Plane reflectors—parallel rays.*—If parallel rays be incident upon a polished plane reflecting surface, they will be reflected parallel; for since they are parallel, they will make equal angles with the perpendiculars to the surface at their points of incidence, and the planes of these angles will also be parallel.

The reflected rays will, therefore, also make equal angles with the perpendiculars, and the planes of reflection will be parallel; consequently the reflected rays will be parallel.

This may also be experimentally verified by admitting rays of solar

light into a dark room through two small apertures. Such rays will always be parallel; and if they are received upon a plane mirror, their reflections will be found to be parallel, the rays and the reflections being rendered visible, as already explained.

931. *Divergent rays.*—If a pencil of divergent rays fall upon a plane mirror, the reflected rays will also be divergent, and their focus will be a point behind the mirror similarly placed, and at the same distance as the focus of incident rays is before it.

To demonstrate this, let A B, *fig.* 276., be the reflecting surface. Let F be the focus of the incident pencil from which the rays F A, F B, F C, &c. diverge, and let F A be perpendicular to the reflecting surface A B. If we take A F′ on the continuation of F A equal to A F, and draw the lines F′ B B′ and F′ C C′, then it can easily be perceived that the lines B B′ and C C′ make angles with the reflecting surface, and therefore with the perpendicular to it, equal to the angles which the incident rays F B and F C make with it respectively; for since A F is equal to A F′, F B will be equal to F′ B, and F C will be equal to F′ C; consequently the angles B F A and B F′ A will be equal, as will also the angles C F A and C F′ A. But the angles B F A and C F A are the angles of incidence of the two rays F B and F C; and since the angles B F′ A and C F′ A are respectively equal to them, and lie on opposite sides of the perpendicular, they will be the angles of reflection; consequently, the ray F B will be reflected in the direction B B′, and the ray F C in the direction C C′. These two rays, therefore, will be reflected from the points B and C as if they had originally radiated from F′ as a focus; and in the same way it may be shown that the other rays of a pencil diverging from F will be reflected from the mirror as if they had diverged from F′. But F′ is the point on the other side of the mirror which is placed similarly and at the same distance from the mirror as the point F is in front of it.

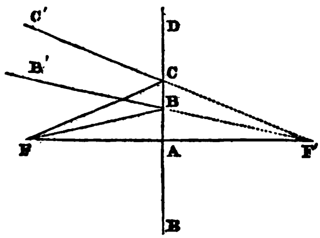

Fig. 276.

932. *Image of an object formed by a plane reflector.*—It follows from what has been just explained, that an object placed before a plane reflector will have an image at the same distance behind the reflector as the object is before it, for the rays which diverge from each point of the object will after reflection, according to what has been shown, diverge from a point holding a corresponding position behind the reflector, and if received after reflection by the eye of an observer will produce the same effect as if they had actually diverged from such point. All the rays, therefore, proceeding from the object, will after reflection follow those directions which they would follow had

they proceeded from a series of points, on the surface of a similar object placed behind the reflector at the same distance as the object itself is before it, and consequently they will produce the same effect on the organs of vision as would be produced by a similar object placed as far behind the mirror as the object itself is before it.

The position of the different parts of the image formed in a plane reflector will be exactly determined by supposing perpendiculars drawn from every point on the object to the reflector, and these perpendiculars to be continued beyond the reflector to distances equal to those of the points from which they are drawn before it. The extremities of the perpendiculars so continued will then determine the corresponding points of the image.

It follows from this, that the images of objects in a plane reflector appear *erect*, that is to say, the top of the image corresponds with the top of the object, and the bottom of the image with the bottom of the object. But considered laterally with regard to the object itself, they will be *inverted*, that is to say, the left will become the right, and the right the left. This will be easily understood by considering that if a person stand with his face to a plane reflector, in a vertical position, his image will be presented with the face towards him, and the image of his right hand will be on the right side of his image as he views it, but will be on the left side of the image itself, and the same will apply to every other part of the image in reference to the object. There is, therefore, *lateral inversion*.

This effect is rendered strikingly manifest by holding before a reflector a printed book. On the image of the book all the letters will be reversed.

It follows also, from what has been explained, that if an object be not parallel to a reflector, but forms an angle with it, the image will form a like angle with it, and will form double that angle with the direction of the object.

Let A B, *fig.* 277., be a plane reflector, before which an object C D is placed. From C draw the perpendicular C O, and continue it from O to C′, so that O C′ shall be equal to O C. In like manner, draw the perpendicular D P, and continue it so that P D′ shall be equal to P D. Then the image of C will be at C′, and the image of D at D′, and the image of all the intervening points between C and D will be at points intermediate between C′ and D′, so that C′ D′ shall be inclined to the reflector at the same angle as C D is inclined to it, and the object and the image will be inclined to each other at twice the angle at which either is inclined to the reflector.

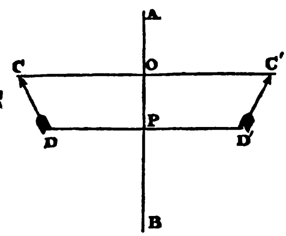

Fig. 277.

Hence, if an object in a horizontal position be reflected by a reflector forming an angle of 45° with the horizon, its image will be in a vertical position; and if the object being in a vertical position be reflected by such a mirror, its image will be in a horizontal position.

If a reflector be placed at an angle of 45° with a wall, the image of the wall will be at right angles with the wall itself.

If a reflector be horizontal, the image of any vertical object seen in it will be inverted. Examples of this are rendered familiar by the effect of the calm surface of water. The country on the bank of a calm river or lake is seen inverted on its surface.

933. *Series of images formed by two plane reflectors.*—If an object be placed between two parallel plane reflectors, a series of images will be produced lying on the straight line drawn through the object perpendicular to the reflector. This effect is seen in rooms where mirrors are placed on opposite and parallel walls, with a lustre or other object suspended between them. An interminable range of lustres is seen in each mirror, which lose themselves in the distance and by reason of their faintness. This increased faintness by multiplied reflection arises from the loss of light caused in each successive reflection, and also from the increased apparent distance of the image.

Let A B and C D, *fig.* 278., be two parallel reflectors; let O be an

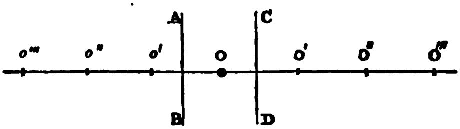

Fig. 278.

object placed midway between them. An image of O will be formed at O′ as far behind C D as O is before it, and another image will be formed at o' as far behind A B as O is before it. The image O′ becoming an object to the mirror A B will form in it another image o'' as far behind A B as O′ is before it, and in like manner the image o' becoming an object to the mirror C D will form an image O″ as far behind C D as o' is before it. The images O″ and o'' will again become objects to the mirrors A B and C D respectively; and two other images will be formed at equal distances beyond these latter. In the same way we shall have, by each pair of images becoming objects to the respective mirrors, an indefinite series of equidistant images.

The distance between each successive pair of images will be equal to the distance of the object O from either of the images O′ or o', and consequently to the distance between the mirrors.

934. *Images repeated by inclined reflectors.*—A variety of interesting optical phenomena are produced by the multiplied reflection

of plane mirrors inclined to each other at different angles. As all these phenomena may be explained upon the same principle, it will suffice here to give a single example.

Let A B, A C, *fig.* 279., be two reflectors, inclined to each other at a right angle, and let O be an object placed at a point between them, equally distant from each. From O draw O M and O N perpendicular to A C and A B, and produce O M to O′ so that M O′ will be equal to M O; and produce O N to O″, so that N O″ shall be equal to N O. Two images of the point O will be formed at O′ and O″. The image O′ becoming an object to the mirror A B will have an image at O‴ just as far behind A B as O′ is before it; and, in like manner, the image O″ becoming an object to the reflector A C will have an image just as far behind A C as O″ is before it; but, in the present case, this latter image of O″ in the reflector A C will coincide with the image of O′ in the reflector A B, and will appear at O‴. Thus, the mirrors will present three images of the object O, which are placed at the angles of a square, of which the point A is the centre.

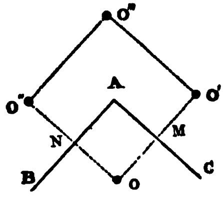

Fig. 279.

In the same manner, if the reflectors A B and A C be placed at an angle which is the eighth part of 360°, there will be formed seven images of the point O, which, with the point O, will be placed at the eight angles of a regular octagon of which the point A where the mirrors meet, will be the centre; and like results will be found by giving the mirrors other inclinations.

935. *The kaleidoscope.*—The optical effects of the kaleidoscope depend upon this principle. Two plates of common looking-glass are fixed in a tube forming an angle of 45°, or some other aliquot part of 360°, with each other; semi-transparent objects of various colours are loosely thrown between them, and shut in by means of plates of glass at the ends, one of which is ground, so as to be semi-transparent. The images of the coloured fragments between the mirrors are multiplied so as to form a polygon as just described, and thus a regularity is given to their arrangement, however irregular their disposition may be between the mirrors. The effect of this instrument may be varied by a provision for varying the inclination of the mirrors.

936. *Optical toy by multiplied reflection.*— An amusing optical toy is represented in *fig.* 280., by means of which objects may be seen, notwithstanding the interposition of any opaque screen between them and the eye. The rays proceeding from the object P entering the tube *d* strike on the mirror *l* placed at an angle of 45°, and are reflected downwards vertically to the mirror *h*, also placed at 45°, from which they are reflected horizontally to the mirror *g* placed at 45°, from which they are again reflected vertically to the mirror *k*

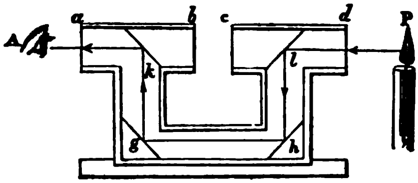

Fig. 280.

placed at 45°, from which they are reflected horizontally to it at A. The eye thus sees the object after four reflections, the rays which render it visible having travelled round the rectangular tube *l h g k a*.

937. *Formation of images by reflecting surfaces in general.* — In order that a reflector should produce a distinct image of an object placed before it, it is necessary that the rays diverging from each point of the object should, after reflection, diverge from, or converge to, some common point.

Thus, the surface of the object may be considered as an assemblage of foci of an infinite number of pencils of incident rays. Each of these pencils will, by reflection, be converted into other pencils having other foci, the assemblage of which will determine the form and magnitude of the image of the object produced by the reflector. In the case of a plane reflector, it has been shown that the assemblage of these foci corresponds in form and magnitude to the object, and therefore, in this case, the image is equal, and in all respects similar to the object; but this does not always happen.

938. *Magnified, diminished, or distorted images.* — The pencils of incident rays may be converted by reflection into pencils of reflected rays having different foci, but the assemblage of these foci may not correspond with the points forming the surface of the object. They may be similar to it in form, but greater in magnitude, in which case the reflector is said to magnify the object; or they may be similar to it in form and less in magnitude, in which case the reflector is said to diminish the object. In fine, they may assume such a form as to present the object in altered proportions. Thus, while the proportion of the vertical dimensions is preserved, that of the horizontal dimensions may be increased or diminished, or *vice versâ;* or either of these dimensions may be generally increased at various points of the image. In such case, the reflector is said to present a distorted image.

939. *Cases in which no image is formed.* — Since to produce a distinct image of any point in an object, it is necessary that the rays diverging from that point should be reflected, so as to diverge from some other point, if after reflection they have no common point of in-

tersection, the point of the object from which they originally diverged can have no distinct image.

In this case the effect of the reflection will be to produce upon the vision a confused impression of the colour of the object, without any distinct form.

940. *Conditions under which the reflected rays shall have a common focus.* — In order, therefore, that a polished surface should reflect the rays which diverging from any point are incident upon it exactly to or from another point, it is necessary that the surface should be of such a nature that lines drawn from the two points in question to any one point on the surface shall make equal angles with the surface. No surface possesses this property except one whose section made by a plane passing through the two points is an ellipse, the two points being its foci. It follows, therefore, that if a pencil of light have its focus at one of the foci of an ellipse, the rays which diverging from such focus strike upon the ellipse, or upon any surface with which the ellipse would coincide, will be reflected to the other focus.

941. *Elliptic reflector.* — To render this more clear, let A C B D, *fig.* 281., be an ellipse whose foci are F and F′. Then, according to what has been explained, if two lines be drawn from F and F′ to any one point, such as P, in the ellipse, they will make equal angles with the ellipse; and, consequently, if F P be a ray of light forming part of a pencil of rays whose focus is F, it will be reflected along the line P F′ to the other focus.

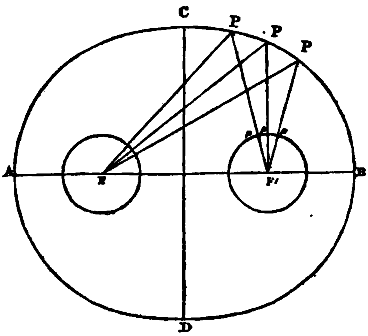

Fig. 281.

Now if we suppose a reflecting surface so formed that the ellipse by turning round the line A B as an axis will everywhere coincide with it, this surface is called an *ellipsoid;* and if it were a polished and reflecting surface, it would be called an *elliptic reflector.*

It is evident that it is not necessary that such a surface should form a complete ellipsoid. Any portion of it upon which a pencil of rays passing from one of the foci would fall, would reflect such pencil so as to make it converge to the other focus. In this case the pencil proceeding from the focus in which the luminous point is placed, would be a diverging pencil, and that which is reflected to the other focus would be a converging pencil.

942. *Parabolic reflectors.* — It has been shown (807.) that a parabola has a property in virtue of which a line drawn from any point in it, such as P, *fig.* 282., to a point F called its focus, and another,

P M, parallel to its axis, make equal angles with the curve. It follows from this, that if the parabola possessed the power of reflecting light, rays diverging from its focus F would be reflected parallel to its axis V M; and, on the other hand, if rays directed along lines parallel to its axis were incident on the parabola, they would be reflected in the form of a pencil converging to its focus.

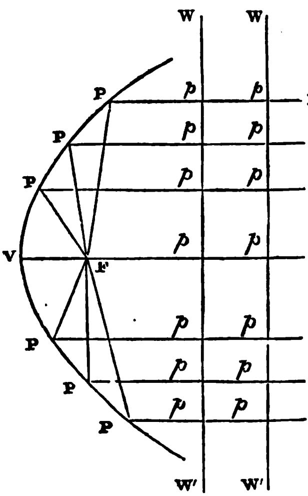

Fig. 282.

If we suppose the parabola to revolve round its axis V M, a surface with which it would everywhere coincide as it revolves is called a *paraboloid*; and if such a surface were polished so as to reflect light regularly, it would form a parabolic reflector. It follows, therefore, that if a luminous point be placed in the focus of such a reflector, its rays after reflection will be parallel to the axis; and, on the other hand, if rays strike upon the reflector in directions parallel to its axis, they will be reflected to its focus.

943. *Experimental verification of these properties in the case of an elliptic reflector.* — These remarkable properties of elliptic and parabolic reflectors may be easily verified by experiment. Let A B C, *fig.* 283., be the section of an elliptic reflector made by a plane passing through its focus F, the other focus being at F′. Let a luminous point, such as a small flame, or still better the light produced by two charcoal points when a galvanic current passes through them, be placed at the focus F.

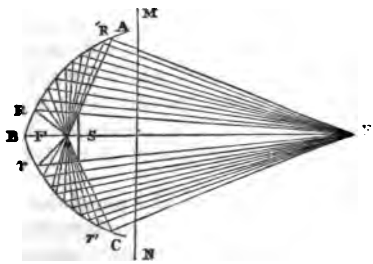

Fig. 283.

Let straight lines be imagined to be drawn from F′ through the extremities of a circular screen S, meeting the reflector at R and *r*, and from the luminous point F draw the lines F R and F *r*. It is clear from what has been stated that a ray of light passing from F to R will be reflected from R to F′; and one passing from F to *r* will be reflected from *r* to F′, both grazing the edge of the screen S; and the same will be true for all rays passing from F which are incident upon

a circle traced on the reflector whose diameter would be a line joining R and r.

The rays proceeding from F, and incident between the points R and r, will after reflection strike upon the screen S, and will thus be prevented from proceeding towards the point F′.

From the point F draw the lines F R′ and F r′ passing the extremities of the screen S. It is clear that the rays passing from F between the lines F R′ and F r′ will be intercepted by the screen.

Thus it follows that all the rays which strike upon the reflector, and which are not intercepted by the screen S, are included on the one side by the lines F R and F R′, and on the other by the lines F r and F r′.

Now, according to what has been explained, all the rays incident upon the surface of the reflector would, after reflection, converge to the point F′, as represented in the figure. To verify this fact, let a white screen M N be placed between F′ and S, at right angles to the line F′ S. The reflected light will appear upon this screen when held near to S, as an illuminated disk with a small circular dark spot in its centre, this dark spot corresponding to the space from which the light both direct and reflected is excluded by the small screen S. If the screen M N be now gradually moved towards F′, being kept perpendicular to the line S F′, the illuminated disk will gradually diminish in diameter, as will also the dark circular spot in its centre, and this diminution will continue until the screen arrives at the point F′, when the illuminated disk will be reduced to a small light spot, and the dark spot in its centre will disappear.

This experiment may be further varied by placing the screen M N as near the reflector as possible, and piercing several holes in it within the area of the illuminated disk. The rays of light passing through these holes will severally converge to the point F′, as may be shown by holding another screen beyond M N, by means of which the course of the rays may be traced, since their light will produce light spots upon this screen. As it is moved towards F′, these light spots will gradually approach each other, and when it arrives at F′ they will coalesce and form a single spot.

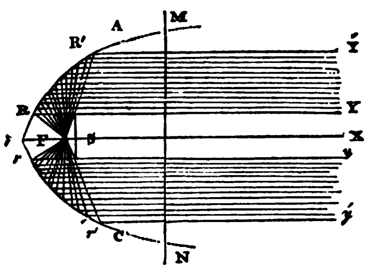

Fig. 284.

944. *Case of a parabolic reflector.* — The reflecting property of a parabolic reflector may be experimentally exhibited by a like expedient. Let A B C, *fig.* 284., represent a section of the reflector, the focus being F. Let a luminous point be placed at F, and a small circular screen S, as before, be placed perpendicular to the axis, and near the point F. It may be shown, as in the case of the elliptic re-

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flector, that the rays F R′ and F r′, which graze the screen, will be reflected in the direction R′ Y′ and r′ y′, parallel to the axis B X; and, in like manner, that the rays F R and F r, which, after reflection, graze the screen, will also be reflected in the direction R Y and r y parallel to the axis.

Hence it follows that the reflected light will be excluded from a cylindrical space, of which the screen S is the circular base, and whose axis coincides with the axis B X of the reflector.

It also appears that no light diverging from the focus F will strike the reflector beyond the points R′ and r′. The light reflected will therefore be included between two cylindrical surfaces, having the axis of the parabola as their common axis, the sides of the exterior cylinder being R′ Y′ and r′ y′, and those of the interior cylinder being R Y and r y.

It is easy to verify these phenomena. Let a white screen M N be held as before at right angles to the axis B X, an illuminated disk will appear upon it, whose diameter will be equal to the line R′ r′, having a small dark spot in the centre, equal in magnitude to the screen S. If the screen M N be moved towards or from the screen S, this illuminated disk will continue of the same magnitude, having a dark spot in the centre constantly of the same magnitude also. Thus it appears that the reflected rays must follow the course already described.

The experiment may be further varied, as in the case of the ellipse, by piercing several holes in the screen M N, through which distinct rays shall pass. These rays being received upon another screen behind M N, will produce upon it luminous spots, and if then either screen be moved towards or from M N, these spots will maintain always the same relative position.

If, in the case of the elliptic reflector, the luminous point be placed at F′, *fig.* 283, instead of F, then the effects will take place in an inverse order, the incident rays being in this case what the reflected rays were in the former, and *vice versâ*; and the phenomena may be verified by a like expedient. If a small circular screen be held between S and B at right angles to the axis, it will be found that the rays reflected from the elliptic surface will be inclosed between two conical surfaces, one of which is bounded by F R′ and F r′, and the other by F R and F r. The light will be excluded from the cone whose base is the screen S, and whose vertex is at F; and also from the cone whose base is R r, and whose vertex is also at F.

In the same manner, all the effects will be inverted if a cylinder of rays parallel to the axis be directed upon a parabolic reflector. In this case, the reflected rays will be included between the conical surface bounded by the lines F R′ and F r′, *fig.* 284., and the conical surface bounded by the lines F R and F r.

This may be in like manner experimentally verified by means of

a white screen moved before the screen S in the vertex B of the reflector.

945. *Parabolic reflectors useful as burning reflectors.*—In consequence of this property, parabolic reflectors are well adapted for collecting the rays of the sun or moon into a focus. Owing to the enormous distance of these objects, compared with any magnitudes which can be subject to experiment, all pencils proceeding from them may be considered as parallel.

If, then, a parabolic reflector be placed so that its axis shall be directed towards the sun, the rays of the sun reflected by it will be collected in its focus; and as their heating power will then be proportionally augmented, the apparatus may be used as a burning reflector.

946. *Experiment with two parabolic reflectors.*—If two parabolic reflectors be placed at any distance asunder, their axes coinciding, the rays proceeding from a luminous point placed in the focus of one will, after two reflections, be collected into the focus of the other.

Thus, if A B and A′ B′, *fig.* 285., be the two parabolic reflectors, the light proceeding from a luminous point at F will be reflected by the surface A B in lines parallel to V V′, and striking upon the reflector A′ B′ will converge to the focus F′.

This is precisely similar to, and explicable on the same principles as the phenomena of echo explained in 879.; all that has been ex-

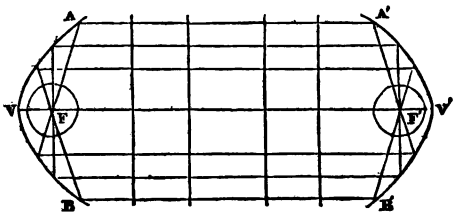

Fig. 285.

plained above in reference to elliptic reflectors is also analogous to the phenomena of echo explained in 879.

Thus the reflection of light is in all respects analogous to the reflection of sound, and subject to the same laws.

947. *Reflection by elliptic or parabolic surfaces when the luminous point is not in the focus.*—If, in the preceding experiments, the luminous point be moved from the position of the focus F, and be placed either nearer to or further from the reflector, or above or below the focus, the reflected rays will no longer converge to a common point after reflection by an elliptic surface, nor will they proceed in parallel directions after reflection by a parabolic surface. These effects may be verified experimentally by the same expedients as before.

If, when the luminous point is placed before the reflector out of the focus F, the screen M N be moved as before, the reflected rays will pro-

duce upon it as before an illuminated disk; but this disk will not be reduced to a luminous point by moving the screen from the reflector; it will diminish in magnitude to a certain limit, and then increase, but will not in any case be reduced to a point.

In the same manner with the parabolic reflector, when the light is placed out of the focus, the illuminated disk produced upon the screen will not continue to be of the same magnitude, but will either increase or diminish, according as the luminous point is placed within or beyond the focus. In the latter case, however, although the illuminated disk will diminish, it will not be reduced to a point, but after being reduced to a certain magnitude, it will again increase, and in all these cases the disk will be much more regular in its outline than in the former case.

It appears, therefore, that an elliptical reflector will only convert rays diverging from a determinate point into rays converging to another determinate point, when the former of these points is at one of the foci; and a parabolic reflector will only convert diverging rays into parallel rays when these rays diverge from the focus, and will only convert parallel rays into rays converging to a determinate point when these parallel rays are parallel to the axis.

948. *Spherical reflectors.* — The form of reflecting surface, however, which is most easy of construction, and most convenient in practice, and consequently which is most generally used, is the *spherical reflector.*

The spherical reflector is a surface which may be conceived to be formed by the arc of a circle less in magnitude than a semicircle revolving round that diameter which passes through its middle point.

Thus, let us suppose A B C, *fig.* 286., to be such an arc, B being its middle point, and O its centre. Taking the line B O X as an axis of revolution, let the arc be imagined to rotate round it.— Now let a surface be conceived, which with the arc as it revolves would be everywhere in exact contact. Such a surface is that of a spherical reflector. If the concave side of it be the polished side, it is called a *concave reflector*, the solidity and thickness being then on the convex side; but if the solidity be included within the concavity, and the convex side be polished, then the reflector is said to be *convex.*

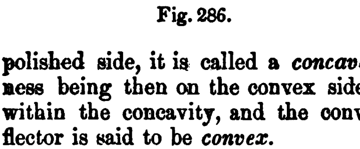

Fig. 286.

These two classes of spherical reflectors, *concave* and *convex*, have distinct properties, which will be explained in succession.

The point B, which is the middle point of the generating arc, is called the *vertex* of the reflector; and the point O, the centre of the

generating arc, is called its *centre*. The length A C of the generating arc itself, expressed in degrees, is called the *opening of the reflector*. Consequently, the angle which the axis O B makes with the radius O A drawn to the edge of the reflector is half the opening. The right line B O X, drawn through the vertex and the centre of the reflector, is called the *axis of the reflector.*

Since all radii of a circle are at right angles to the circumference at the point where they meet it, it follows also that the radii of a spherical surface are at right angles to such surface. Hence it follows, that all radii of a spherical reflector, such as O R, O R′, O R″, &c., are respectively at right angles to the surface of the reflector.

These definitions and consequences are equally applicable to concave and convex reflectors.

When a pencil of rays proceeding from any luminous point or illuminated object is incident upon a spherical reflector, that ray of the pencil which passes through the centre O of the reflector is called the *axis of the pencil.* Thus, if a pencil of rays diverging from the point I, *fig.* 287., 288., be incident upon the reflector A B C, the axis

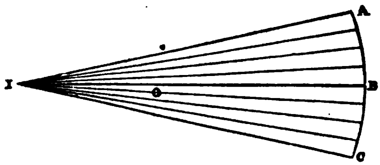

Fig. 287.

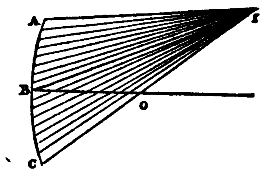

Fig. 288.

of that pencil will in such case be the line I O passing through the centre O of the reflector, and meeting the surface.

In the case represented in *fig.* 287., the axis of the pencil coincides with the axis of the reflector; but in the case represented in *fig.* 288., it is inclined to it at the angle B O C. A pencil, such as that represented in *fig.* 287., is called the *principal pencil*, and the line I B the principal axis. The pencil represented in *fig.* 288. is called the *secondary pencil*, and the axis I O a secondary axis.

It is clear, from mere inspection of the diagram, that the axis of the principal pencil is the axis of the reflector. But in the case of the secondary pencil, represented in *fig.* 288., the axis I O C of the pencil is not in the centre of the rays which strike the reflector, being more on the side B A than on the side B C.

The axis of a pencil of parallel rays is defined in the same manner; a principal pencil of parallel rays being one whose direction is parallel to, and whose axis coincides with the axis of the reflector, and a

secondary pencil of parallel rays being one whose rays and axis are inclined to the axis of the reflector.

A principal pencil of parallel rays is represented in *fig.* 289., B O X being its axis; and a secondary pencil of parallel rays is represented in *fig.* 290., X O B′ being its axis.

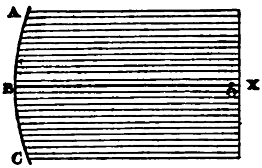

Fig. 289.

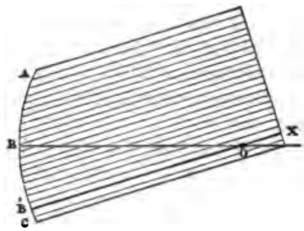

Fig. 290.

949. *Reflection of parallel rays by spherical surfaces.*—Let us first consider the case of a principal pencil of parallel rays.

Let R Y and *r y fig.* 291., be two rays of the pencil at equal dis-

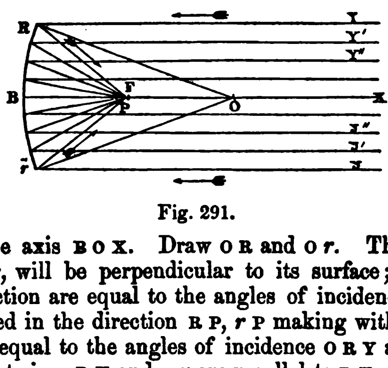

Fig. 291.

tances from the axis B O X. Draw O R and O *r*. These being radii of the reflector, will be perpendicular to its surface; and since the angles of reflection are equal to the angles of incidence, the reflected rays will proceed in the direction R P, *r* P making with the lines O R and O *r* angles equal to the angles of incidence O R Y and O *r y*. But it is evident that since R Y and *r y* are parallel to B X, the angles O R Y and O *r y* are equal to the angles R O P and *r* O P. From this it follows that P R, P O, and P *r* are equal to each other.

Since the two sides of a triangle taken together must be greater than its base, P R and P O taken together are greater than the radius O R of the reflector, and consequently O P must be greater than half of O B. If then F be the middle point of O B, the point P will be between F and B, and this will be the case at whatever point of the reflector the rays R Y and *r y* are incident.

Now, if two other parallel rays R′ Y′ and *r′ y′* be taken, in like

manner equally distant from B X, but nearer to it than R Y and *r y*, it can be shown that they will be reflected to a common point in the axis O B between P and F. In the same manner, if two other parallel rays R'' Y'' and *r'' y''*, still equally distant from the axis B X, but nearer to it than R' Y' and *r' y'*, be reflected, they will converge to a common point, still nearer to the middle point F of the axis O B, but still between F and B; in a word, the nearer such rays are to the axis B X, the nearer will be their common point of convergence after reflection to the middle point F; but however near they may be to B X, they cannot converge to any point beyond F in the direction of the centre O.

It is evident, therefore, from these results, that parallel rays incident upon a spherical surface do not after reflection converge to any common point, since each cylindrical surface formed by such rays converges to a different point upon the axis; nevertheless, it appears, that all these points of convergence are included within a small space P F upon the axis, provided that the reflector have not great extent; and it is found, that if the reflector do not extend to more than about 5° or 6° on each side of its vertex, all the parallel rays reflected from it will converge so nearly to the middle point F of the radius O B passing through its vertex, that, for practical purposes, the reflector may be considered as possessing the properties of a parabola already explained, and the reflected rays may be considered as vertically convergent to a common point. This common point will be F, the middle point of the radius O B, which forms the axis of the reflector, and which is parallel to the incident rays.

If a secondary pencil of parallel rays be incident on the reflector, as represented in *fig*. 292., the focus to which its rays will be reflected will be the middle point F of the radius O B', which forms the secondary axis.

All the reasoning which has been applied to the principal pencil, *fig*. 291., will be equally applicable in this case.

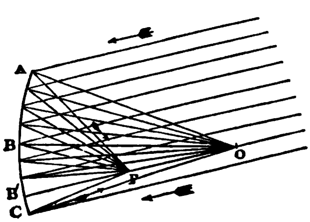

Fig. 292

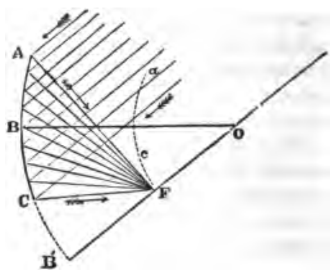

Fig. 293.

If a secondary pencil be inclined to the axis O B, at an angle greater than half the opening of the reflector, its axis will not meet the reflecting surface. This case is represented in *fig.* 293., where the line O F B′ drawn through the centre parallel to the rays of the pencil pass below the limit C of the reflector. In such a case, nevertheless, the focus of the reflected rays is determined in the same manner as it would be if the reflector extended to B′, and, accordingly, the rays reflected from A C will converge to a focus at F, the middle point of O B′.

950. *Principal focus of spherical reflector at the middle point of the radius.* — If, therefore, any number of pencils of parallel rays, principal and secondary, are incident upon the same reflector, their several foci will lie at the middle point of the radii of the reflector which coincide respectively with their several axes; and if an infinite number of such pencils fall at the same time on the reflector, their foci will form a circular arc *a c*, *fig.* 293., whose centre is the centre of the reflector O, and whose radius is O F, one-half the radius of the reflector.

951. *Experimental verification.*—All these effects may be experimentally verified by means of screens, in a manner similar in all respects to that which has been already explained in the case of a parabolic reflector. Thus it can be shown, that if the opening of a reflector be much greater than 20°, parallel rays will not be reflected converging to a common point; and, on the other hand, if a luminous point be placed at F, *fig.* 292., the reflected rays will not be parallel; but if the opening do not exceed 20° or thereabouts, parallel rays will be sensibly convergent to the point F after reflection, and rays diverging from F will be reflected in directions sensibly parallel.

The focus to which parallel rays converge after reflection is called the principal focus of the reflector.

It follows, therefore, from what has been stated, that the principal focus of a spherical reflector is the middle point of that radius which is parallel to the incident rays; and the principal foci for secondary pencils of parallel rays lie in a spherical surface *a c*, *fig.* 293., whose centre is the centre of the reflector, and whose radius is half the radius of the reflector.

952. *Aberration of sphericity.* — When the opening of a spherical reflector exceeds the limit already stated of about 20°, parallel rays falling on that part of its surface which is more than 10° from its vertex will be reflected sensibly distant from the principal focus, and consequently the entire pencil of rays whose base is the reflector will not have a common point of convergence. Those which are incident upon the reflector within a distance of 10° from its vertex will converge sensibly to the principal focus; but those beyond that limit will converge to points more or less distant from the principal focus, according as these points of incidence, more or less, exceed a distance of 10° from the vertex of the reflector.

This departure from correct convergence, produced by the too great magnitude of the reflecting surface, is called the *aberration of sphericity*, or spherical aberration.

To convey a more exact idea of the form and curvature of a spherical reflector which has the effect of effacing spherical aberration, such a reflector is represented in *fig*. 294., where A C is an arc 20° in length, representing the vertical section of the reflector, B being its vertex, O its centre, and F its principal focus. Rays falling on A C parallel to O B would be reflected sensibly to the point F; but if the reflector were greater in the opening, as, for example, if it extended to A′ and C′, being 20° on each side of the vertex B, then the parallel rays incident at its extreme points A′ and C′ would be reflected to *f*, a point between F and B. In such cases, the space *f* F would be that within which all the rays incident between A and A′, and between C and C′, would be collected. This space *f* F would then be the extremity of the aberration of sphericity due to a reflector 40° in magnitude.

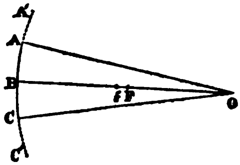

Fig. 294.

The spherical aberration of a secondary pencil will be greater than that of a principal pencil; for in the case of the secondary pencil represented in *fig*. 293, the axis of which is in the direction of O B′, the aberration will be the same as if the opening of the reflector were twice the arc A B′; and in proportion as the angle formed by the axis of the secondary pencil O B′ with the axis of the reflector O B is increased, this cause of aberration will be also increased. Thus in the secondary pencil represented in *fig*. 293, the aberration would be the same as if the opening of the reflector were twice the angle A O B′.

In fine, the aberration attending any secondary pencil will always be the same as that which would be produced with a principal pencil by a reflector whose opening would be equal to the opening of the proposed reflector, added to twice the angle formed by the axis of the reflector and the axis of the secondary pencil. Thus, in the case represented in *fig*. 293, the aberration of the secondary pencil is the same as would be produced upon a principal pencil by a reflector having an opening equal to twice A B′.

953. *Case of convex reflectors.*—In what precedes, the case of concave reflectors only has been contemplated. The same conclusions, however, will be applicable, with but little qualification, to the case of convex reflectors.

Let such a reflector be represented by A C, *fig*. 295, a pencil of rays parallel to the axis B X being incident upon it. The extreme

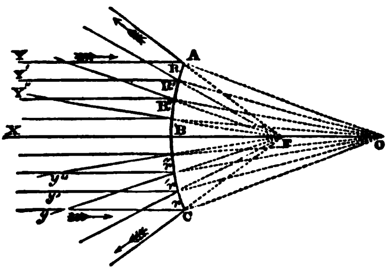

Fig. 295.

rays R Y and *r y*, equidistant from B X, will be reflected from R and *r*, as if they had diverged from F, the middle point of O B, provided R and *r* be not more distant than 10° from B. In the same manner, the rays R′ Y′ and *r′ y′*, and also the rays R″ Y″ and *r″ y″*, and, in a word, all rays between the extreme rays and the axis, will be reflected as if they had diverged from F. This point F, being the middle point of the radius O B, is therefore, as in the case of the concave reflector, the principal focus.

A difference is presented here in the two cases, which suggests a distinction to which we shall often have occasion to recur in other instances. In the case of the concave reflector represented in *fig.* 291, the principal focus is a point to which the reflected rays do actually converge, and where, as has been shown, the light is concentrated. In the case, however, of the convex reflector represented in *fig.* 295, the rays diverging from the surface diverge as if they had originally been united at F. This point F is, therefore, in such case, not a point, as in the case of a concave reflector, where the rays do actually coalesce, but a point where they would coalesce if they had been continued backwards from the points on the surface of the reflector.

954. *Foci real and imaginary.*—A focus like the former, where the rays do actually converge, is called a real focus, and sometimes a physical focus; whereas a focus like the latter, in which the rays do not actually converge, but which merely forms the point of convergence of their directions, is called an imaginary focus. In the case already explained of plane reflectors, the focus of reflection of a divergent pencil is an imaginary focus; and, on the other hand, of a convergent pencil is a real or physical focus.

955. *Images formed by concave reflectors.*—If an object be placed before a concave reflector at so great a distance from it that all pencils of rays passing from such object would be considered as parallel, an image of such object will be formed at the principal focus of the reflector; that is to say, midway between its centre and its surface.

Let A C, *fig.* 296, be such a reflector, B being its vertex, O its centre, and F the principal focus. Let L M be an object, placed at so great a distance from the reflector, that the divergence of a pencil of rays passing from any point upon it, and having the reflector as their

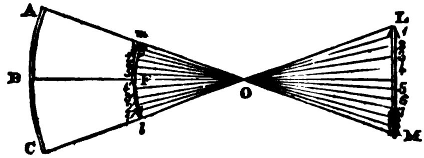

Fig. 296.

base, shall be so small that the rays may be considered as practically parallel.

Let L O *l* be the axis of the secondary pencil passing from L, and M O *m* the axis of the secondary pencil passing from M, *l* and *m* being respectively the middle points of the radii, and therefore the foci tc which the pencils proceeding from L and M respectively are collected after reflection. Images, therefore, of the points L and M respectively will be produced at *l* and *m*.

In the same manner, the pencils proceeding from the several points marked 1, 2, 3, 4, 5, &c., will converge, after reflection, to the corresponding points marked 1′, 2′, 3′, 4′, 5′, &c., which are the middle points of the several radii which are in the direction of the axes of the several pencils. At these points, therefore, images will be formed of the corresponding points in the object, and the assemblage of these images will form a complete image of the object in an inverted position, midway between the centre O and the surface A B C of the reflector.

It is evident that the points forming the image *m l* will lie in a spherical surface, whose centre is O, and whose radius is half the radius of the reflector. If, therefore, the object be a straight line, its image will be the arc of a circle; and if the object be a plane surface, its image will be a spherical surface.

In the case represented in *fig.* 296., the central point of the object is placed in the direction of the axis of the reflector, and the central point of the image lies consequently also in the axis, and the image is at right angles to the axis of the reflector and is bisected by it.

It will be explained hereafter that the apparent visual magnitude of an object is determined by the angle formed by two straight lines drawn from the eye to the extremities of the object. Thus if the eye were placed at O, the centre of the reflector, the angle L O M would be the apparent magnitude of the object. The full import and propriety of this term will be explained more fully hereafter, but for the present it will be convenient to use it in the sense just explained.

It is evident, then, that the apparent magnitude of the object L M, as viewed from the centre of the reflector O, is the same as the apparent magnitude of its image *l m* viewed from the same point, since the lines drawn from the limits of the object and the image intersect each other at the point O.

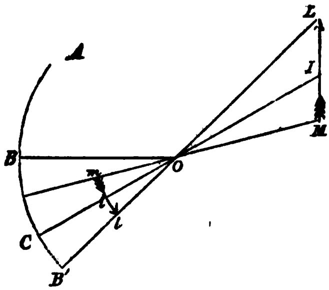

Fig. 297.

It is evident also that the linear magnitude of the image will be just so much less than the linear magnitude of the object, as one-half the radius of the reflector O F is less than the distance of the object from the centre O.

956. *Case in which the object is not placed on the axis of the reflector.* — The case in which the axis of the reflector is not directed to the centre of the object is represented in *fig.* 297.

In this case the image of the object L M is produced at *l m*, between the axes of the secondary pencils, proceeding from the extremities of the object L M, and at the middle points of the radii which coincide with the axes.

In the case of a convex reflector, let L M, *fig.* 298., be the object, placed, as before, at such a distance that each pencil of rays proceed-

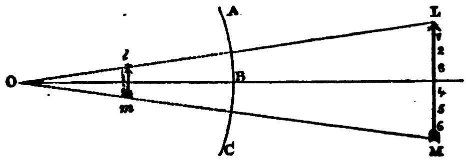

Fig. 298.

ing from a point in the object to the reflector may be considered as parallel. Let L O and M O be the axes of the pencils proceeding from the extreme points of the object. After reflection, the rays of these pencils will diverge as if they had proceeded from the points *l*, *m* respectively, which are the middle points of the radius of the reflector; and therefore, if such rays were received by the eye of an observer, they would produce the same effect on vision as if they had proceeded from the points *l*, *m*, and consequently the points L M of the object would appear as if they were placed at *l*, *m*. In the same manner, it may be shown that the intermediate points 1, 2, 3, 4, 5 of the object will appear as if they were at the intermediate points 1, 2, 3, 4, 5 of the radii, which are in the direction of their respective pencils. Thus an eye directed to the reflector, receiving the rays of the reflected pencils, will see the object as if it were on a spherical surface, of which the centre is O, and of which the radius is one-half the radius of the reflector.

The image *l m* in this case, though not formed by the real interse-

tion of the rays of light, is not the less present to vision. The eye receives the rays exactly as it would receive them if they had actually diverged from the points *l*, 1, 2, 3, 4, 5, *m*, and consequently the effect on vision is the same as if a real image of the object were placed at *l m*.

It is evident from the figure, that in this case the image is erect, and not inverted, as in the case of the concave reflector.

All that is said, however, of the relative magnitudes of the image and object in the case of the concave reflector, will be equally applicable here. Thus, to an eye placed at O, the apparent magnitude of the object L M, and of its image *l m*, will be the same; and the real linear magnitude of the image will be just so much less than that of the object, as one-half the radius of the reflector is less than the distance of the object.

957. *Experimental verification of these principles.*—The phenomena which have been just explained in the case of the reflection of very distant objects produced by concave and convex reflectors, may easily be verified experimentally.

If a concave reflector be directed towards the sun or moon, an image of either of those objects will be found at its principal focus, and such image may be rendered apparent by holding at its principal focus, and at right angles to the radius directed to the object, a small semi-transparent screen, which may be formed of ground glass or oiled paper.

A small image will be seen upon the screen, the diameter of which will bear the same proportion to the *real* diameter of the sun or moon, as half the radius of the reflector bears to the distance of one or other of these objects.

The effects of a convex reflector can be still more easily made manifest. When a convex reflector is presented to any distant objects, their images will be seen in it, and will appear as if they were behind the reflector.

They will be less in magnitude than the objects in the proportion in which half the radius of the reflector is less than the distance of the objects, and they will appear as if they were painted on a spherical surface, having its centre at the centre of the reflector, and having half the reflector for its radius.

958. *Geometrical principles on which the explanation of the phenomena depends.*—Before proceeding to explain the effects produced by spherical reflectors on diverging and converging pencils, it will be convenient here briefly to state some principles derived from geometry, to which it will be necessary frequently to recur in explanation of the effects produced on pencils of rays by spherical surfaces on which they are incident, whether these surfaces belong to opaque bodies or transparent media.

The magnitude of angles is easily explained by stating the degrees

and parts of degrees of which they consist. It may also be often more conveniently expressed by stating the ratio which the arc which bounds them bears to the radius. Thus an angle B A C, *fig.* 299., will be perfectly defined if the ratio which the arc B C bears to its radius A B be stated. Any other angle, such as *b a c*, the arc of which *b c* bears the same ratio to the radius *b a*, will necessarily have the same magnitude. This principle may be rendered still clearer, if, with A as a centre, several arcs, such as B′ C′, B″ C″, B‴ C‴, &c. be drawn subtending some angle A. It is demonstrated in geometry, and made evident from the figure, that the arcs B′ C′, B″ C″, B‴ C‴, bear respectively the same ratio to their radii A B′, A B″, A B‴, as the arc B C bears to its radius A B.

On this principle, the magnitude of an angle may with great convenience be expressed by a fraction, whose numerator is its arc, and whose denominator is its radius. Thus the angle A, *fig.* 299.,

may be expressed by $\frac{\mathrm{B\,C}}{\mathrm{B\,A}}$, or $\frac{\mathrm{B'\,C'}}{\mathrm{B'\,A}}$, or $\frac{\mathrm{B''\,C''}}{\mathrm{B''\,A}}$, &c.

If the angles be very small, perpendiculars drawn from the extremity of either side, including them upon the other, may be considered

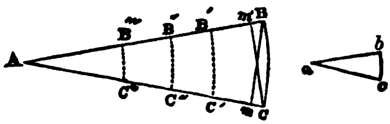

Fig. 299.

as equal to the arc. Thus, in *fig.* 299., the perpendiculars B *m* and C *m′* may be regarded as equal to the arc B C, provided the angle A do not exceed a few degrees.

In the case of such angles, therefore, their magnitude may be easily expressed by a fraction whose numerator is the perpendicular, and whose denominator is the radius.

Thus the angle A, being small, will be expressed by $\frac{\mathrm{B}\,m}{\mathrm{B\,A}}$ or by $\frac{\mathrm{C}\,m'}{\mathrm{C\,A}}$.

REFLECTION OF DIVERGENT AND CONVERGENT RAYS BY SPHERICAL SURFACES.

959. *Concave reflectors.* — Let I, *fig.* 300., be the focus of a diverging pencil of rays, incident upon a concave reflector A B C, the point I being supposed to be upon the axis of the reflector. Draw I A and I C, representing the extreme rays of the pencil. Draw O A and

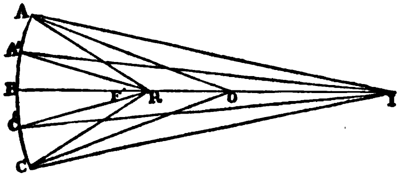

Fig. 300.

O C, the radii, to the points of incidence. The angles O A I and O C I will then be the angles of incidence; and these will evidently be equal, because the three sides of the two triangles are respectively equal.

To find the direction of the respective rays, it would be only necessary to draw from A and C lines which make with A O and C O angles equal to the angles of incidence.

Let these lines be A R and C R. The two rays I A and I C will therefore be reflected converging, and will meet at R.

By the principles of geometry,* the angle O A R of reflection is equal to the difference between the angles A R B and A O B, and the angle O A I of incidence is equal to the difference between the angles A O B and A I B.

Now, let f express the distance I B of the focus of incident rays from the vertex, and f' the distance R B of the focus of reflected rays from the same point, and let r express the radius O B of the surface. We shall then have, according to what has been explained,—

$$\text{OAI} = \frac{\text{AB}}{r} - \frac{\text{AB}}{f},$$

$$\text{OAR} = \frac{\text{AB}}{f'} - \frac{\text{AB}}{r}.$$

But since the two angles are equal, we shall have

$$\frac{\text{AB}}{r} - \frac{\text{AB}}{f} = \frac{\text{AB}}{f'} - \frac{\text{AB}}{r}.$$

Omitting the common numerator A B, we shall then have

$$\frac{1}{r} - \frac{1}{f} = \frac{1}{f'} - \frac{1}{r};$$

and consequently we shall have

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{r} \cdot\cdot (\text{A}).$$

The same formula is applicable to rays incident at every point between A or C and the vertex B; so that rays reflected from all such points will converge to a common point on the axis, whose distance from B will be determined by the value of f', found by the preceding formula.

* Euclid, Book 1. Prop. 32.

The formula (A) is of the utmost importance, and may be both understood and remembered with the greatest facility.

It may be expressed in common language as follows :—

If the fractions, whose numerator is 1, and whose denominators are the numbers expressing the distances of the foci of incident and reflected rays from the vertex, be added together, their sum will be equal to a fraction, whose numerator is 2, and whose denominator is the radius of the reflecting surface.

960. *Rule to determine the conjugate foci in concave spherical reflectors.*—By this formula (A) the position of the focus of reflected rays can always be found when that of the incident rays is known. We have only to subtract the fraction whose numerator is 1, and whose denominator is the distance of the focus of incident rays from the vertex, that is to say, the fraction $\frac{1}{f}$ from the fraction whose numerator is 2, and whose denominator is the radius, and the remainder will be equal to a fraction whose numerator is 1, and whose denominator is the distance of the focus of reflected rays from the vertex. It is evident that by a like process the focus of incident rays can be found whenever the focus of reflected rays is known.

Since the two fractions $\frac{1}{f}$ and $\frac{1}{f'}$, added together always produce the same sum, it follows that whatever circumstances increase one must diminish the other; and hence it follows that the more distant the focus of incident rays I is from the reflector, the nearer the focus of reflected rays R will be to it, and *vice versâ;* because as I B increases, R B must diminish, and *vice versâ*, as has been just explained.

If the focus I were removed to an infinite distance, then the fraction $\frac{1}{f}$ would be infinitely small, and consequently the other fraction $\frac{1}{f'}$, would be equal to $\frac{2}{r}$, and consequently f' would be equal to $\frac{1}{2}r$; that is to say, the focus of reflected rays would then be coincident with the principal focus, which is only what might have been anticipated, because if the focus of incident rays I be removed to an infinite distance, the pencil of incident rays having the reflector for its base must be parallel.

But in order to produce this effect, it is not necessary that the focus of the pencil of incident rays should be either infinitely or even very considerably distant. Let us suppose that the distance I B, which is here expressed by f, is only one hundred times the length of the radius of the reflector, and let half the radius, or the distance of the principal focus from the vertex, be expressed by F. Then we shall have

$$f = 200\,F.$$

Consequently we shall have

$$\frac{1}{f} + \frac{1}{200\,F} = \frac{1}{F};$$

and therefore

$$\frac{1}{f'} = \frac{1}{F} - \frac{1}{200\,F} = \frac{199}{200\,F},$$

and therefore

$$f' = \frac{200\,F}{199} = F + \frac{1}{199} \times F;$$

that is to say, the distance of the focus of reflected rays from the vertex will exceed the distance of the principal focus by the 199th part of half the radius, or nearly the 400th part of the radius of the reflector, an insignificant quantity.

It follows, therefore, that whenever the distance of an object from the reflector is not less than 100 times its radius, all pencils proceeding from it may be regarded as parallel, and therefore as coincident with the principal focus of the reflector.

It follows also from the preceding formula, that when the focus of incident rays is beyond the centre, the conjugate focus of reflected rays will be between the centre and the principal focus; and that when the focus of incident rays is between the centre and the principal focus, the conjugate focus of reflected rays will be beyond the centre.

In the preceding cases, we have supposed the focus of incident rays to be situate at some point beyond the principal focus of the reflector.

Let us now consider the case in which the focus of incident rays I, *fig*. 301., is placed between the principal focus F and the vertex.

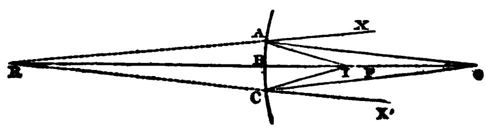

Fig. 301.

Let I A and I C, as before, be the two extreme rays of the pencil, and draw the radii A O and C O. To find the direction of the reflected rays, it is only necessary to draw from A and C two lines, which shall make with O A and O C angles equal to those which A I and C I make with them. Let this direction be A X and C X′. It follows, therefore, that in this case the reflected rays will diverge instead of converging, as in the former case, and that their point of divergence will be at R,

upon the axis behind the reflector; consequently the focus will be an imaginary focus.

By geometrical principles already referred to,* the angle of incidence I A O is equal to the difference between the angles A I B and A O B, and the angle of reflection X A O is equal to the sum of the angles A R B and A O B; and since the angles formed by O A, I A, and R A with the axis O R are so small as to come within the scope of the principles already expressed, we shall have

$$\text{I A O} = \frac{\text{A B}}{f} - \frac{\text{A B}}{r}$$

$$\text{X A O} = \frac{\text{A B}}{f'} + \frac{\text{A B}}{r},$$

where f and f' express, as in the former case, the distances of the foci of incidence and reflection respectively from the vertex B.

We shall therefore have

$$\frac{\text{A B}}{f} - \frac{\text{A B}}{r} = \frac{\text{A B}}{f'} + \frac{\text{A B}}{r};$$

and omitting the common numerator A B, we shall have

$$\frac{1}{f} - \frac{1}{f'} = \frac{2}{r} \quad . \quad . \text{ (B)},$$

a formula which is identical with formula (A), p. 48., only that $\frac{1}{f'}$ in it is negative, which indicates that the focus of reflected rays is imaginary and behind the reflector.

In the formula (B) it is not the sum of the two fractions $\frac{1}{f'}$ and $\frac{1}{f}$, but their difference, which is equal to $\frac{2}{r}$.

Analogous results, however, follow from this formula, which may be expressed in common language as follows:—

When the focus of rays incident upon a concave reflector is placed between its principal focus and the vertex, the difference between the fraction whose numerator is 1 and whose denominator is the distance of the focus of incident rays from the vertex, and the fraction whose numerator is 1 and whose denominator is the distance of the focus of reflected rays from the vertex, will be equal to the fraction whose numerator is 2 and whose denominator is the radius of the reflecting surface.

Since the difference between these two fractions is always the same, however they may separately vary, it follows, that when one increases, the other must increase to the same extent. Hence it follows, that f and f' increase and diminish together; and therefore it also follows, that as the focus of incident rays I approaches the vertex B, the focus

* Euclid, Book 1. Prop. 32.

of reflected rays R must also approach it; and as the focus of incident rays I recedes from the vertex, the focus of reflected rays R must also recede from it.

When the focus of incident rays I arrives at the principal focus F, the focus of reflected rays R recedes to an infinite distance.

961. *Case of converging incident rays.*—If rays fall on the reflector converging to a point R behind it, they will be reflected converging to the point I. In this case, the focus of incident rays being behind the reflector will be imaginary, and the focus of reflected rays being before it will be real. The relative positions of the two foci, however, will be determined in the same manner exactly as if I were the focus of incidence, and R the focus of reflection. It may be useful to observe in general, that the conjugate foci are in all cases interchangeable.

If the focus of incidence become the focus of reflection, the focus of reflection will become the focus of incidence, and *vice versâ.*

962. *Convex reflectors.*—The effects attending diverging or converging rays incident upon convex reflectors, are in all respects analogous to those which have been just established respecting concave reflectors.

It is only necessary to observe, that converging rays upon a convex reflector are analogous to diverging rays upon a concave reflector; and diverging rays upon a convex reflector are analogous to converging rays upon a concave reflector.

Thus, if A C, *fig.* 300., instead of representing a concave, represent a convex reflector, and a pencil of rays be supposed to be incident upon it, which if not intercepted would converge upon the point I, those rays after reflection will diverge from the point R. The conjugate foci will be in this case precisely similar, and determined by similar conditions as in the former case, except that the incident rays are convergent, while the reflected rays are divergent, the contrary being the case in a concave reflector.

In like manner, if the reflector A C, *fig.* 301., be a convex instead of a concave reflector, a pencil of rays incident upon it, which if not intercepted would converge to I, will be reflected converging to R. In this case, the focus of incident rays will be imaginary, and the focus of reflected rays real, contrary to what was shown for a concave reflector; but the relative position of the two foci will be determined as before.

Images of near objects formed by spherical reflectors.—The manner has been explained in which images are formed by spherical reflectors of objects whose distance is so great that the pencils of rays proceeding from them may be considered as consisting of parallel rays. It is in this and like cases important, that the student should not confound the directions of the pencils themselves with the directions of the rays which form them. Thus, the pencils of rays pro-

ceeding from points upon the surface of the sun or moon are pencils of parallel rays, because the distance of the foci of such pencils from the observer is incomparably great compared with any surface which can form the base of the pencil. Thus, the surface of the largest reflector is as nothing compared with the distance of any point in the sun; and consequently, the rays which form a pencil, whose vertex is a point in the sun, and whose base is the surface of such a reflector, may be practically considered as parallel; but this parallelism must not be applied to the direction of the pencils themselves which proceed from different points in the sun. The directions of these pencils, or, to speak strictly, those of their respective axes, are not parallel, the axes of the extreme pencils forming an angle with each other equal to the apparent diameter of the sun; and the same observations would be applicable to any other object whose distance is so great that a pencil of rays proceeding from it may be regarded as parallel.

These observations being premised, we shall now explain the manner in which images are formed by spherical reflectors of objects which are not so distant that the rays of the pencils proceeding from points in them can be regarded as parallel.

Let A B C, *fig.* 302., be a concave reflector, whose centre is O, and whose vertex is B. Let L M be an object, whose form we shall for

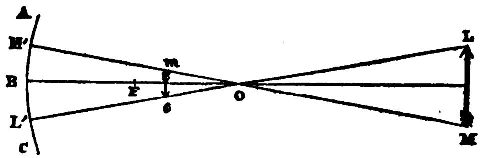

Fig. 302.

the present assume to be that of an arc of a circle whose centre is O. Let L L′ and M M′ be the axes of the extreme secondary pencils proceeding from this object, and let *l* and *m* be the foci of reflection conjugate to the points L and M. An image of the point L will be formed at *l*, and an image of the point M will be formed at *m*, and images of all the intermediate points between L and M will be formed at intermediate points of an arc drawn from *l* to *m*, having O as a centre.

Since the lowest point of the image corresponds to the highest point of the object, and *vice versâ*, the image will in this case be inverted with respect to the object, and the linear magnitude of the image will bear to that of the object the same proportion as O *l* bears to O L.

These results follow in the same manner as in the case of the images of distant objects already explained.

The distance O l is determined when O L is known by the formulæ (A) and (B), p. 48. and p. 51.; that is to say, the position and magnitude of the image will be determined when the position and magnitude of the object are known.

In this case, the object L M has been supposed to have the form of a circular arc, and its image to have a similar form. If the object form part of a spherical surface whose centre is O, the image would have a like form; but if the object were a straight line or flat surface, then the image would be more or less curved, and would consequently be distorted. But as, in general, the angle O, under which the object or image would be seen from the centre, is small, this curvature may be disregarded, and we may assume that the image will be similar to the object.

963. *Spherical aberration of reflectors.* — The pencils of rays proceeding from or to the incident focus will be reflected to a common point, only on the condition that the opening of the reflector is limited, as was explained in the case of parallel rays. If it be not so limited, then the extreme rays of the pencil will converge to points sensibly different from those which are within such limit of distance of the vertex already defined, and hence will arise a spherical aberration.

If even the reflector be sufficiently limited in its opening, a sensible spherical aberration will arise from the secondary pencils which proceed from the borders of the object, and are inclined at the greatest angles to the axis of the reflector, for in this case the angle of divergence of such pencils will, as has been already explained, exceed that limit which would efface the spherical aberration. Hence it arises that images produced by spherical reflectors when the objects are too great, are indistinct towards the borders, the pencils which form each part of the image not being brought to the same focus, and consequently producing a confused effect.

964. *Case in which the object is placed between the principal focus and the reflector.* — In what precedes, the position of the object before a concave reflector has been considered as being either beyond the centre or between the centre and the principal focus F. Let us now consider the position of the object to be at L M, *fig.* 303., between the principal focus F and the reflector. In this case the image l m will be behind the reflector at the points which form the foci conjugate to the several points of the object L M.

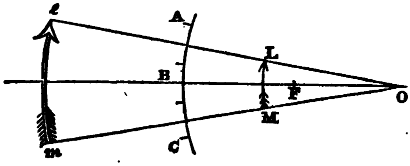

Fig. 303.

The image will in this case evidently be erect with respect to the

object, and will be greater in magnitude than the object in the proportion of O *l* to O L.

If the reflector be convex, the object L M, *fig.* 304., will have its image at the points *l*, *m*, which are the foci conjugate to the points at L M, and those points will, according to what has been already explained, lie between the reflector and the principal focus F.

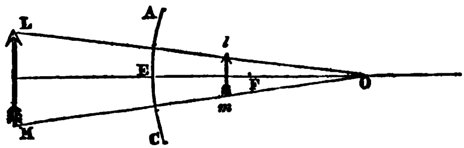

Fig. 304.

The rays proceeding from the several points of the object L M will, after reflection, diverge as if they had proceeded from the corresponding points of *l m*, and will produce upon the vision the same effects as if an object had been actually placed at *l m*.

The image in this case, therefore, will be erect, and it will be less than the object in the proportion of O *l* to O L. In this manner is explained the effect familiar to every one, that convex reflectors exhibit a diminished picture of the object placed before them.

All the preceding observations on the effect of spherical aberration, and the indistinctness incident to the borders of the image, will be equally applicable in the present case.

965. *Case in which the object is not placed in the axis of the reflector.*—In the preceding example, the object has been supposed to be placed so that its centre coincides with the axis of the reflector. The image, however, is determined on like principles, whatever other position it may have.

Thus, let L M, *fig.* 305., be the object, A B C being the reflector, O its centre, and F its principal focus. From the extremities of the object draw lines L O and M O through the centre O of the reflector to meet the continuation of the section of the reflector at P and Q. Let *l* be the focus conjugate to L, and *m* the focus conjugate to M, determined according to the principles and formulæ already established. Images, therefore, of the points L and M will be formed at *l m*, and images of all the intermediate points of the object will in like manner be formed between *l* and *m*, so that an inverted image of the object will be formed at *l m*.

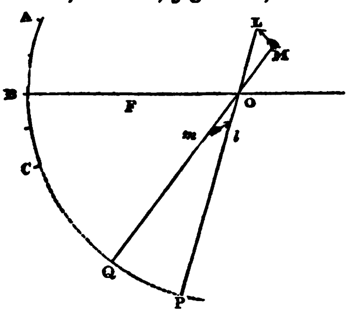

Fig. 305.

In like manner, if the object be placed at *l m*, its image will be formed at L M.

966. *Experimental verifications.* — All the preceding results may be verified experimentally by means analogous to those already explained. Thus, if the flame of a candle be placed at L M, *fig.* 302., outside the centre of a concave reflector, and a small semi-transparent screen, such as a piece of ground glass or oiled paper, be held at *l m*, an inverted image of the candle will be seen upon it; and, on the other hand, if the candle be placed at *l m*, and the screen held at L M, the image will be again seen. If any object, such as one's hand, be presented between the principal focus F and a concave reflector, as at L M, *fig.* 303., a magnified image of the hand will be seen at *l m*.

Amusing optical deceptions are often exhibited with concave reflectors founded on this principle. Thus, a hand presenting a dagger is held between O and F, *fig.* 302., when immediately a magnified image of the hand and dagger is presented outwards at L M.

If a candle be held at L M, *fig.* 305., opposite the upper edge of a concave reflector, an inverted image of the candle may be exhibited on a screen at *l m*, opposite the lower edge.

967. *Cylindrical and conical reflectors.* — A cylindrical surface is circular in one direction, and rectilinear in the other, these directions being at right angles to each other. A sheet of paper, or a plate of metal bent into the form of a circle, will be a cylindrical surface.

It may be polished either on the concave or convex side, thus presenting the varieties of a concave or convex cylindrical reflector.

If a cylindrical reflector be placed vertically before an object, its effects upon the vertical dimensions will be the same as those of a plane reflector, and its effects upon the horizontal dimensions the same as those of a spherical reflector. An image, therefore, will be presented, which will be identical in form with the object in all its vertical dimensions, but enlarged, diminished, or reversed in its horizontal dimensions in the same manner as it would be in a spherical reflector.

If a cylindrical reflector be placed with its axis horizontal before a vertical object, it will have the same effect as a plane reflector on the horizontal dimensions, and as a spherical reflector on the vertical dimensions.

The horizontal dimensions, therefore, will be preserved in the image, while the vertical dimensions will be enlarged, diminished, or reversed, in the same manner as would be the case with a spherical reflector.

A conical reflector, whether concave or convex, is circular in all sections made at right angles to its axis, and rectilinear in all sections made by planes through its axis. It will therefore, if placed with its axis vertical, have the effect of an inclined plane reflector on the vertical dimensions of an object, and will have the effect of a spheri-

cal reflector on the horizontal dimensions; but each horizontal section will be differently magnified or diminished, according to the position of such section with reference to the axis of the cone, since the circular section of the cone will diminish in approaching the axis, and increase in receding from it. An infinite variety of amusing deceptions are thus produced.

CHAP. VI.

REFLECTION FROM IMPERFECTLY POLISHED SURFACES.

968. *A perfectly reflecting surface would be invisible.*—If the surface of an opaque body were perfectly polished, and capable of reflecting regularly all the light incident upon it, such surface would itself be invisible.

The images of all objects placed before it would appear in the position and with the form and magnitude determined in the last chapter; and an observer receiving the reflected light would perceive nothing but such images.

Thus, a plane reflector of that kind placed vertically against the wall of a room, would appear to the eye merely as an opening leading into another room, precisely similar and similarly furnished and illuminated; and an observer would only be prevented from attempting to walk through such an opening by encountering his own image as he would approach it.

969. *No such surfaces exist.*—But such a reflector as this has no practical existence, for there is no surface natural or artificial possessing the power of reflecting all the light incident upon it regularly. The absence of complete polish is one of the principal causes of this.

970. *How the surfaces of reflectors are rendered visible.*—The consequence is, that even the most polished surfaces reflect a certain portion of the light incident upon them irregularly; that is to say, the material points, the assemblage of which forms such surfaces, becoming separately illuminated form so many radiant points, from which pencils of light diverge, and render such surfaces visible exactly in the same manner, though much more faintly than is the case with unpolished surfaces. The quantity of light which is thus irregularly reflected, and which therefore renders the reflecting surface itself more or less visible, diminishes in the same proportion as the perfection of the polish of the surface increases.

The most perfectly polished surfaces, which serve as reflectors, are certain alloys of metal known as speculum metal. These are used

generally for the metallic specula of telescopes, microscopes, and other optical instruments.

971. *How light incident on any opaque surface is disposed of.* — When light falls therefore on any imperfectly polished and opaque surface, it is disposed of in three ways. 1°. A part is regularly reflected, and forms the optical image of the object from which it proceeds. 2°. A part is irregularly reflected, and renders the surface of the reflector perceivable. 3°. A part is absorbed by the surface, and, consequently, not reflected. The smaller the proportion of the light subject to the two last-mentioned effects, the more perfect will be the reflector.

The quantity of light regularly reflected by a given surface also varies with the angle of incidence. When the angle of incidence is nothing, and consequently the light falls perpendicularly on such a surface, a less proportion of it is regularly reflected, and a greater proportion irregularly reflected and absorbed, than when the angle of incidence has some magnitude: and, consequently, the light falls more or less obliquely; and in general, as the angle of incidence increases, the quantity of light reflected regularly is augmented, and, consequently, the quantities reflected irregularly and absorbed are diminished.

The following is given by Bouguer as the proportion of the light regularly reflected from different reflecting surfaces, at different angles of incidence :—

972. *Table showing the proportion of light incident on reflecting surfaces which are regularly reflected at different angles of incidence.*

Species of reflecting Surface.	Angle of Incidence.	Number of Rays incident.	No. of Rays regularly reflected.	No. of Rays irregularly reflected and absorbed.
Water	89° 30′	1000	721	279
	75° 0′	1000	211	789
	60°	1000	65	935
	30° to 0°	1000	18	982
Glass	85°	1000	543	457
	75°	1000	300	700
	60°	1000	112	888
	30° to 0°	1000	25	975
Black marble polished	80° 45′	1000	600	400
	75°	1000	156	844
	60°	1000	51	949
	30° to 0°	1000	23	977
Metallic reflectors	Great angles.	1000	700	300
	Small angles.	1000	600	400

In the preceding table, the light is understood to pass from air to the several media indicated in the first column. The law by which the quantity of light regularly reflected varies according to the density or other physical qualities of the media, has not been ascertained.

It is however certain, that it depends upon the qualities of the medium from which the light passes, as well as those of the medium into which it passes.

973. *Effect of angle of incidence on the quantity of light regu larly reflected.*—The angle of incidence has often so much effec upon the quantity of light regularly reflected, that it will sometimes happen that a surface which reflects no light regularly when the angle of incidence is nothing, reflects a considerable quantity when such angle has much magnitude. Thus, a surface of unpolished glass produces no image of an object by reflection when the rays fall on it nearly perpendicularly; but if the flame of a candle be held in such a position that the rays fall upon the surface at a very small angle, a distinct image of it will be seen. Similar phenomena will be observed with surfaces of wood, of common woven stuff, and of paper blackened by smoke.

974. *How light incident on the surface of a transparent body is disposed of.*—When light is incident upon the surface of a transparent body, such as glass or water, it is disposed of as follows:—1°. A part is regularly reflected, and produces an optical image of the object from which the light proceeds. 2°. A part is irregularly reflected, and renders the surface visible. 3°. A part is absorbed, and, consequently, neither reflected nor transmitted. 4°. A part is transmitted through the transparent medium.

If light be incident upon the surface of a transparent medium bounded by parallel surfaces, such as a flat plate of glass, all the circumstances above mentioned will take place both at its entrance at the one surface and its escape from the other. Light will be reflected regularly and irregularly at both surfaces; light will be absorbed at both, and light will be transmitted from both. The quantity of light, therefore, transmitted in such a case from the second surface will be less than the quantity of light incident upon the first surface by the sum of all the light regularly and irregularly reflected from the first surface, and all the light regularly and irregularly reflected from the second surface, and all the light absorbed at both surfaces in its transit through the medium.

975. *How light is affected in passing through the atmosphere.*—Even when the transparent medium consists of the same substances, these effects take place if the substance composing it varies in density. The successive strata of the atmosphere present an example of this.

It has been already explained that in ascending in the atmosphere the succeeding strata of air gradually diminish in density. The light, therefore, of the sun and other celestial bodies in passing through the

atmosphere is transmitted through a succession of strata of increasing density, and is subject consequently to all the effects just explained. Light is gradually absorbed and reflected by the successive strata of air through which it passes, and consequently the direct solar light which arrives at the surface of the earth is less in quantity considerably than the light originally incident upon the superior surface of the atmosphere. A portion, however, of the light irregularly reflected from the successive strata of the atmosphere arrives at the earth from these strata, as has been already explained, in the same manner as light is received from the surface of any opaque illuminated body. A portion, however, of the light which enters the air is absolutely absorbed by it, and, as has been already stated, a certain depth might be assigned to the atmosphere, which would completely intercept the solar light. It is calculated that seven feet thickness of water is sufficient to intercept one-half of the light transmitted through it.

976. *Blackened glass reflectors.* — A reflecting surface convenient for certain optical purposes is produced by blackening one side of a plate of glass. By this means the light transmitted through the plate is absorbed by the blackened surface on the other side, and light is prevented from being transmitted from the opposite side by the opaque coating; consequently, the only light regularly reflected in this case will be that which is reflected from the superior surface.

977. *Effect of a common looking-glass explained.* — The effects of a common looking-glass are produced by the reflection of the metallic surface attached to the back of the glass, and not by the glass itself. The effect may be explained as follows: — A portion of the light incident upon the anterior surface is regularly reflected, and another portion irregularly. The former produces an image of the object placed before the glass visible in it; the other renders the surface of the glass itself visible. Another and much greater portion, however, of the light incident upon the anterior surface penetrates the plate, and arrives at the posterior surface. This surface, coated with an amalgam produced by the combination of tinfoil and quicksilver, has an intense metallic lustre, and possesses therefore strong reflecting power. The chief part of the light, therefore, which passes through the plate of glass is regularly reflected by this metallic surface, and returning to the eye, produces a strong image of the objects placed before the glass. There are, therefore, strictly speaking, two such images formed: first, a faint one by the light reflected regularly from the anterior surface; and, secondly, a vivid one by the light reflected regularly from the metallic surface. One of these images will be before the other, at a distance equal to twice the thickness of the glass.

In good mirrors which are well silvered, the superior brilliancy of the image produced by the metallic surface will render the faint image produced by the anterior surface of the glass invisible; but in glasses badly silvered, the two images may be easily seen.

CHAP. VII.

REFRACTION OF LIGHT.

978. *Refraction of light explained.* — When a ray of light, after passing through a transparent medium, enters another of a different density, or possessing other physical properties, it will change its direction at the point which separates the two media, and consequently the direction it follows in the second medium will form a certain angle with that which it has followed in the first medium. The ray is as it were broken at the common surface of the two media, which has caused this phenomenon to be called *refraction*.

Let A B, *fig.* 306., be the surface which separates the two media. Let I be the point at which a ray E I is incident, and let I R be the course which this ray takes after entering the second medium. Let N I N′ be a perpendicular to the surface A B, drawn through the point of incidence I. A B is called the *refracting surface*, E I N is called the *angle of incidence*, and R I N′ is called the *angle of refraction*.

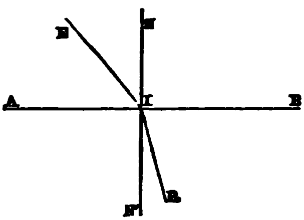

Fig. 306.

979. *Law of refraction.* — The following law of refraction has been established by experiment: —

I. The angles of refraction and incidence are in the same plane perpendicular to the refracting surface.

II. The sine of the angle of incidence has to the sine of the angle of refraction always the same ratio for the same medium.

It will appear hereafter that, under certain circumstances, a single ray of light entering a refracting medium will be divided into several, which follow different directions; but for the present we shall limit our observations to such light only as after refraction follows a single direction. To such light the above law is strictly applicable.

To explain the preceding law more fully, and to indicate the manner of verifying it by experiment, let A M B be a piece of glass, having the form of a semi-cylinder, as represented in *fig.* 307. Let C be the centre, and A B the diameter of the semi-cylinder. Let the semicircle A O B be imagined to be drawn on a vertical card, so as to complete the circle. Let O C M be the diameter perpendicular to A B, and let the surface A B be covered with an opaque card, with a small hole to admit light at C.

If the flame of a candle, or any other bright object, be held at O, it will be visible to an eye placed at M. It follows, therefore, that a

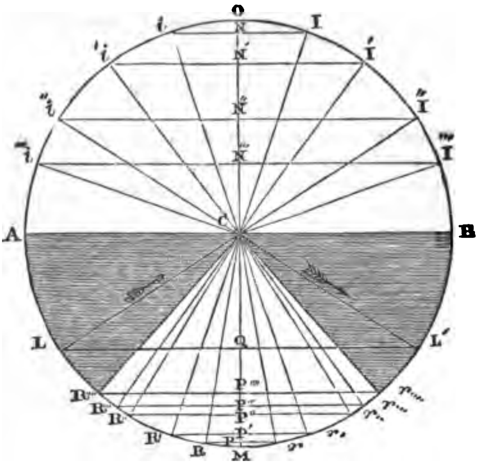

Fig. 307.

ray of light striking the refracting surface in a direction perpendicular to it, such as O C, will suffer no change of direction after it enters it, but will proceed in the same straight line C M as it would have done if it had passed through no refracting medium. Let the luminous point be now transferred to f, and let the line I N be drawn perpendicular to C O. This line I N is the sine of the angle of incidence I C O. Let the eye be now moved along the arc M A from M towards A, until it see the luminous point I.

Let R be the place at which the luminous point thus becomes visible, C R will then be the direction of the refracted ray. Draw R P perpendicular to C M. This line R P will be the sine of the angle of refraction R C M.

Now if I N and R P be respectively measured, it will be found that R P is exactly two-thirds of I N. Therefore, in this case, the sine of the angle of incidence will be to the sine of the angle of refraction as 3 to 2, that is to say, we shall have

$$\frac{\text{I N}}{\text{R P}} = \frac{3}{2}.$$

Let the luminous point be now moved to I′, and let the eye be moved towards A until it see it. Let R′ be the point at which it becomes visible; C R′ will then be the refracted ray, I′ C being the incident ray.

Draw I′ N′ perpendicular to C O, and R′ P′ perpendicular to C M; I′ N′ will then be the sine of the angle of incidence, and R′ P′ will be the sine of the angle of refraction. If these two lines be respectively

measured, it will be found that R′ P′ will be two-thirds of I′ N′; so that we shall have, as before,

$$\frac{\text{I}'\,\text{N}'}{\text{R}'\,\text{P}'}=\frac{3}{2}.$$

In the same manner, if the luminous point be moved to any other point, such as I″, and the eye be moved towards A until it see it, the lines I″ C and C R″ will be the incident and refracted rays, I″ N″ and R″ P″ will be sines of the angles of incidence and refraction respectively; and we shall find, as before, by measurement, that

$$\frac{\text{I}''\,\text{N}''}{\text{R}''\,\text{P}''}=\frac{3}{2}.$$

Thus, in general, in whatever manner the position of the luminous point may be viewed, it will always be found that the sine of the angle of incidence will be to the sine of the angle of refraction as 3 to 2, that is to say, in one constant ratio.

In this case, the incident ray is supposed to pass through air, and the refracted ray through glass. If the semi-cylinder A M B, instead of glass, be water, then the ratio of the sine of the angle of incidence to the sine of the angle of refraction will be 4 to 3, so that we shall have

$$\frac{\text{I N}}{\text{R P}}=\frac{4}{3},$$

$$\frac{\text{I}'\,\text{N}'}{\text{R}'\,\text{P}'}=\frac{4}{3},$$

$$\frac{\text{I}''\,\text{N}''}{\text{R}''\,\text{P}''}=\frac{4}{3},$$

and so on.

Thus each transparent medium has its own particular refracting power, but for the same transparent medium the ratio of the sines of the angles of incidence and refraction is always the same.

980. *Index of refraction.*—The number which thus expresses the ratio of the sine of the angle of incidence to the sine of the angle of refraction, and which in the case of air and glass is $\frac{3}{2}$ or 1·5, and in the case of air and water is $\frac{4}{3}$ or 1·333, is called the ***index of refraction.***

From what has been stated, it is evident that each transparent medium will have its own index of refraction, which constitutes one of its most important physical properties.

981. *Case of light passing from denser into rarer medium.*—If the luminous point, instead of being moved along the arc O B, be moved along the arc M A, and the eye be transferred to the arc O B, then the incident ray will pass through the denser medium, and

the refracted ray through the rarer medium. In this case it will be found that the direction of the incident and refracted rays, described in the former case, will be interchanged. Thus, if the luminous point be applied at M, it will be visible at O, showing that a ray of light incident perpendicularly on the surface of a rarer medium, will suffer no change in its direction. If the luminous point be placed at R, it will be visible at I, showing that if R C be the incident ray, C I will be the refracted ray; and in the same manner, if the luminous point be placed at R′ and R″, it will be visible at I′ and I″.

982. *Directions of incident and refracted rays interchangeable.* — Hence it follows, that if a ray of light passing from one transparent medium into another transparent medium be refracted in a particular direction, a ray of light passing from the latter into the former in the direction in which it was refracted, will, after entering the former, follow the direction in which the former ray was incident; or in general it may be stated that the direction of the incident and refracted rays passing between the media are interchangeable.

983. *Indices of refraction between two media in contrary directions reciprocals.* — It follows from this that the indices of refraction between the media are reciprocals; that is to say, if the index of refraction from air into glass be $\frac{3}{2}$, the index of refraction from glass into air will be $\frac{2}{3}$; the latter number being what is called in arithmetic the reciprocal of the former. In the same manner, the index of refraction from air into water being $\frac{4}{3}$, the index of refraction from water into air will be $\frac{3}{4}$.

It appears in the two cases which have been stated of water and glass, that when a ray passes from air into either of these media it will be bent *towards* the perpendicular; and that, on the other hand, when it passes out of either of these media into air, it will be bent *from* the perpendicular. This will be evident by reference to *fig.* 307. The rays I C, I′ C, I″ C entering water or glass are bent in the directions C R, C R′, C R″ *towards* the perpendicular C M; and, on the other hand, the rays R C, R′ C, R″ C, passing from glass or water into air, are bent in the directions C I, C I′, C I″ *from* the perpendicular C O.

984. *Rays not always bent towards perpendicular in entering a denser medium.* — This result being too hastily generalized, is sometimes announced as follows: — When a ray of light passes from a rarer into a denser medium, it is bent towards the perpendicular, and from a denser into a rarer from the perpendicular, which is by no means generally true.

Such a proposition is based upon the supposition that the refracting power always increases with the density; whereas numerous instances will be produced in which media of greater density have a less refracting power.

985. *Index of refraction increases with the refracting power.* — The refracting power is estimated by the index of refraction, one

medium being said to have a greater or less refracting power, according as its index of refraction is greater or less than that of the other. Thus, glass is said to have a greater refracting power than water, because its index of refraction being 1·50, is greater than the index of refraction of water, which is 1·33.

The propriety of this test of the refracting power will be easily understood. If the index of refraction of one medium be greater than that of another, the angle of refraction which corresponds to a given angle of incidence will be smaller in the former than in the latter; and consequently, the same incident ray would be bent more out of its course in the one case than in the other, that is to say, it would be more refracted.

986. *But not in proportion to it.*—Although, however, the refracting power of a transparent medium increases with every increase of its index of refraction, this power does not increase in proportion to such index, but in proportion to a number found by subtracting 1 from the square of the index. Thus, in the case of glass, where the index of refraction is $\frac{3}{2}$, its square is $\frac{9}{4}$, from which 1 being subtracted leaves $\frac{5}{4}$, which represents the refracting power. In the same manner, the index of water being $\frac{4}{3}$, its square is $\frac{16}{9}$, from which 1 being subtracted leaves $\frac{7}{9}$, which represents the refracting power of water; or, in general, if n be the index, n^2-1 will represent the refracting power.

The principle upon which this number n^2-1 is shown to be proportional to the refracting power, does not admit of an explanation sufficiently elementary for this work. We must therefore adopt it as a datum without demonstration.

In the following table are given the indices of refraction of those transparent substances which are of most usual occurrence.

987. *Table of the indices of refraction for light passing from a vacuum into various media.*

SOLIDS AND LIQUIDS.

Chromate of lead (maximum)	2·974
" (minimum)	2·500
Sulphur, native	2·115
Carbonate of lead (maximum)	2·084
" (minimum)	1·813
Felspar (Spinelli)	1·764
Chrysoberyl	1·760
Nitrate of lead	1·758
Carbonate of strontia (maximum)	1·700
" (minimum)	1·543
Boracite	1·701
Aragonite (ordinary* refraction)	1·693
" (extraordinary* refraction)	1·535

* Ordinary and extraordinary refraction will be explained in Chap XVIII.

Calcareous spar (ordinary refraction)	1·654
" (extraordinary refraction)	1·483
Sulphate of baryta	1·647
" (ordinary refraction)	1·620
" (extraordinary refraction)	1·635
Colourless topaz	1·610
Topaz of Brazil (extraordinary refraction)	1·640
" (ordinary refraction)	1·633
Anhydrite (extraordinary refraction)	1·622
" (ordinary refraction)	1·577
Euclase (extraordinary refraction)	1·663
" (ordinary refraction)	1·643
Flint-glass (maximum)	1·605
" (minimum)	1·576
Quartz (ordinary refraction)	1·548
" (extraordinary refraction)	1·558
Crown-glass (maximum)	1·534
" (minimum)	1·525
Sulphate of lime	1·525
Saltpetre (nitrate of potassa) (maximum)	1·514
" (minimum)	1·335
Sulphate of potassa	1·509
"	1·495
Sulphate of ammonia and magnesia	1·483
Carbonate of potassa	1·482
Spermaceti, melted	1·446
Albumen	1·360
Ether	1·358
Aqueous humour of eye	1·337
Vitreous do	1·339
External coating of the crystalline	1·377
Middle coating do	1·379
Central coating do	1·399
Entire crystalline	1·384
Water	1·336
Ice	1·310
Vacuum	1·000

GASES.

Atmospheric air	1·000,294
Oxygen	1·000,272
Hydrogen	1·000,138
Nitrogen	1·000,300
Ammonia	1·000,385
Carbonic acid	1·000,449
Chlorine	1·000,772
Hydrochloric acid	1·000,449
Nitrous oxide	1·000,503
Nitrous gas	1·000,303
Carbonic oxide	1·000,340
Cyanogen	1·000,834
Olefiant gas	1·000,678
Light carburetted hydrogen	1·000,443
Muriatic ether (vapour)	1·001,095
Hydrocyanic acid	1·000,451

Chloro-carbonic acid (phosgene gas)	1·001,159
Sulphurous acid	1·000,665
Sulphuretted hydrogen	1·000,644
Sulphuric ether (vapour)	1·001,530
Vapour of sulphuret of carbon	1·001,500
Protophosphuret of hydrogen	1·000,789

988. *How to find the index of refraction from one medium to another.* — The indices of refraction given in the preceding table relate to rays of light passing from a vacuum into the several media indicated. If it be required to find the index of refraction for a ray passing from one medium to another, it is only necessary to divide the index of the medium into which the ray is supposed to pass by the index of the medium from which it passes, and the quotient will be the required index. Thus, if it be desired to determine the index of refraction for a ray passing from atmospheric air into any medium indicated in the table, it will be only necessary to divide the index of the medium whose relative index is required by 1·000,294, the index of refraction of atmospheric air.

989. *Course of a ray passing through a succession of media with parallel surfaces.* — It follows from this, that if a ray pass from any medium successively through several transparent media with parallel surfaces, its course in the last of the series will be the same as it would be if it had been incident directly on the surface of the last without having passed through the preceding media. This is easily proved: for let I be the angle of incidence upon the surface of the first medium, and R the angle of refraction. This angle R will be the angle of incidence on the second medium, in which the angle of refraction is R′. This angle of refraction R′ will be the angle of incidence on the surface of the third medium, in which the angle of refraction is R″.

If n be the index of refraction of the original medium through which the ray passes, and n', n'', and n''' be the indices of refraction of the three successive media by which it is refracted, then the index of refraction from the original medium into the first will be $\frac{n'}{n}$ and consequently we shall have

$$\frac{\sin. \text{I}}{\sin. \text{R}} = \frac{n'}{n};$$

and in like manner we shall have

$$\frac{\sin. \text{R}}{\sin. \text{R}'} = \frac{n''}{n'}, \qquad \frac{\sin. \text{R}'}{\sin. \text{R}''} = \frac{n'''}{n''}.$$

By multiplying all these together we shall have

$$\frac{\sin. \text{I}}{\sin. \text{R}''} = \frac{n'''}{n};$$

which is the index of refraction from the original medium through which the ray passed to the last medium by which it has been refracted. The angle of refraction, therefore, R″, in this latter medium, would be the same if the original ray had been directly incident upon it with the same angle of incidence.

990. *A ray having passed through several parallel surfaces, emerges parallel to its incidence.* — It follows from this, that if a ray of light, after passing through several successive media separated by parallel surfaces, pass finally into the medium from which it was originally incident, it will issue in a direction parallel to the original ray. Thus, in the preceding example, if the original ray of light, after passing successively through the three media, issue again into the medium through which it originally passed, its direction will be parallel to its original direction; for, according to what has been already proved, its course, after passing through the three media and not the fourth, will be the same as if it passed directly from the first medium into the fourth; but in this case the first medium being the same as the fourth, the ray would not be deflected from its course. It must therefore, after passing through the parallel media, preserve its original direction.

991. *Why objects are distinctly seen through window glass.* — It is for this reason that plates of glass with parallel surfaces, such as window glass, produce no distortion in the objects seen through them; the rays from such objects, after passing through the glass, preserve their original direction.

992. *The angle of refraction in passing from a rarer into a denser medium has a limit of magnitude which it cannot exceed.* — The law of refraction which has been just explained and illustrated is attended with some remarkable consequences in the transmission of light through media of different refracting powers.

Let A B, *fig.* 308., represent, as before, the surface which separates a medium of air A O B from a medium of glass A M B. According to what has been already explained, any incident ray, such as I C, will be deflected towards the perpendicular C M, so that its angle of refraction shall have a sine equal to two-thirds of that of its angle of incidence. Now, let us suppose the angle of incidence gradually to increase, so as to approach to a right angle. It is evident that the sine of the angle of incidence I N will also gradually increase until it approach to equality with the radius C B. This will be evident on inspecting the diagram, in which I′ N′, I″ N″, I‴ N‴, &c. are the sines of the successive angles of incidence; and if we suppose the direction of the incident ray to approximate as closely as possible to that of the line B C, the sine of the angle of incidence will approach as close as possible to the magnitude of B C.

Now, let us consider what corresponding change the angles of refraction will suffer. Their sines will be respectively, in the case of

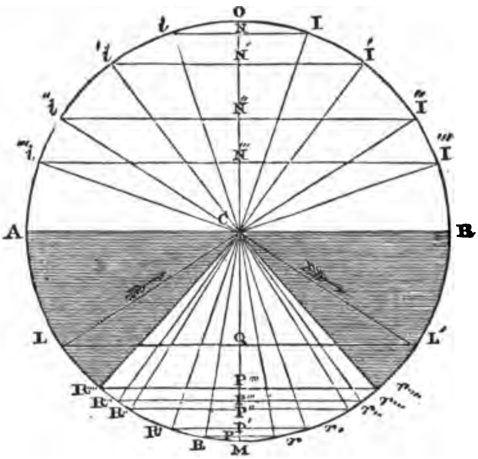

Fig. 308.

glass here supposed, two-thirds of the sines of the angles of incidence; thus the sine R P of the angle of refraction corresponding to I C will be two-thirds of I N; the sine R′ P′ of the angle of refraction corresponding to I′ C will be two-thirds of I′ N′; the sine R″ P″ of the angle of refraction corresponding to I″ C will be two-thirds of I″ N″; and so on. When the incident ray approaches to coincidence with B C, the sine of the angle of incidence will approach to equality with B C, and consequently the sine of the angle of refraction will be equal to two-thirds of B C. If, therefore, it were possible that a ray passing directly from B to C could enter the glass at C, such ray would have an angle of refraction whose sine would be two-thirds of the radius B C. Now, if we draw C R'''' to such a point that the sine of the angle of refraction R'''' P'''' shall be two-thirds of the radius B C, it is evident that all the incident rays whose directions lie between O C and B C will be refracted in directions lying between C R'''' and C M.

In like manner it may be shown, that all incident rays whose directions lie between O C and A C will be also included after refraction beween the lines C M and C r'''', corresponding in position to C R''''.

Thus it appears that rays of light converging from all directions to the point C, will be after refraction included within a cone whose angle is R'''' C r''''.

Hence follows the remarkable consequence, that light entering the glass at C, from whatever direction it may proceed, will be totally excluded from the space A C R'''' and B C r'''', all such light being included, as has been observed, within the cone whose angle is R'''' C r''''.

993. *Experimental verification of this.* — This may be verified

experimentally in the following manner. Let an opaque covering be placed on the surface A B, a small circular aperture being left uncovered at C.

Let a light be moved round the semicircle B O A. This light will enter the aperture C, and will successively illuminate the points of the arc R'''' M r''''.

Commencing from B, it will produce an illuminated spot near R''''; as it is moved successively from B to O, it will illuminate the points successively from R'''' to M; and as it is moved successively from O to A, it will illuminate successively the points from M to r''''.

In the same manner it will be found, that if the luminous point be placed at R'''', its light, after passing from the point C, will fall near B, taking the direction C B. If the light be moved successively over the parts of the arc R'''' M, it will successively illuminate the points of the arc from B to O; and being moved in like manner from M to r'''', it will successively illuminate the points of the arc from O to A.

994. *The angle of incidence at which refraction can take place from a denser to a rarer medium, has a limit which corresponds to that of the angle of refraction in the contrary direction.*— Now a question arises as to what will happen if the light be placed between R'''' and A; for since, being at R'''', the sine of the angle of incidence R'''' P'''', is two-thirds of C B, this sine will be more than two-thirds of C B if the luminous point be placed between R'''' and A; and consequently it would follow, by the law of refraction, that the sine of the corresponding angle of refraction must be greater than the radius B C.

But since no angle can have a sine greater than the radius, it would follow that there can be no angle of refraction, and consequently that there can be no refraction, for a ray which shall make with the refracting surface at C a greater angle of incidence than R'''' C M. What then, it will be asked, becomes of such a ray, as, for example, the ray L C, making an angle of incidence L C M, whose sine L Q is greater than two-thirds of the radius C B?

995. *Total reflection takes place at and beyond this limit.*—The answer is, that such a ray being incapable of refraction at C will be reflected, and that such reflection will follow the common law of regular reflection, so that the ray L C will be reflected in the direction C L', making the angle of reflection L' C M equal to the angle of incidence L C M.

Thus it follows, that all rays which meet the point C, in any direction included between R'''' C and A C, will be reflected from C in corresponding directions between r'''' C and B C, according to the common laws of reflection.

This may be verified by observation; for if the flame of a candle be moved from R'''' to A, it will be seen in corresponding positions by an eye moved in the same way from r'''' to B, and it will be seen with

a splendour of reflection far exceeding that produced by any artificially polished surface.

996. ***Angle of total reflection determines the limit of possible transmission.*** — Hence it is that this species of reflection has been called *total reflection*. The angle R''' C M, which limits the direction of the rays capable of being transmitted from C into the superior medium, and of being reflected, is called the *limit of possible transmission*.

The rays C R''' and C r''' separate the rays which are capable of refraction at C, from those which are reflected at C.

As in the case of glass, the limit of possible transmission is one whose sine is two-thirds of the radius; so in the case of water, it would be three-fourths of the radius, and, in general, it would be an angle whose sine is the reciprocal of the index of refraction.

It follows, therefore, that this limit of possible transmission diminishes as the refracting power of the medium increases.

Since the angle whose sine is $\frac{3}{4}$ is 48° 28', and the angle whose sine $\frac{2}{3}$ is 41° 49', it follows that these are the limits of possible transmission for water or glass into air.

997. *Table showing the limits of possible transmission, corresponding to the different transparent bodies expressed in the first column.*

Names of Media.	Index of Refraction.	Limit of Transmission.
Chromate of lead	2·926	19° 59'
Diamond	2·470	23 53
Sulphur	2·040	29 21
Zircon	2·015	29 45
Garnet	1·815	33 27
Felspar	1·812	33 30
Sapphire	1·768	34 26
Ruby	1·779	34 12
Topaz	1·610	38 24
Flint-glass	1·600	38 41
Crown-glass	1·533	40 43
Quartz	1·548	40 15
Alum	1·457	43 21
Water	1·336	48 28

The properties here described may be illustrated experimentally by the apparatus represented in *fig.* 309.; let *a b c d* represent a glass vessel filled with water or any other transparent liquid. In the bottom is inserted a glass receiver, open at the bottom, and having a tube such as a lamp-chimney carried upwards and continued above the surface of the liquid. If the flame of a lamp or candle be placed

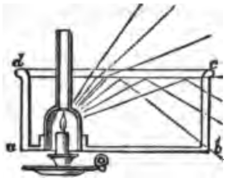

Fig. 309.

in this receiver, as represented in the figure, rays from it penetrating the liquid, and proceeding towards the surface *d c*, will strike this surface with various obliquities. Rays which strike it under angles of incidence within the limits of transmission will issue into the air above the surface of the liquid, while those which strike it at greater angles of incidence, will be reflected, and will penetrate the sides of the glass vessel *b c*.

An eye placed outside *b c* will see the candle reflected on that part of the surface *d c*, upon which the rays fall at angles of incidence exceeding the limit of transmission; and an eye placed above the surface will see the flame, in the direction of the reflected rays, striking the surface with obliquities within the limit of transmission.

CHAP. VIII.

REFRACTION OF PLANE SURFACES.

HAVING explained the principles which determine the change of direction which a single ray of light suffers when it passes from one transparent medium to another, we shall now proceed to show the effects produced by pencils of rays, whether parallel, diverging, or converging, which are incident upon plane surfaces.

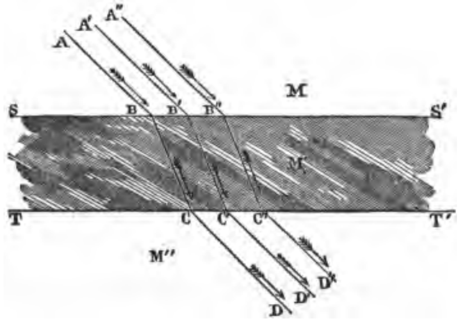

Fig. 310.

998. *Parallel rays.*—If a pencil of parallel rays be incident upon a plane surface S S', *fig.* 310., which separates two refracting media

M and M′, the rays of the pencil, provided they enter the medium M′ at all, will continue to be parallel.

Whether the rays of the pencil enter the medium M′, will be determined by the relative refracting powers of the two media M and M′, and the magnitude of the angle of incidence of the pencil upon the surface S S′.

If the medium M′ be more refracting than the medium M, then the pencil will enter the medium M′, whatever be the angle of incidence; but if the medium M′ be less refracting than the medium M, then the pencil will enter the medium M′ only when the angle of incidence is less than the limit of transmission. If it be greater than that limit, it will be reflected from the surface S S′, according to the common laws of reflection.

If a pencil of parallel rays be incident successively upon parallel plane surfaces separating different media, its rays will, if transmitted at all through them, preserve their parallelism; for, from what has been already proved, the pencil, if parallel in the medium M, will be parallel in the medium M′; and being parallel in the medium M′, it will for the same reason be parallel in the medium M″; and the same will be true for every successive medium through which the pencil passes, provided the surface separating the media be parallel.

But whether the pencil be transmitted at all through the successive media, will depend, as before, upon the relative refracting powers of the media and the angles of incidence. If, for example, at any surface, such as T T′, the medium M″ have less refracting power than the medium M′, the pencil will only enter it provided the angle at which the rays strike the surface T T′ be less than the limit of transmission, otherwise the rays will be reflected.

If a refracting medium M′, bounded by parallel planes, have the same medium at each side of it, as, for example, if the medium M′ be a plate of glass, and the media M and M″ be both the atmosphere, the pencil of rays A B, after passing through the medium M′, will emerge in the direction C D, C′ D′, C″ D″, parallel to the original direction A B, A′ B′, A″ B″, &c.

This has been already proved for a single ray, and will therefore be equally true for any number of parallel rays.

999. *Parallel rays incident on a succession of parallel surfaces.* —If a pencil of parallel rays, after passing through a succession of media bounded by parallel surfaces, be incident upon the surface of a less refracting medium, at an angle greater than the limit of transmission, it will be reflected, and after reflection will return through the several media, making angles with the other surfaces equal to those which it produced passing through them, but on the other side of the perpendicular.

For example, let A B, *fig.* 311., be a ray of the incident pencil, and let it be successively refracted by the media M, M′, M″ in the directions

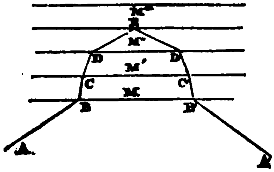

Fig. 311.

B C, C D, and D E; and let it be supposed that, the medium M''' having a less refracting power than the medium M'', the ray D E is incident upon its surface at an angle greater than the angle of transmission.

This ray will consequently be reflected in the direction E D', making an angle with the surface at E equal to that which D E makes with it. The rays E D' and E D, being equally inclined to the surface separating the media M'' and M', will be refracted by the medium M' in the direction D' C', inclined at the same angle as D C to the surface D D', but on the other side of the perpendicular; and in the same way, in passing through the medium M, it will take a direction C' B' inclined to C C' at the same angle as the ray C B is inclined to it. In fine, it will issue from the medium M in the direction B' A', inclined to the surface B B', at the same angle as the incident ray A B is inclined to such surface.

If an eye were placed, therefore, at A', it would see the object from which the ray A B proceeds in the direction A' B', the phenomenon being in all respects similar to that of common reflection.

1000. *Mirage, Fata Morgana, &c. explained.*—These principles serve to explain several atmospheric phenomena, such as Mirage, the Fata Morgana, &c.

In climates subject to sudden and extreme vicissitudes of temperature, the strata of air are often affected in an irregular manner as to their density, and consequently as to their refracting power. If it happen that rays proceeding from a distant object directed upwards after passing through a denser be incident upon the surface of a rarer stratum of air, and that the angle of incidence in this case exceeds the limit of transmission, the ray will be reflected downwards; and if it be received by the eye of an observer, an inverted image of the object will be seen at an elevation much greater than the object itself.

To explain this, let S, *fig.* 312., be an object, which if viewed from E would be seen in the direction E S.

Let M and M' be two atmospheric strata, of which M' is much more rare than M, and let the ray S M be incident upon the surface separating these strata at an angle greater than the angle of transmission. Such ray will in this case be reflected in the direction M E, making with the surface an angle equal to that which S M makes with it. The eye, therefore, will see an image of S, exactly as it would if the surface separating M and M' were a mirror, and consequently the image S' of the object S will be inverted. If no opaque obstacle lie in the line E S. the object S and the inverted image will be seen at the same time; but if any object be interposed between the eye and S, such

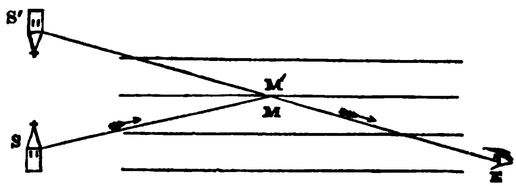

Fig. 312.

as a building, or elevated ground, or the curvature of the earth, then the object S will be invisible, while its inverted image S' will be seen.

It sometimes happens that the reflection takes place from a lower stratum of air towards the eye in an upper stratum, and in such case the inverted image is seen below the object.

1001. *Curious examples of these phenomena.*—Various fantastic optical effects of this kind are recorded as having been observed during the campaign of the French army in Egypt. On this occasion, a corps of savans accompanied the army, in consequence of which, the particulars of the phenomena were accurately observed and explained.

When the surface of the sands was heated by the sun, the land seemed terminated at a certain point by a general inundation. Villages standing at elevated points seemed like islands in the middle of a lake, and under each village appeared an inverted image of it. As the spectator approached the boundary of the apparent inundation, the waters seemed to retire, and the same illusion appeared round the next village.

1002. *Case in which parallel rays are incident successively on surfaces not parallel.*—If a pencil of parallel rays be transmitted successively through several transparent media bounded by plane surfaces which are not parallel, its rays will preserve their parallelism throughout its entire course, whether they strike the successive surfaces at an angle within the limit of transmission or not.

If they strike them at angles within the limit of transmission, they will pass successively through the media, and the preservation of their mutual parallelism may be established by the same reasoning as was applied to parallel surfaces; for the angles of incidence of the parallel rays upon the surface of the first medium being equal, the angles of refraction will also be equal, and therefore the rays through the first medium will be parallel. They will therefore be incident at equal angles on the surfaces of the two media, and the angle of refraction through the strata within the limits of transmission will be also equal, and therefore the rays in passing through the second medium will be parallel; and the same will be true of every successive medium through which the rays would be transmitted. But if they

strike upon the surface of any medium at an angle beyond the limit of transmission, they will be reflected, and being reflected at the same angle at which they are incident, the reflected rays must be parallel. In returning successively through the media they will be subject to the like observation, and will therefore preserve their parallelism whether they be refracted or reflected.

In these observations it is assumed that all the rays composing the parallel pencil are equally refrangible by the same refracting medium, and to such only the above inferences are applicable. It will, however, appear hereafter that certain pencils may be composed of rays which are differently refrangible, a case not contemplated here.

1003. *Refraction by prisms.* — The deflection of a pencil from its original course by its successive transmission through refracting surfaces which are not parallel, is attended with consequences of great importance in the theory of light, and it will therefore be necessary here to explain these effects with some detail.

If two plane surfaces be not parallel, they may be considered as forming two sides of a triangular prism, which is a solid, having five sides, three of which are rectangular, and the two ends triangular. Such a solid is represented in *fig.* 313. A B C and A′ B′ C′ are the triangular ends, which are at right angles to the length of the prism,

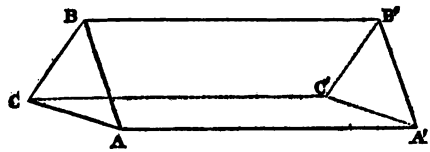

Fig. 313.

and therefore parallel to each other. The three rectangular sides are A B B′ A′, B C C′ B′, and A C C′ A′.

1004. *The refracting angle — designations of prisms.* — The refracting angle of the prism is that angle through the sides of which the refracted light passes. Thus, if the light enter at any point of the side A B B′ A′, and emerge from a point of the side B C C′ B′, then the angle of the prism whose edge is B B′ is called the *refracting angle*, and the opposite side A C C′ A′ is called the base of the prism.

Triangular prisms are distinguished according to the properties of the triangles which form their ends. Thus, if the triangle A B C be equilateral, the prism is said to be equilateral; if it be right-angled, the prism is said to be rectangular; if the sides A B and B C of the refracting angle be equal, the prism is said to be isosceles; and so forth.

1005. *Manner of mounting prisms for optical experiments.* — It is usual to mount such prisms for optical purposes on a pillar, as represented in *fig.* 314., having a sliding tube *t* with a tightening screw, by which the elevation may be regulated at pleasure, and a knee-joint at *g*, by which any desired inclination may be given to the prism.

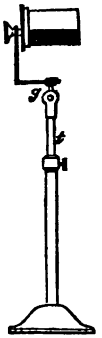

Fig. 314.

By the combination of these arrangements, the apparatus may always be adjusted, so that a pencil may be received in any desired direction with reference to its refracting angle.

If the transparent medium composing the prism be a solid, the prism may be formed by cutting and polishing the solid in the form required; if it be a liquid, the prism may be formed of glass plates hollow, so as to be filled by the liquid.

1006. *Effect produced on parallel rays by a prism.*— Let a pencil of parallel rays be supposed to be incident at O, *fig.* 315., upon one side A B of the refracting angle A B C of a prism. Let it be required to determine under what conditions such a pencil entering the prism and traversing it will be transmitted through the other side B C.

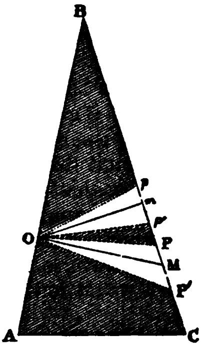

Fig. 315.

We shall here assume that the refracting power of the prism is greater than that of the surrounding medium. This being the case, the pencil incident upon the surface A B will enter the prism, whatever be its angle of incidence. From O draw O M perpendicular to A B, and O *m* perpendicular to B C; draw O P and O P′, making with O M the angles P O M and P′ O M, each equal to the limit of transmission; and also draw the lines O *p* and O *p*′, making angles with O *m* also equal respectively to the limit of transmission. It is evident, from what has been already explained, that in whatever direction the incident ray would fall at O, it will, when refracted, fall within the angle P O P′.

It follows also from what has been explained, that no ray proceeding from O and incident upon the surface B C can be transmitted through it unless it fall between *p* and *p*′, that is, within the angle *p* O *p*′.

It is evident, then, that if these two angles P O P′, and *p* O *p*′ lie altogether outside each other, as represented in *fig.* 315., no ray incident at O could pass through the surface B C; and that, consequently, every such ray must be reflected by such surface. In order that any of the rays transmitted through the prism, and therefore falling within

the angle P O P′, should be transmitted, it would be necessary that the angle p O p', or some part of it, should fall upon or within the angle P O P′.

To determine the conditions which would ensure such a result, we are to consider that the lines O M and O m, which are perpendicular respectively to the sides of the refracting angle, must form with each other the same angle, that is, the angle m O M must be equal to the refracting angle B.

This angle m O M is, as represented in *fig.* 315., equal to the sum of the three angles M O P, m O p', and p' O P. Therefore, the angle p' O P will be equal to the angle m O M, diminished by twice the limit of transmission, because the two angles m O p' and M O P are respectively equal to the limit of transmission.

It follows, therefore, that the angle p' O P, which separates the rays transmitted through the prism from the direction of those rays which it would be possible to transmit through the surface B C, is equal to the difference between the refracting angle B, and twice the limit of transmission. If, therefore, the refracting angle of the prism be greater than twice the limit of transmission, the rays which enter the prism cannot be transmitted through the second surface of the refracting angle, but will be reflected by it. If the angle M O m be equal to twice the limit of transmission, then the commencement O P of the rays which pass through the prism will coincide with the commencement O p' of those rays which it would be possible to transmit through the surface B C. This case is represented in *fig.* 316. In this case, none of the rays which pass through the prism can be transmitted through the surface B C, and the line O P is the limit which separates the two cones of rays, one consisting of the rays which traverse the prism, and the other including those directions which would render their transmission through the second surface possible.

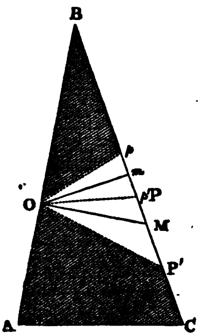

Fig. 316.

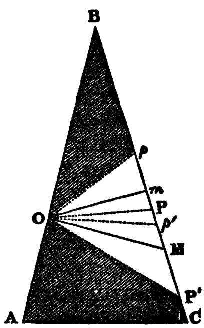

Fig. 317.

If, in fine, the angle m O M, as represented in *fig.* 317., be less than twice the limit of transmission, then a portion of the cone p O p' will lie within the cone P O P′ and all the refracted rays which are included between O P and O p' will fulfil the condition of transmission, and will consequently pass through the surface B C; but all the others which strike the surface B C between P′ and p', will be reflected.

The rays, therefore, incident at the point O, which are capable of being transmitted through the two surfaces B A and B C of the prism, will be those whose angles of refraction are greater than p' O M, and less than P O M.

But if L express the limit of transmission, and B the refracting angle of the prism, we shall have

$$p' \text{ O M} = \text{L} - p' \text{ O P} = \text{B} - \text{L}.$$

The condition, therefore, of transmission at the two surfaces is that the refracting angle of the prism shall be less than twice the limit of transmission, and the rays which in this case are capable of transmission are those whose angles of refraction at the first surface are greater than the difference between the refracting angle of the prism and the limit of transmission.

To explain the course of a ray which, passing through the prism, fulfils these conditions of transmission, let A B C, *fig.* 318., be the refracting angle, and P O the incident ray.

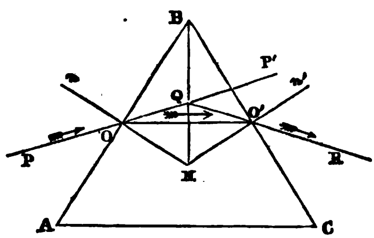

Fig. 318.

The prism being supposed to be more dense than the surrounding medium, or to have a greater refracting power, the ray P O, in passing through it, will be bent towards the perpendicular O N, so that the angle of refraction O′ O N will be less than the angle of incidence at O. Thus the refracted ray will be bent out of its course through the angle Q O O′, which is the first deviation of the refracted ray from its original direction. The refracted ray O O′ being incident on the second surface at O′ at the angle O O′ N, will pass through this surface, and will emerge in the direction O′ R deflected from the perpendicular.

Since O Q is the direction of the original incident ray P O, and Q R the direction of the emergent ray O′ R, it follows that the total deviation of the ray from the original direction produced by the two refractions is the angle P′ Q R.

1007. *Condition on which the deviation of the refracted ray shall be a minimum.* — If the angle of incidence of the original ray P O be

such that the refracted ray O O′ shall make equal angles with the sides of the prism, that is to say, so that the angles B O O′ and B O′ O shall be equal, then the deviation of the emergent ray O′ R from its original direction will be less than it would be for any other angle of incidence of the original ray P O.

In this case it is easy to see that the angles which the incident and emergent rays P O and O′ R make with the sides of the prism, and with the refracted ray O O′, are equal; for since the angles B O O′ and B O′ O are equal, the angles N O O′ and N O′ O are also equal.

But

$$\frac{\sin. \text{P O } n}{\sin. \text{N O O}'} = \text{index of refraction},$$

$$\frac{\sin. \text{R O}' n'}{\sin. \text{N O}' \text{O}} = \text{index of refraction}.$$

But since the angles N O O′ and N O′ O are equal, it follows that the angles P O *n* and R O′ *n*′ are consequently also equal. Therefore the incident and emergent rays make equal angles with the perpendicular to the two surfaces, and therefore with the two surfaces themselves.

It is easy to show experimentally that in this case the deviation of the direction of the emergent from that of the incident ray is a minimum, for the direction of these rays can be determined by observation and the deviation directly measured. By turning the prism on its axis, so as to vary the angle which the first surface makes with the incident ray by increasing or diminishing it, it will be found that the deviation of the direction of the emergent from that of the incident ray will be augmented in whatever way the prism may be turned from that position in which the incident and emergent rays are equally inclined to the sides of the prism.

1008. *How this supplies means of determining the index of refraction.*—Means are thus obtained, by observing the minimum deviation produced upon a ray transmitted through a prism, of determining, by a simple observation, the index of refraction; for the angle of refraction N O O′, being equal to the angle N B O, is one-half the refracting angle of the prism, and the angle of incidence P O *n* is equal to the angle of refraction N O O′, or one-half the angle of the prism, together with the angle O′ O Q, or one-half the deviation O′ Q P′. Thus, if I be the angle of incidence, and R the angle of refraction at the first surface O, and if B be the refracting angle of the prism, and D the angle of deviation, we shall have

$$\text{I} = \tfrac{1}{2}\text{D} + \tfrac{1}{2}\text{B},$$

$$\text{R} = \tfrac{1}{2}\text{B}.$$

Therefore we shall have

$$\frac{\sin. \frac{1}{2}(D + B)}{\sin. \frac{1}{2} B} = \text{index of refraction.}$$

By knowing, therefore, the angle of the prism, and by measuring the angle of minimum deviation, the index of refraction of the material composing the prism can be found.

If the ray transmitted through the prism do not fulfil the conditions of transmission at the second surface, it will be reflected, and will therefore return to the first surface, and pass through it into the medium from which it came, or will return to the base, and be transmitted through it, or reflected by it, according as the angle at which it strikes it is within the limit of transmission or not.

In the case represented in *fig.* 319., the incident ray P O striking upon the surface B C at O′, is reflected by it and passes to the base at O″, through which it is transmitted.

1009. ***Rectangular prism used as reflector.*** — A rectangular isosceles prism of glass is often used for an oblique reflector. Such a prism is represented in *fig.* 320. The sides A B and A C being equal, the angles A B C and A C B must be each 45°. If a parallel pencil of rays, of which P O is one, is incident upon B A perpendicularly, it will enter the medium of the prism without refraction, and will proceed to the surface B C, on which it will be incident at O′ at an angle of 45°. Now, the limit of transmission of glass being but 40°, such a ray must suffer total reflection, and will accordingly be reflected from B C at an angle of 45°, that is, in the direction of O′ R, at right angles to the original direction P O′.

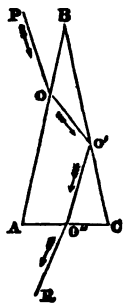

Fig. 319.

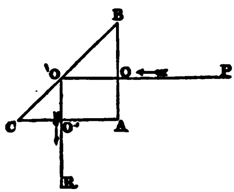

Fig. 320.

An object, therefore, placed at R would be seen by an eye placed at P in the direction P O′, and an object placed at P would be seen by an eye placed at R in the direction R O′.

1010. *Diverging rays refracted at plane surfaces.*—Let I, *figs.* 321., 322., be the focus from which a pencil of diverging rays proceeds, and is incident upon the refracting surface A B C, separating the media M and M′.

Let I B be that ray of the pencil which being perpendicular to the surface is its axis, and will therefore pass into the medium M′ without having its direction changed. Let I D be two other rays equidistant from B falling obliquely on the surface so near the point B as to bring them within the scope of the principle explained in 958. Let D E be the directions of the refracted rays which being continued backwards meet the line B I at R. *Fig.* 321. represents the case in which the medium M′ is more dense than M, and in which, therefore, the refracted rays are deflected towards the perpendicular. *Fig.* 322. represents the case in which the medium M′ is less dense than M, and where, therefore, the refracted rays are deflected from the perpendicular.

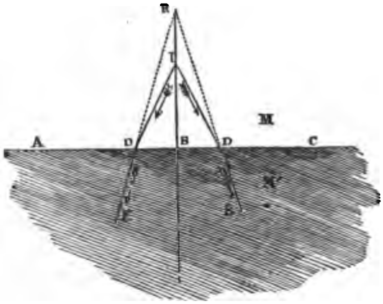

Fig. 321.

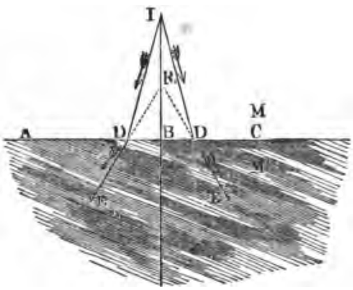

Fig. 322.

In the former case, the point B falls above I, in the latter below it. The point R will then be the focus at which the rays I B and D E, or their continuations, meet. This will therefore be the focus of the refracted rays. The angle D I B which the incident ray makes with the perpendicular I B, is equal to the angle of incidence; and the angle D R B, which the direction of the refracted ray makes with the perpendicular, is the angle of refraction.

Let the distance I B of the focus of incident rays from the surface be expressed by f and R B, that of the focus of refracted rays from the surface by f'.

Since the angles which R D and I D make with R I B are so small as to come within the scope of the principle expressed in 958., we shall have

$$\text{I} = \frac{\text{DB}}{f}, \qquad \text{R} = \frac{\text{DB}}{f'};$$

and consequently,

$$\frac{\text{I}}{\text{R}} = \frac{f'}{f}.$$

But since the angles I and R are small, their sines, by the principle explained in 958., may be taken to be equal to the angles themselves; and, consequently, we shall have, by the common law of refraction, $\frac{\text{I}}{\text{R}}$ equal to the index of refraction n. Thus we shall have

$$\frac{f'}{f} = n, \quad f' = n \times f \quad . \quad . \quad (\text{C}).$$

In this case, n is the index of refraction of the rays proceeding from the medium M to the medium M′, and is consequently greater than 1 when M′ is more dense than M, and less than 1 when M′ is less dense than M.

The formula (C) is equivalent to a statement that the distance of the foci of refraction and incidence from the refracting surface is in the proportion of the index of refraction to 1; that is to say,

$$f' : f :: n : 1.$$

1011. *Convergent rays incident on plane surfaces.* — The cases represented in *figs.* 321. and 322. are those of diverging rays. Let us now consider the case of converging rays. Let the rays E D be incident upon the surface A B C, *figs.* 323., 324., converging to the point I.

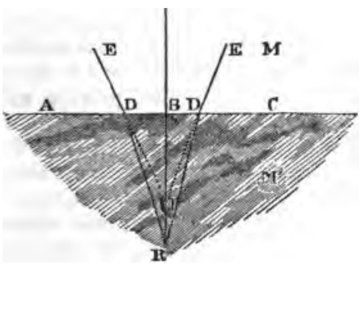

Fig. 323.

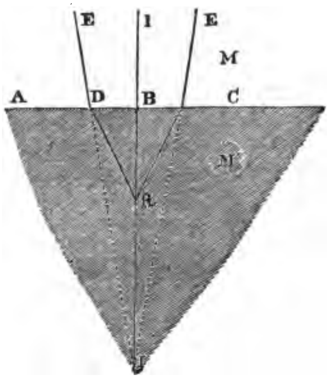

Fig. 324.

If the medium M′ be more dense than M, the rays being deflected towards the perpendicular would meet the axis B I at the point R, more distant than I from B; and if M′ be less dense than M, being deflected from the perpendicular they will meet the axis at the point R, less distant from the surface than I. In this case, the same reasoning will be applicable as in the former, and the same formula (C) for the determination of the relative distances of I and R from B will result.

If I, *figs.* 321., 322., be any point in an object seen by an eye placed within the medium M′, the point I will appear at R, because the rays D E proceeding from it enter the eye as if they came from R. The point will therefore seem to be more distant from the surface A C than it really is in the case represented in *fig.* 321., and less distant in that represented in *fig.* 322.

1012. *Why water or glass appears shallower than it is.* – This explains a familiar effect, that when objects sunk in water are viewed by an eye placed above the surface, they appear to be less deep than they are, in the proportion of 3 to 4, this being the index of refraction for water. If thick plates of glass with parallel surfaces be placed in contact with any visible object, as a letter written upon white paper, such object will appear, when seen through the glass, to be at a depth below the surface only of two-thirds the thickness of the glass, the index of refraction for glass being $\frac{3}{2}$.

If a straight wand be immersed in water in a direction perpendicular to the surface, the immersed part will appear to be only three-fourths of its real length, for every point of it will appear to be nearer to the surface than it really is, in the proportion of 3 to 4. If the wand be immersed in a direction oblique to the surface, it will appear to be broken at the point where it meets the surface, the part immersed forming an angle with the part not immersed.

Let A C, *fig.* 325., represent in this case the surface of the water, and let L B L′ be the real direction of the rod, B L′ being the part immersed. From any point P, draw P M perpendicular to the surface A C, and let M p be equal to three-fourths M P. The point P will therefore appear as if it were at p; and the same will be true for all points of the rod from B to L′. The rod, therefore, which really passes from B to L′, will appear as if it passed from B to l', this line B l' being apparently at a distance from the surface of three-fourths the distance B L′.

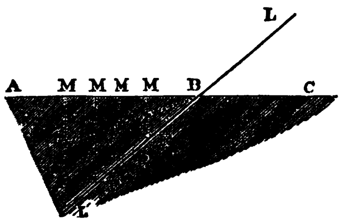

Fig. 325.

1013. *Refracting and refractive power explained.* — Much confusion and consequent obscurity prevails in the works of writers on optics of all countries, arising from the uncertain and varying use of

the terms refracting or refractive power, as applied to the effect of transparent media upon light transmitted through them.

It is evident that if rays of light incident at the same angle on the surfaces of two media be more deflected from their original course in passing through one than in passing through the other, the *refracting power* of the former is properly said to be greater than the *refracting power* of the latter. But it is not enough for the purposes of science merely to determine the *inequality* of refracting power. It is necessary to assign numerically the amount or degree of such inequality, or, in other words, to assign the *numerical ratio* of the refracting powers of the two media.

In some works the *index of refraction* is adopted as the expression of the *refracting power*. Thus the first table in the Appendix to Sir David Brewster's Optics is entitled "Table of *Refracting Powers* of Bodies;" the table being, in fact, a table of the *indices of refraction.*

The correct measure of the refracting power of a medium is, however, not the index of refraction itself, but the number which is found by subtracting 1 from the square of that index. Thus, if n express the index of refraction, n^2-1 would express the refracting power.

This measure of the refracting power is based upon a principle of physics not easily rendered intelligible without more mathematical knowledge than is expected from readers of a volume so elementary as the present. In the corpuscular theory of light, the number n^2-1 expresses the increment of the square of the velocity of light in passing from the one medium to the other; and in the undulatory theory it depends on the relative degrees of density of the luminous ether in the two media. In each case there are mathematical reasons for assuming it as the measure of the refractive power.

Taking the *refractive power* in this sense, it may be expressed for any medium, either on the supposition that light passes from a vacuum into such medium, or that it passes from one transparent medium to another. If the refractive powers of two media be given, on the supposition that light passes from a vacuum into each of them, the refractive power, where light passes from one medium to the other, can be found by dividing their refractive powers from a vacuum one by the other. Thus the refractive power of glass from vacuum being 1·326, and that of water 0·785, the refractive power of glass, in reference to water, will be

$$\frac{1\cdot326}{0\cdot785}=1\cdot690.$$

1014. *Absolute refractive power explained.*—The term "absolute refracting power" has been adopted to express the ratio of the refracting power of a body to its density. Thus, if D express the

density of a medium, and A express its absolute refracting power, we shall have

$$A = \frac{n^2 - 1}{D}.$$

When an elastic fluid or gaseous substance suffers a change of density, its refracting power undergoes a corresponding change, increasing with the density; but in this case the "absolute refracting power" remains sensibly constant, the index of refraction varying in such a manner that $n^2 - 1$ increases or diminishes in the same ratio as the density.

CHAP. IX.

REFRACTION AT SPHERICAL SURFACES.

1015. *The radius of a spherical surface taken as the perpendicular to which all rays are referred.*—It has been already explained that a ray of light incident upon a curved surface suffers the same effect, whether by refraction or reflection, as it would suffer if it were incident upon a plane surface touching the curved surface at the point of incidence; and consequently the perpendicular to which such ray before or after refraction must be referred, will be the normal to the curved surface at the point of incidence. But as the curved surfaces which are chiefly considered in optical researches are spherical, this normal is always the line drawn through the centre of the sphere of which such curved surface forms a part. When a ray of light, therefore, is incident upon any spherical surface separating two media having different refracting powers, its angles of incidence and refraction are those which the incident and refracted rays respectively make with the radius of the surface which passes through the point of incidence.

Thus if A B C, *fig.* 326., be such a surface, of which O is the centre, a ray of light Y P, being incident upon it at P, and refracted in the direction P F, the angle of incidence will be the angle which Y P

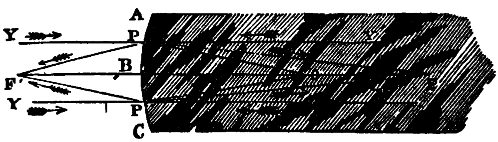

Fig. 326.

makes with the continuation of O P, and the angle of refraction will be O P F. The sine of the angle of incidence will be, according to the common law of refraction, equal to the sine of the angle of refraction multiplied by the index of refraction.

We shall first consider the case of pencils of parallel rays incident on spherical surfaces; and, secondly, that of divergent or convergent rays.

It may be here premised once for all, that in what follows such pencils of rays only will be considered as have angles of incidence or refraction so small as to come within the scope of the principle explained in 958., so that in these cases the angles of incidence and refraction themselves may be substiuted for their sines, and *vice versâ*; and the arcs which subtend these angles, and the perpendiculars drawn from the extremity of either of their sides to the other, may indifferently be taken for each other. The retention of this in the memory of the reader will save the necessity of frequent repetition and recurrence to the same principle.

1016. *Parallel rays.*—Let Y P, *fig.* 326., be two rays of a parallel pencil whose axis is F O B, and which is incident at P upon a spherical surface A B C, whose centre is O.

There are two cases presenting different conditions:

I. When the denser medium is on the concave, and the rarer on the convex side of the refracting surface:

II. When the denser medium is on the convex, and the rarer medium on the concave side of the refracting surface.

1017. *First case. Convex surface of denser medium.*—The rays Y P, *fig.* 326., incident at P, entering a denser medium, will be deflected towards the perpendicular O P, and will consequently meet at a point F beyond O. The angle P O B is equal to the angle of incidence. Let this be called I. The angle O P F is the angle of refraction, which we shall call R.

By the common principles of geometry (Euclid, book 1. prop. 32.), we have

$$\text{R} = \text{I} - \text{B F P}.$$

If the distance B F, of the focus F, from the vertex B be expressed by F, and the radius B O by r, we shall have

$$\text{I} = \frac{\text{B P}}{r}, \text{R} = \frac{\text{B P}}{r} - \frac{\text{B P}}{\text{F}}.$$

But since I is equal to $n \times$ R, we shall have

$$\frac{\text{B P}}{r} = n \times \frac{\text{B P}}{r} - n \times \frac{\text{B P}}{\text{F}}.$$

Omitting the common numerator B P, we shall have

$$\frac{1}{r}=\frac{n}{r}-\frac{n}{F};$$

and consequently

$$F=\frac{n\,r}{n-1}\quad . \ . \ (A).$$

1018. *To find the distances of the principal focus from the surface and the centre.*—By this formula, when the index of refraction n, and the radius r of the surface A B C, are known, the distance of the point F from B can always be computed, as it is only necessary to multiply the radius by the index of refraction, and to divide the product by the same index diminished by 1.

To find the distance of the focus F from the centre O, it is only necessary to subtract from the formula expressing its distance from B, the radius r. Thus we have

$$F\,O=\frac{n\times r}{n-1}-r=\frac{r}{n-1}\quad . \ . \ (B).$$

1019. *Case in which the rays pass from the denser into the rarer medium.*—In the case contemplated above, the rays Y P pass from the rarer to the denser medium. If they pass in the contrary direction, that is to say, in the direction Y′ P, then the index n from the denser to the rarer medium will be less than 1, and the expression for F, formula (A), will be negative, showing that in this case the focus lies to the left of the vertex B at F′. The same formula, however, expresses its distance from B, only that the index of refraction n is in this case the reciprocal of the index for the rays passing in the contrary direction. If, then, we express by n' the index of refraction from the denser to the rarer medium, the distance of F′ from B will be expressed by

$$F'=\frac{n'\times r}{n'-1}.$$

It is easy to show that the distance F′ B of the focus of the rays Y′ P from the vertex B is equal to the distance F O of the focus F of the rays Y P from the centre. To show this, it is only necessary to substitute $\frac{1}{n}$ for n', which is its equivalent, and we find

$$F'=\frac{r}{1-n},$$

which is the same as the expression already found for the distance of F from O, but having a different sign, inasmuch as it lies at a different side of the vertex B

1020. *Relative position of the two principal foci.* — The two foci F and F′ of parallel rays incident upon the refracting surface A B C in

opposite directions, are called the *principal foci*, one F of the convex surface, and the other F′ of the concave surface.

It follows from what has been just proved that the distance of each of these foci from the vertex B is equal to the distance of the other from the centre O.

It follows, also, from what has been here proved, that parallel rays, whether incident upon the convex surface of a denser, or the concave surface of a rarer medium, will be refracted, converging to a point upon the axis in the other medium, determined by the formulæ above obtained.

1021. *Second case. Concave surface of a denser medium.*— The formulæ (A) and (B) are equally applicable to the case in which the denser medium is on the convex side of the surface A B C. It is only necessary, in this case, to consider that the value of n, for the rays Y P, is less than 1. This condition shows that the value of F, given by the formula (A), is negative, and consequently that the focus will lie to the left of the vertex B, as at F′. Now, since the rays Y P, after passing the surface A B C, have their focus at F′, they must be divergent, and the focus F′ will be imaginary.

In like manner, if the rays pass from the rarer to the denser medium, in the direction Y′ P, the value of F will be positive, because in this case n will be greater than 1, and consequently the focus will lie to the right of the vertex B, as at F, the rays diverging from it being those which, by refraction, pass into the medium to the left of the surface A B C. The focus F, therefore, in this case, is also imaginary.

The same *fig.* 326., therefore, will represent the circumstances attending the case in which the denser medium is at the convex side of the surface, the only difference being that in this latter case F is the focus of the rays Y′ P, and F′ the focus of the rays Y P. The distances of F and F′ from B and O respectively will be the same as in the former case.

1022. *Case of parallel rays passing from air to glass, or* vice versâ.—To illustrate the application of the preceding formulæ, let us suppose, for example, that the denser medium is glass, and the rarer air, and that consequently the value of n, for rays passing from the rarer to the denser, is $\frac{3}{2}$, and its value for rays passing from the denser to the rarer is $\frac{2}{3}$.

We have, consequently, in the case represented in *fig.* 326.,

$$\mathrm{FB} = \frac{n\,r}{n-1} = 3\,r;$$

that is to say, the distance of the principal focus of the parallel rays

Y P from B is three times the radius O B, and consequently its distance F O from O is twice its radius.

In like manner, to find the distance F′ B, we have

$$n' = \frac{2}{3},$$

and consequently,

$$F' = -2r;$$

that is to say, the distance F′ B is equal to twice the radius, and is negative, since it lies to the left of B.

In like manner, it will follow that when the surface of the denser medium is concave, B F′ and F O are each equal to twice the radius O B.

1023. *Rays diverging from the principal focus of the convex surface of a denser, or the concave surface of a rarer medium, or converging to the principal focus of the convex surface of a rarer, or the concave surface of a denser medium, are refracted parallel.* — Since the directions of the incident and refracted rays are in all cases reciprocal and interchangeable, it follows that if, in the first case, where the denser medium is on the concave side of the surface, rays are supposed to diverge from either of the foci F or F′, *fig.* 326., they will be refracted parallel to the axis F B in the other medium; and in the second case, if rays be incident upon the refracting surface in directions converging to F or F′, they will be refracted parallel to the axis in the other medium.

It may be asked what utility there can be in considering the case of incident rays converging, inasmuch as rays which proceed from all objects, whether shining by their own light, or rendered visible by light received from a luminary, must be divergent, each point of such objects being a radiant point, which is the focus of a pencil of rays radiating or diverging from it in all directions.

It is true that the rays which proceed immediately from any objects are divergent, and therefore, in the first instance, all pencils of rays which are incident upon reflecting or refracting surfaces are necessarily divergent pencils; but in optical researches and experiments, pencils of rays frequently pass successively from one reflecting or refracting surface to another, and in these cases pencils which were originally divergent, often are rendered convergent, and in this form become pencils incident upon other reflecting or refracting surfaces. In such cases the pencils have imaginary foci behind the surface upon which they are incident, such foci being the points to which they would actually converge if their direction were not changed by the reflecting or refracting surfaces which intercept them.

1024. *Convergent and divergent surfaces defined.* — It appears from the preceding investigation that a spherical refracting surface, having a denser medium on its concave side, always renders parallel rays convergent, in whatever direction they are incident upon it; and

that, on the contrary, a spherical surface, having a denser medium at its convex side, always renders parallel rays divergent in whatever direction they are incident upon it. As these two surfaces possess these distinguishing optical properties, it will be convenient to express the former as a convergent refracting surface, and the latter as a divergent refracting surface.

1025. *Effect of a spherical refracting surface on diverging and converging rays.* — Having explained the conditions which determine the position of the foci of parallel rays incident on spherical reflecting surfaces, we shall now proceed to investigate those by which the focus to which diverging or converging pencils of incident rays are refracted is determined.

Let A B C, *figs.* 327., 328., be a spherical refracting surface, of

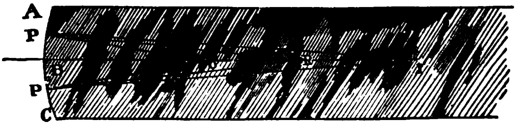

Fig. 327.

Fig. 328.

which the centre is O, and the vertex B. Let I be the focus of the pencil of incident rays, whether diverging or converging; and let R be the conjugate focus of refracted rays, so that the incident pencil may after refraction be converted into another pencil, diverging from or converging to the point R. The angle O P I will be the angle of incidence, and the angle O P R the angle of refraction.

Let the radius B O be expressed as before by r, and let I B and R B be expressed respectively by f and f'.

We shall have, by the principles of geometry,* *fig.* 327.,

$$OPI = BOP - BIP = \frac{BP}{r} - \frac{BP}{f},$$

$$OPR = BOP - BRP = \frac{BP}{r} - \frac{BP}{f'}.$$

* Euclid, Book 1. Prop. 32.

But since the angle of incidence, being small, is equal to the angle of refraction multiplied by the index of refraction, we shall have

$$\frac{\text{B P}}{r}-\frac{\text{B P}}{f}=n\times\left(\frac{\text{B P}}{r}-\frac{\text{B P}}{f'}\right).$$

Omitting the common numerator B P, we shall have

$$\frac{1}{r}-\frac{1}{f}=n\times\left(\frac{1}{r}-\frac{1}{f'}\right).$$

From this we infer,

$$\frac{1}{f}-\frac{n}{f'}=\frac{1-n}{r} \quad . \quad . \quad \text{(C)}.$$

1026. *How to find the focus of refraction when the focus of incidence is given.*—By this formula, when the distance of the focus of incident rays from the vertex, the radius of the surface, and the index of refraction, that is f, n, and r, are known, the position of the focus of refracted rays, that is, its distance f' from the vertex, can always be determined. It is only necessary to observe, that when the value of f' obtained from the formula (C) is positive, it is to be measured to the right of the vertex B, and consequently lies on the concave side of the surface; and that when negative it should be measured to the left of B, and consequently lies on the convex side of the surface.

When the focus of incident rays I lies to the right of B, and therefore on the concave side of the surface, the distance f is positive; but if I lie to the left of B, or on the convex side of the surface, then f in the formula (C) must be taken negatively. The index n is understood in all cases to be the index of refraction of the medium from which the ray proceeds to the medium into which it passes; and is consequently greater than unity when the latter is denser, and less when it is rarer than the former.

With this qualification, the formula (C) will determine the relative position of conjugate foci in every possible case, whether of convergent or divergent rays, and at whichever side of the surface the denser medium may lie.

As an example of the application of this formula, let us take the most common case of a pencil of rays passing from air into glass.

If the pencil be divergent and the refracting surface be convex, as represented in *fig.* 328., the distance of I B, the focus of incident rays, from the vertex, will be negative, and the value of n will be $\frac{3}{2}$. Hence the formula (C) will become

$$-\frac{1}{f}-\frac{3}{2f'}=\frac{-1}{2r}.$$

From whence we infer,

$$f' = \frac{3\,f \times r}{f - 2\,r} \quad . \quad . \quad (\text{D}).$$

If I B, or f, therefore, be greater than twice the radius, f' will be positive, and will therefore lie within the surface A B C at a distance from B determined by the formula (D). In this case the rays diverging from I, *fig.* 328., will be made to converge after refraction to R.

But if the distance I B or f be less than twice the radius, then the preceding value of f' will be negative, and must consequently be taken to the left of B, as at R′, *fig.* 328. Consequently, in this case, rays after refraction will diverge, as if they had proceeded from R′.

In fine, if I B be equal to $2\,r$, then the value of f' will be infinite, which indicates that in such case the refracted rays are parallel, their points of intersection being at an infinite distance.

By like reasoning, the position of the focus of refracted rays which corresponds to every other variety of position of the focus of incident rays may be determined.

Principal and secondary pencils. — In the preceding observations, the focus of incident rays is supposed to be placed upon the axis of the spherical surface. Such pencil is, as in the case of reflectors, called the *principal pencil*, and the axis the *principal axis*.

When the focus of a pencil of rays is not on the axis of the refracting surface, or if it be a parallel pencil when its rays are not parallel to such axis, it is called a *secondary pencil;* and its axis, which is the ray passing through the centre of the refracting surface, is called a *secondary axis*.

The focus of refracted rays of a secondary pencil lies upon its axis, and is determined in the same manner as in the case of a principal pencil. The rays, however, from such a pencil will only be refracted to the same point provided the distance of its extreme rays from the axis, measured on the spherical surface, does not exceed a few degrees. If the rays be refracted beyond this limit, they will not be collected into a single point, but will, as in the case of reflectors, be dispersed over a certain space, and produce an aberration of sphericity.

CHAP. X.

PROPERTIES OF LENSES.

1027. *Lens defined.* — When a transparent medium is included between two curved surfaces, or a curved surface and a plane surface, it is called a *lens*.

Lenses are of various species, according to the characters of the curved surfaces which bound them; but those which are almost exclusively used in optical instruments and in optical experiments, are bounded by spherical surfaces, and to these, therefore, we shall here limit our observations.

Spherical surfaces, combined with each other and with plane surfaces, produce the following six species of lens, which are denominated converging and diverging lenses, because, as will be explained hereafter, the first class render a pencil of parallel rays incident upon them convergent, and the second class render such a pencil divergent.

1028. *Three forms of converging lenses, — meniscus, double convex, and plano-convex.* — Converging lenses are of the three following species:—

I. *The meniscus.* The form of this lens may be conceived to be produced as follows: —

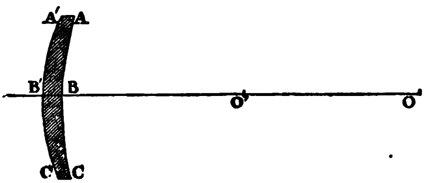

Fig. 329.

Let A B C and A′ B′ C′, *fig.* 329., be two circular arcs, whose middle points are B and B′, and whose centres are O and O′, the radius O B being greater than the radius O′ B′. Let the two arcs be supposed to revolve round a line O O′ B B′ as an axis, and they will in their revolution produce a solid of the form of the meniscus lens.

It is evident from this that the convexity A′ B′ C′ of such a lens is greater than its concavity A B C, the radius O′ B′ of the convexity being less than the radius O B of the concavity.

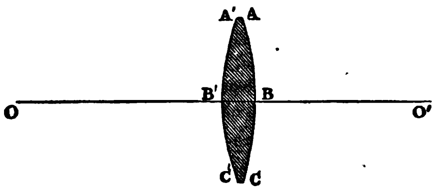

Fig. 330.

II. *Double convex lens.* The form of this lens may in like manner be conceived to be produced as follows:—

Two circular arcs, A B C and A′ B′ C′, *fig.* 330., whose middle points are B and B′, and whose centres are O and O′, being conceived to revolve round a line O B′ B O′ as an axis, will, by their revolution, produce the form of this lens. The convexities of the sides will be equal or unequal according as the radii O B and O′ B′ are equal or unequal.

III. *Plano-convex lens.* The form of this lens may be conceived to be produced as follows: —

Let A′ B′ C′, *fig.* 331., be a circular arc, whose middle point is B′,

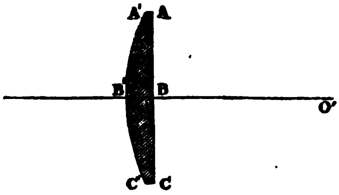

Fig. 331.

and whose centre is O'; and let A B C be a straight line at right angles to B' O', whose middle point is B. If a figure thus formed revolve round the line O' B' as an axis, it will produce the form of a plano-convex lens, the side A B C being plane, and the side A' B' C' being convex.

1029. *Three forms of diverging lenses, — concavo-convex, double concave, and plano-concave.* — Diverging lenses are of the three following species: —

I. *Concavo-convex lens.* To form this lens, as before, proceed as follows:—

Let A B C and A' B' C', *fig.* 332., be two circular arcs, whose middle points are B and B', whose centres are O and O', and whose radii are O B and O' B'; the latter being greater than the former. If this be

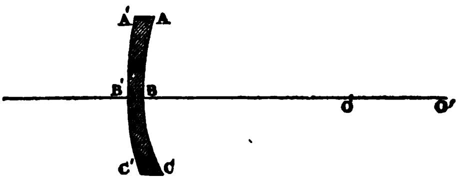

Fig. 332.

supposed to revolve round the line O' O B B' as an axis, it will produce the form of a concavo-convex lens. Since the radius of the concave side A B C is less than the radius of the convex side A' B' C', the concavity will be greater than the convexity.

II. *Double concave lens.* The form of this lens may be supposed to be produced as follows:—

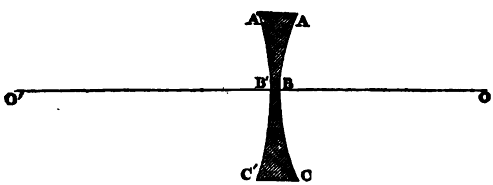

Fig. 333.

Let A B C and A' B' C', *fig.* 333., be two circular arcs, whose middle points are B and B', and whose centres are O and O'. Let this figure be supposed to revolve round the line O O' as an axis, and it will pro-

duce the form of a double concave lens. The concavities will be equal or unequal, according as the radii O B and O′ B′ are equal or unequal.

III. *Plano-concave lens.* This lens may be conceived to be produced as follows:—

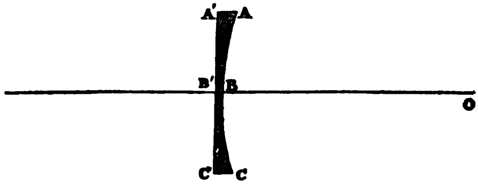

Fig. 334.

Let A B C, *fig.* 334., be a circular arc, whose middle point is B, and whose centre is O. Now let A′ B′ C′ be a straight line perpendicular to O B, whose middle point is B′. Let this figure be supposed to revolve round O B B′ as an axis, and it will produce the form of a plano-concave lens.

1030. *The axis of a lens.*—In all these forms of lens the line O B B′ is called the *axis of the lens.*

1031. *The effect produced by a lens on incident rays.*—To determine the effect produced on a pencil of rays by a lens, we shall first take the case of the meniscus.

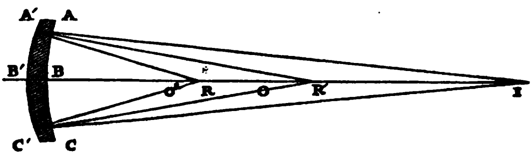

Fig. 335.

Let O, *fig.* 335., be the centre, and O B the radius of the concave surface A B C. Let O′ be the centre, and O′ B′ be the radius of the convex surface A′ B′ C′. Let I be the focus of a pencil of rays incident upon the surface A B C. Let R′ be the focus to which the rays of this pencil would be refracted by the surface A B C, independently of the surface A′ B′ C′.

The pencil whose focus is this point R′ will then be incident upon the second surface A′ B′ C′ of the lens, and the rays from this pencil being again refracted by the second surface will have another focus R, which will be the definitive focus of the rays after refraction by both surfaces of the lens.

In this, and in all other cases of lens, it will be necessary that the

thickness B B' of the lens may be disregarded, being inconsiderable compared with the other magnitudes which enter into computation.

Now let the distances of the foci I, R', and R from the middle point B or B' of the lens be expressed respectively by f, f'', and f'; and let the radii O B and O' B' be expressed by r and r'; we shall then have, by what has been already explained respecting refracting surfaces, the following conditions:

$$\frac{1}{f} - \frac{n}{f''} = \frac{1-n}{r}.$$

$$\frac{1}{f''} - \frac{n'}{f'} = \frac{1-n'}{r'}.$$

In this case n is the index of refraction from air into the medium of the lens, and n' is the index of refraction from the medium of the lens into air.

By what has been already explained, these two indices are reciprocals, and consequently their product is equal to unity, so that we shall have $n \times n' = 1$.

Now if we multiply the latter equation by n, we shall have

$$\frac{n}{f''} - \frac{n \times n'}{f'} = \frac{n - n \times n'}{r'};$$

but since $n \times n' = 1$, this will become

$$\frac{n}{f''} - \frac{1}{f'} = \frac{n-1}{r'};$$

by combining this with the first equation we shall have

$$\frac{1}{f} - \frac{1}{f'} = \frac{1-n}{r} - \frac{1-n}{r'} \quad . \quad . \quad (\text{E}).$$

By these conditions the distance f' can always be determined when f, r, r', and n are known; that is to say, the position of the focus of refracted rays can always be determined when the position of the focus of incident rays, the radii of the lens, and the index of refraction are known.

This formula (E), by a due attention to the signs of the quantities which compose it, may be applied to lenses of every species. If the focus of incident rays lie to the right of the lens, as in *fig.* 335., f must be taken to be positive; if to the left of the lens, f must be taken negatively. If the centre of either surface lie to the right of the lens, the radius will be taken positively; and if to the left of the lens, it will be taken negatively. If one of the surfaces of the lens be a plane surface, it may be considered as having an infinite radius; and accordingly, the term of the equation (E) in the denominator of which such radius enters will become equal to 0, and will therefore disappear from the equation.

When the value of f', which determines the distance of the focus

of refracted rays from B, will have been found by the equation (E), it must be taken to the right of the point B if it be positive, and to the left if it be negative.

1032. *To determine the principal focus of a lens.* — If the incident rays whose focus is I be refracted parallel, then the distance f' of the focus of refraction from B will be infinite, and consequently, we shall have $\frac{1}{f'} = 0$. Now, in this case, I will be the *principal focus* for parallel rays incident upon the surface A′ B′ C′. Let this be expressed by F, and we shall have by the equation (E)

$$\frac{1}{F} = \frac{1-n}{r} - \frac{1-n}{r'},$$

from which we infer,

$$F = \frac{r\,r'}{(1-n)\,(r'-r)} \quad . \quad . \quad (F).$$

a formula by which the distance of the focus of parallel rays incident upon A′ B′ C′ can always be calculated.

If the incident rays be parallel, their focus I will be at an infinite distance, and we shall have $\frac{1}{f} = 0$. In this case, the focus R will be the principal focus of the parallel rays incident upon the surface A B C.

Let the distance of this focus from B be expressed by F′, and we shall find as before from equation (E),

$$F' = -\frac{r\,r'}{(1-n)\,(r'-r)} \quad . \quad . \quad (G).$$

Thus it appears that F and F′ differ in nothing save in their sign, the one being positive and the other negative; the inference from which is, that parallel rays, whether incident on the one or the other surface of a lens, will be refracted to points equally distant from the lens, but on opposite sides of it.

1033. *The focal length of a lens.* — The common distance of these principal foci from the lens is called the *focal distance* or *focal length of the lens.*

1034. *The meniscus, double convex, and plano-convex, are convergent lenses.* — If the lens be a meniscus, and composed of a refracting substance more dense than air, it will render a parallel pencil incident upon either of its surfaces convergent, and its principal foci will consequently be real. This follows as a consequence from the formulæ (F) and (G); for in the case of a meniscus, r' is less than r, and, consequently, the value of F given by the formula (F) is positive, and the value of F′ given by the formula (G) is negative; consequently, the focus of parallel rays incident upon A′ B′ C′ lies to the

right of the lens, and the focus of parallel rays incident on A B C lies to the left of it. Parallel rays are therefore rendered convergent after refraction, and the foci are real in whichever direction they may pass through such a lens.

It is easy to show, that the same will be true for double convex and plano-convex lenses. In the case of double convex lenses, the radius r is negative and r' positive; the consequence of which is, that the value of F is positive, and F′ negative. In the case of plano-convex lenses, the radius r is infinite, and the formulæ (F) and (G) become

$$F = +\frac{r'}{n-1}, F = -\frac{r'}{n-1}.$$

Thus it appears, that in all the three forms of convergent lens, parallel rays, whether incident on the one surface or on the other, are refracted, converging to a focus on the other side of the lens; and the foci in all such cases are consequently real.

1035. *Concavo-convex, double concave, and plano-concave, are divergent lenses.* — It is easy to show, by the same formulæ, that parallel rays incident on every species of divergent lens will be refracted diverging from a point on the same side of the lens as that at which they are incident.

In the case of the concavo-convex lens, the radius r' is greater than the radius r; and since n is greater than 1, the value of F (given in the formula F) will be negative, and the value of F′ (given in the formula G) positive. Thus it appears that the principal focus of parallel rays incident on the surface A B C, *fig.* 332., will be to the right of B, and the principal focus of the rays incident on the surface A′ B′ C′ to the left of B, the foci in each case being at the same side of the lens with the incident rays; and, consequently, being in such case imaginary.

In the case of the double concave lens, the radius r' is negative; and since n is greater than 1, the value of F will be negative, and that of F′ positive.

In the case of the plano-concave lens, the value of r' is infinite, and since n is greater than 1, F will be negative, and F′ positive.

Thus it appears that in all the forms of divergent lenses, parallel rays incident upon their surfaces are refracted, diverging from a focus on the same side of the lens as that at which they are incident.

It is from this property that the two classes of convergent and divergent lenses have received their denomination; and it is evident, therefore, that the meniscus and plano-convex lens are optically equivalent to a double convex lens, and that the concavo-convex and plano-concave lens are optically equivalent to a double concave lens.

1036. *Case of a lens with equal radii and convexities in the same direction.* — Among the varieties presented by the preceding formulæ,

there is an exceptional case which requires notice. If the radii of the two surfaces of a lens be equal, and their centres be both at the same side of the lens, the lens will hold an intermediate place between a meniscus and a concavo-convex. In the former, the radius of the convex surface is less than that of the concave surface; and in the latter, the radius of the concave surface is less than that of the convex surface. These radii might, however, be in each case as nearly equal as possible, the lenses actually retaining their specific characters. Each species, therefore, would approach indefinitely to an intermediate lens whose surfaces would have equal radii.

It is evident that the condition which would render equal the radii r and r', and give them the same sign, would render both the focal distances F and F′ infinite, their denominators being nothing.

To comprehend this it is only necessary to consider that in the case of the meniscus and the concavo-convex lens, the more nearly equal the radii r and r' are, the less will be the denominators of the values of F and F′; and, consequently, the greater will be these values themselves, and if we suppose the difference between the radii to be infinitely diminished, the values of F and F′ will be infinitely increased. These conditions lead to the inference that if the radii of the two surfaces be equal, the focus of parallel rays incident upon these two surfaces will be infinitely distant from the lens, that is to say, parallel rays will be refracted parallel.

Thus it appears that a lens formed by spherical surfaces, whose radii are equal, and whose centres lie at the same side of the lens, will have no effect on the direction of rays proceeding through it, and that such lens will be equivalent to transparent plates with parallel surfaces.

An example of such a lens as this is presented in the usual form of a watch-glass.

1037. *Lenses may be solid or liquid.*—Lenses may be composed of any transparent substance, whether solid or liquid.

If they be composed of a solid, such as glass, rock crystal, or diamond, they must be ground to the required form, and have their surfaces polished; if they be composed of liquid, they must then be included between two lenses, such as has been just described, having themselves no refracting power, and having the form required to be given to the liquid lens.

Thus, two watch-glasses, placed with their concavities towards each other, and so inclosed at the sides as to be capable of holding a liquid, would form a double convex liquid lens. If their convexities were presented towards each other, they would form a double concave liquid lens.

1038. *Rules for finding the focal length of lenses of glass.*—The material almost invariably used for the formation of lenses in optical instruments being glass, it will be useful here to give the

principal formulæ, showing the position of the focus in lenses of this material.

In the case of glass, the index of refraction, the incident rays being supposed to pass from air into that medium, is $\frac{3}{2}$: the formulæ (E) and (F) therefore, in this case, become

$$\frac{1}{f} - \frac{1}{f} = \frac{1}{2r'} - \frac{1}{2r} \quad . \quad (E').$$

$$F = \frac{2rr'}{r - r'} \quad . \quad . \quad . \quad (F').$$

By the latter formula, the focal length of a glass lens can always be found.

In its application, however, it is necessary to observe that when the convexities of the surface of the lens are turned in opposite directions, as in the cases of double convex and double concave lenses, the denominator will be the *sum* of the radii; and if they are turned in the same direction, as in the case of the meniscus, and the concavo-convex lens, it will be the *difference* of the radii. The following general rule will always serve for the determination of the focus when both surfaces of the lens are spherical

RULE.

Divide twice the product of the radii by their difference for the meniscus and concavo-convex lenses, and by their sum for the double convex and double concave lenses. The quotient will in each case be the focal length sought.

To find the focus of a plano-convex or a plano-concave lens, we are to consider that it has been already proved that the focal length is given by the formula

$$F = \frac{r}{n - 1}.$$

and since n is $\frac{3}{2}$, we shall have

$$F = 2r;$$

that is to say, the focal length of a plano-convex or plano-concave lens is double the radius of the convexity or concavity.

If a double convex or double concave lens have equal radii, then the formula (F') becomes

$$F = r.$$

The focal length, therefore, of such a lens is equal to the radius of either surface.

For the same class of lens the formula (E′) becomes

$$\frac{1}{f} - \frac{1}{f'} = \frac{1}{r};$$

where r expresses the common magnitude of the radii of the two surfaces. From this we infer,

$$f' = \frac{rf}{r - f};$$

which supplies the following rule for finding the focus of refracted rays, when the focus of incident rays is given.

RULE.

Multiply the common radius of the two surfaces by the distance of the focus of incident rays from the lens, and divide the product by the difference between the radius and the distance of the focus of incident rays from the lens.

If the distance of the focus of incident rays from the lens in this case be less than the radius, the value of f' will be positive, and the focus of refracted rays will lie at the same side of the lens with the focus of incident rays; but if the value of f be greater than r, then the value of f' will be negative, and the focus of refracted rays will lie at the other side of the lens.

1039. *Case of secondary pencils.*—We have here considered those cases only in which the focus of the incident pencil is placed upon the axis of the lens, or of pencils whose rays are parallel to that axis. The focus of the refracted rays may, however, be determined by the same formula for secondary pencils whose axes, passing through the centre of the lens B, are inclined to its axis, provided only the incli nation be not so great as to produce such spherical aberration as may prevent the rays from having an exact, or nearly exact, focus.

1040. *Field of a lens. Opening of a lens.*—If X X′, *fig.* 336., be the axis of the lens, and Y Y be the greatest angle at which the axis of the secondary pencil can be inclined to X X′, so that the rays may have a nearly exact focus, the angle included between the two secondary pencils Y Y′ is called the *field of the lens.*

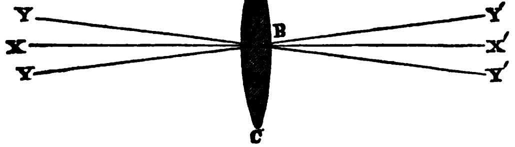

Fig. 336.

The angle formed by lines drawn from the edge of the lens to its principal focus is called the *opening of the lens;* and this opening cannot in general exceed 10° or 12° without producing an aberration of sphericity, which would prevent the rays of the pencil incident upon it from having an exact focus.

1041. *Images formed by lenses.*—The images of objects formed by lenses are explained upon the same principles as have already been applied to the case of spherical surfaces. If an object, whether it be self-luminous like the sun, or receive light from a luminary like the moon, be placed before a lens, each point upon its surface may be considered as a point from which light radiates in all directions. Such a point will be then the focus of a diverging pencil incident upon the lens, the base of the pencil being the surface of the lens.

If the pencils which thus diverge from all points of the object be rendered, after refraction by the lens, convergent, they will have real foci on the other side of the lens, and the assemblage of such foci will form an *image* of the object. But if these pencils, after passing through the lens, be divergent, their foci will be imaginary, and will be placed at the same side of the lens with the object.

These pencils would in such case be received by an eye on the other side of the lens as if they had originally proceeded from these points, which are the foci of the refracted pencils.

The assemblage of these points would thus form an *imaginary image.*

All these circumstances are analogous to those which have been already explained in the case of reflectors. They will, however, be rendered still more intelligible by explaining their application to glass lenses.

1042. *Every lens, whatever be its form, can be represented by a double convex or double concave lens with equal radii.*—Since all converging lenses, having equal focal lengths, are optically equivalent, a double convex lens with equal radii can always be assigned, which is the optical equivalent of any proposed converging lens, whether it be meniscus, double convex with unequal radii, or plano-convex.

Since, in like manner, all diverging lenses having equal focal lengths are optically equivalent, a double concave lens with equal radii may always be assigned, which is the optical equivalent of any proposed diverging lens, whether it be concavo-convex, double concave with unequal radii, or plano-concave.

1043. *Image formed by double convex lens.*—It will therefore be sufficient to investigate the effects of double convex and double concave lenses with equal radii.

Let A B C, *fig.* 337., therefore, be a double convex lens, with equal radii; and let L M be an object, the centre of which is upon the axis of the lens, and placed beyond the principal focus F. Let the distance of this object from B be expressed by f; let the distance of its image

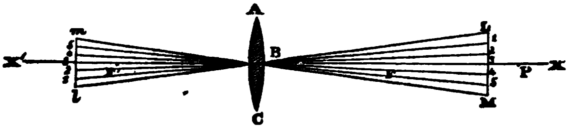

Fig. 337.

be f', and the focal length of the lens, or its radius, be r. By what has been already explained, we shall have

$$\frac{1}{f'} = \frac{1}{f} - \frac{1}{r};$$

and, therefore,

$$f' = \frac{r \times f}{r - f}$$

Since the distance of the object from the lens is supposed to be greater than B F, we shall have f greater than r; and consequently f' will be negative, which indicates that the image of L M will lie on the other side of the lens.

It appears also, by the preceding formula, that the distance f' of the image from the lens will be greater than r, and the image $l\,m$ will therefore lie beyond the point F'.

If we draw L B l, this line will be the secondary axis of the pencil whose focus is at L, and consequently the focus of refracted rays will be at l; so that an image of the point L will be formed at l. In like manner it may be shown, that an image of the point M will be formed at m; and in like manner the images of all the points of the object, such as, 1, 2, 3, 4, 5, between L and M, will be formed at corresponding points 1, 2, 3, 4, 5 between l and m. It is evident, therefore, that in this case the image will be inverted.

1044. *Conditions which determine the magnitude of the image.*—Since the axis of the extreme secondary pencils L l and M m intersect at the centre of the lens, we shall have the following proportion:—

$$LM : lm :: LB : lB;$$

or, which is the same,

$$LM : lm :: f : f';$$

that is to say, the magnitude of the object is to that of its image, as their distances respectively from the lens. The image, therefore, will be greater, equal to, or less than the object, according as f' is greater, equal to, or less than f.

Now, it appears by the preceding formula, that if f be equal to 2 r, f' will also be equal to 2 r; that is, if the distance of the object from the lens be twice the focal length of the lens, the distance of

the image on the other side of the lens will also be twice the focal length; the image, therefore, in this case will be equal to the object.

An object L M being moved towards F, so as to have less distance from the lens than twice its focal length, the value f' given by the preceding formula becomes greater than f, and increases as the object approaches F. It appears, therefore, that as L M approaches F, its image $l\,m$ recedes from F′ in the direction F′ X′, and that consequently the distance of the image from the lens being greater than that of the object, its magnitude will be greater in the same proportion.

When the object L M comes very near F, the denominator in the preceding formula becoming very small, the magnitude of f' becomes very great, and if we suppose L M to arrive at F, then the denominator becomes O, the value of f' becomes infinite, and consequently the image $l\,m$ would recede to an infinite distance from the lens, and in effect cease to exist.

This circumstance is only what might have been anticipated; for when an object L M is placed at F, a pencil of rays proceeding from any point in it, such as L, will, after passing through the lens, become parallel; and having no point of convergence, it cannot form an image of the pencil L; and the same will be true of any point in the object.

But although no image of the object thus placed in the principal focus of the lens will thus be formed, the lens, in such a position with reference to the object, is attended with optical effects of great importance, which will be explained hereafter.

Let us again suppose an object L M placed at a distance from the lens equal to twice its focal length, and the image $l\,m$ placed at an equal distance at the other side of the lens. If we now suppose the object, instead of approaching F, to recede from it in the direction F X, the distance f being greater than $2\,r$, it follows that the magnitude of f' will be less than $2\,r$; and the farther the object L M is removed from the lens, and consequently the more its distance from the lens exceeds twice the focal length, the nearer will the image $l\,m$ approach to F′.

1045. *Images of very distant objects are formed at the principal focus.* — If we suppose the object L M to recede to a distance actually infinite, then the value of f' would become equal to r, and the image $l\,m$ would coincide with the principal focus F′. This would lead to the inference that the image of a distant object can never be in the principal focus of the lens, because such a supposition would involve the condition that the distance of the object must be actually infinite.

But, practically, it is found, that if the diameter of the lens bear an insignificant proportion to the distance of the object from it, the image of the object will be formed at its principal focus. This is easily explained by reference to the conditions which determine the position of the principal focus.

It will be recollected, that the point F′ is the focus to which paral-

lel rays incident upon the lens A B C would be made to converge. Now, if L M be so distant from the lens that the rays proceeding from any point upon it, such as L, and incident upon the lens, may be considered as parallel, these rays will converge to the principal focus.

This will be the case, provided that the diameter of the lens, which is that of the base of the pencil, is incomparably less than the distance of the object from the lens. Thus, let us suppose the diameter of the base to be 4 inches, and the distance of the object to be 80 feet; the distance will in this case be 240 times the diameter of the lens, and the angle of divergence of the pencil would consequently be less than a quarter of a degree. A pencil which has such a divergence as this would be refracted to a point not sensibly different from the principal focus.

It is evident, that as an object recedes to an increasing distance from the lens, the image approaching the principal focus F′ diminishes in magnitude in proportion to the distance from the lens; and when the image is formed at the principal focus, the lines *l* B, *m* B drawn from its extremities to the centre of the lens will form an angle equal to that which would be formed by lines drawn from the extremities of the object to the same point.

1046. *Experimental illustrations.*—All these circumstances admit of easy experimental verification. Let P be a point on the axis at a distance from B equal to 2 B F, so that P F shall be equal to B F.—Let the flame of a candle be held at L M between F and P, the lens A C being inserted in an aperture formed in a screen so as to exclude the light of the candle from the space to the left of the lens. If a white screen be held at right angles to the axis and behind the lens, and be moved to and fro, until a distinct inverted image of the candle shall be seen upon it, its distance from the lens when this takes place will be found to be greater than twice the focal length, and to correspond exactly with that which would be computed by the preceding formula. If the candle be moved towards P, the image will become indistinct upon the screen, but will recover its distinctness by moving the screen towards F′; and if the candle be placed at P, the screen being placed at a distance from B equal to twice B F′, a distinct image will be formed on the screen equal in magnitude to the object. If the candle be moved from P towards X, the screen must be moved towards F′ to preserve the image distinct; and if the candle be gradually moved in the direction P X, the screen must be continually moved towards F′. If the candle be moved to so great a distance from the lens that the diameter of the lens shall have an insignificant proportion to its distance, then a distinct image will be formed on the screen placed at the principal focus F′. If the candle be placed at the principal focus F, then the screen will show no image of it in whatever position it may be placed behind the lens, but will exhibit merely an illuminated disk formed by parallel rays composing the refracted

pencils into which the pencils proceeding from such point of the candle are converted by the lens.

Let us now suppose such object placed at L M, *fig.* 338., between the principal focus F and the lens. In this case, f being less than r, the

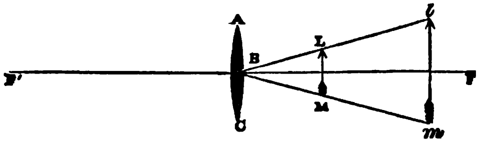

Fig. 338.

value of f' obtained by the preceding formula will be positive, and, consequently, the focus of refracted rays will lie at the same side of the lens with the focus of incident rays. If then the pencil of rays diverging from L pass through the lens, it will after refraction diverge from the point l, more distant from the lens than L. In like manner, the pencil diverging from M will after passing through the lens diverge from m; and the same will be true of all the intermediate points of the object, so that the various pencils which diverge from different points of the object and pass through the lens will after refraction diverge from the corresponding points of $l\,m$. The image, therefore, in this case will be imaginary, and an eye placed to the left of the lens A B C would receive the rays of the various pencils as if they diverged, not from a point of the object L M, but from points of the imaginary image $l\,m$.

The magnitude of the image in this case will be greater than the object in the same proportion as l B is greater than L B.

As the object L M is moved towards F, its distance f from the lens will approach to equality with r, and the denominator of f' in the preceding formula diminishes, and consequently the distance of its image from the lens will be proportionally increased; therefore, as the object L M is moved towards F, its image $l\,m$ will recede indefinitely from the lens, and would become infinite in distance and magnitude when the object arrives at F, which is consistent with what has been already explained of the principal focus.

It appears, therefore, that whenever the object is between the principal focus and the lens, its image will be at a greater distance from the lens on the same side of it, and will be erect, imaginary, and greater than the object.

1047. *Images formed by concave lenses.*—If an object L M, *fig.* 339., be placed before a double concave lens A B C, the focus corresponding to the several points of the object will lie between the object and the lens, at distances determined by the formula

$$f' = \frac{r \times f}{r + f}$$

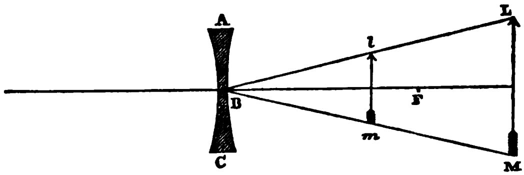

Fig. 339.

It is evident from this formula that f' is less than f, and that consequently the distance of the image *l m* from the lens is less than the distance of the object from it. It appears also that the distances f and f' increase and diminish together, so that when the distance of an object from the lens L M is augmented, the distance of its image *l m* will also be augmented. But the distance of the image from the lens can never be greater than the focal length of the lens, because, as the distance of the object is indefinitely increased, the value of f' obtained from the formula approaches indefinitely to equality with r, though it can only become equal to it when the distance of the object becomes infinite.

1048. *Spherical aberration.* — We have hitherto considered that the pencils of rays proceeding from the lens were brought to an exact focus, and this would be practically the case if the angles of incidence of the extreme rays of the pencils do not exceed a certain limit; but if, from the magnitude of the lens, or the proximity of the object, this be not the case, effects will be produced which have been called *spherical aberration*, which it will be necessary here more clearly to explain.

Let A B C, *fig.* 340., be a plano-convex lens, having its plane side

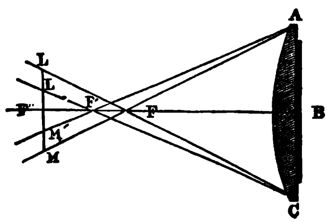

Fig. 340.

Fig. 341.

presented to the incident rays. Let a circular disk of card or sufficiently thick paper be formed, a little less in diameter than the lens, and let it be attached concentrically with the lens upon the plane side, so as to leave a narrow ring of the glass uncovered round the edge of the lens, as represented in *fig.* 341.

If this lens be now presented to a distant object, such as the sun,

None but the extreme rays of each pencil will pass through it, and an image will be formed of the sun by these extreme rays at F, which will therefore be the principal focus of an annulus of parallel rays passing through the edge of the lens. Now let another circular piece of paper or card be cut so as to cover an annular surface surrounding the edge of the lens, and another to cover the central portion of it, so as to leave a ring of the surface uncovered at some distance within the edge, as represented in *fig.* 342. The lens being again presented to the sun, it will be found that an image will be formed at F′, *fig.* 340., somewhat more distant from the lens than F.

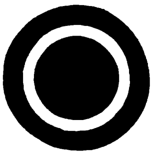

Fig. 342.

Fig. 343.

If, in fine, a disk of card be cut, equal in magnitude with the lens, having a small circular aperture at its centre, as represented in *fig.* 343., and be in the same manner attached to the lens, so as to allow only the central rays of each pencil to pass, an image of the sun will be formed at F″, *fig.* 340., still further from the lens.

It appears, therefore, that those rays of the pencil which are nearest the centre will have a focus further from the lens than those which are more distant from it, and the more distant the rays of each pencil are from the axis of the lens, the nearer their focus will be to the lens.

If the lens being uncovered be therefore presented to the sun, the rays incident near its edge will be refracted to the focus F, and after passing that focus will diverge in the direction F L and F M. The rays incident nearer to the centre will intersect at F′, and will diverge to L′ M′, while the rays nearer the axis will intersect at F″.

1049. *Longitudinal and lateral aberrations.* — The distance F F″ measured on the axis between the focus of the extreme rays which pass through the edge of the lens, and the focus of the central rays along which the foci of all the intermediate rays are placed, is called the *longitudinal aberration:* the point F″, which is the focus of the central rays, is called the *principal focus* of the lens, and the circle whose diameter is L M, over which the rays are spread, is called the *lateral aberration.*

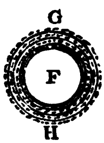

Fig. 344.

1050. *Experimental illustration.* — These effects may be rendered apparent by holding a white screen at F″, at right angles to the axis of the lens. An image of the sun will be formed round F″, and beyond the edge of

this image will be formed a ring or halo of light, growing fainter from the central image outwards, as represented in *fig.* 344.

1051. *Magnitude of spherical aberration in different forms of lenses.*—The magnitude of the spherical aberration varies in the different forms of lenses.

1. In a plano-convex lens with its plane side turned to parallel rays, that is, turned to distant objects if it is to form an image behind it, or turned to the eye if it is to be used in magnifying a near object, the spherical aberration will be $4\frac{1}{2}$ times the thickness.

2. In a plano-convex lens with its convex side turned towards parallel rays, the aberration is only $1\frac{17}{100}$ of its thickness. In using a plano-convex lens, therefore, it should always be so placed that parallel rays either enter the convex surface or emerge from it.

3. In a double convex lens with equal convexities, the aberration is $1\frac{67}{100}$ of its thickness.

4. In a double convex lens, having its radii as 2 to 5, the aberration will be the same as in a plano-convex lens in Rule 1., if the side whose radius is 5 is turned towards parallel rays; and the same as the plano-convex lens in Rule 2., if the side whose radius is 2 is turned to parallel rays.

5. The lens which has the least spherical aberration is a double convex one, whose radii are as 1 to 6. When the face whose radius is 1 is turned towards parallel rays, the aberration is only $1\frac{7}{100}$ of its thickness.

These results are equally true of plano-concave and double concave lenses.

If we suppose the lens of least spherical aberration to have its aberration equal to 1, the aberration of the other lenses will be as follows:—

Best form, as in Rule 5	1·000
Double convex or concave, with equal curvatures	1·561
Plano-convex or concave in best position, as in Rule 2.	1·093
Plano-convex or concave in worst position, as in Rule 1.	4·206

CHAP. XI.

ANALYSIS OF LIGHT.

1052. *Solar light a compound principle.*—In the preceding chapters, light has been regarded, in relation to transparent media, as a simple and uncompounded principle, each ray composing a pencil being subject to the same effects.

That all light is not thus subject to uniform effects, is rendered manifest by the following experiment of Newton:—

Let a pencil of parallel rays of solar light be admitted through a circular opening P, *fig.* 345, about half an inch in diameter, made in

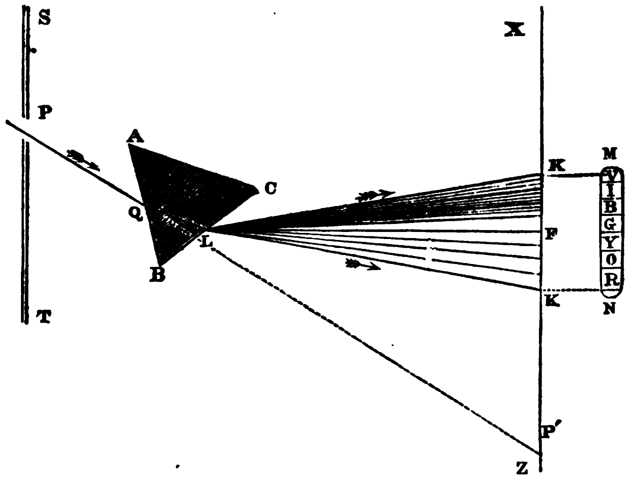

Fig. 345.

a screen or partition S T, all other light being excluded from the space into which the pencil enters. If a white screen X Z be placed parallel to S T, and at a distance from it of about 12 feet, a circular spot of light nearly equal in diameter to the hole will appear upon it at P′, the point where the direction of the pencil meets the screen. Now let a glass prism be placed at A B C, with the edge of its refracting angle B in a horizontal direction, and presented downwards so as to receive the pencil upon its side A B at Q. According to what has been already explained, the pencil would be refracted, in passing through the surface A B, in the direction Q L towards the perpendicular; and it would be again refracted in emerging from the surface C B from the perpendicular in the direction L K. It might therefore be expected that the effect of the prism would be merely to move the spot of light from P′ to some point, such as K, more elevated upon the screen. The phenomenon, however, will be very different. Instead of a spot of light, the screen will present an oblong coloured space, the outline of which is represented at M N as it would appear when viewed in front of the screen.

1053. *The prismatic spectrum.* — The sides of this oblong figure are parallel, straight, and vertical; its ends are semi-circular, and its length consists of a series of seven spaces, vividly coloured, the lowest

space being red, R; the next in ascending orange, O; and the succeeding spaces yellow, Y; green, G; light blue, B; dark blue or indigo, I; and, in fine, violet, V.

These several coloured spaces are neither equal in magnitude nor uniform in colour. The red space R, commencing at the lowest point with a faint red, increases in brilliancy and intensity upwards. The red, losing its intensity, gradually melts into the orange, so that there is no definite line indicating where the red ends and the orange begins. In the same manner, the orange, attaining its greatest intensity near the middle of the space, gradually melts into the yellow; and in the same manner, each of the succeeding colours, having their greatest intensities near the middle of the spaces, melts towards its extremities into the adjacent colours.

The proportion of the whole length occupied by each space will depend upon the sort of glass of which the prism is composed. If it be flint-glass, and the entire length M N be supposed to consist of 360 equal parts, the following will be the length of each succeeding colour, commencing from the red upwards.

Red	56
Orange	27
Yellow	27
Green	46
Blue	48
Indigo	47
Violet	109
	360

It appears, therefore, that the ray of light P Q, after passing through the prism, is not only deflected from its original course P Q P′, but it is resolved into an infinite number of separate rays of light which diverge in a fanlike form, the extreme rays being L K and L K′, the former being directed to the lowest point of the coloured space upon the screen, and the latter to the highest point. The coloured space thus formed upon the screen is called the *prismatic spectrum*.

1054. *Composition of solar light.*—From this experiment the following consequences are inferred:—

1. Solar light is a compound principle, composed of several parts differing from each other in their properties.

2. The several parts composing solar light differ from each other in refrangibility, those rays which are directed to the lowest part of the spectrum being the least refrangible, and those directed to the highest part being the most refrangible; the rays directed to the intermediate parts having intermediate degrees of refrangibility.

3. Rays which are differently refrangible are also differently coloured.

4. The least refrangible rays composing solar light are the red rays, which compose the lowest division R of the spectrum. But these

red rays are not all equally refrangible, nor are they precisely of the same colour. The most refrangible red rays are those which are deflected to the lowest point of the red space R, and the least refrangible are those which are directed to the point where the red melts into the orange. Between these there are an infinite number of red rays having intermediate degrees of refrangibility. The colour of the red rays varies with their refrangibility, the most intense red being that of rays whose refrangibility is intermediate between those of the extreme rays of the red space.

The same observations will be applicable to rays of all the other colours.

5. Each of these components of solar light having a different refrangibility will have for each transparent substance a different index of refraction. Thus the index of refraction of the red rays will be less than the index of refraction of the orange rays, and these latter will be less than the index of refraction of the yellow rays, and so on, the index of refraction of violet rays being greater than for any other colours.

But the rays of each colour being themselves differently refrangible, according as they fall on different parts of the coloured space, they will, strictly speaking, have different indices of refraction. The index of refraction, therefore, of any particular colour must be understood as expressing the index of refraction of the middle or mean ray of that particular colour. Thus, the index of refraction of the red rays will be the index of refraction of the middle ray of the red space; the index of refraction of the orange rays will be the index of refraction of the middle ray of the orange space; and so on.

It must not, however, be supposed that a pencil of solar light consists of separate and distinct rays of the different colours which form the spectrum, so that it might be possible by any mechanical division of such a pencil to resolve it into such rays. Each individual ray of such a pencil is composed of all the rays of the spectrum, just as the gases oxygen and hydrogen, which are the chemical constituents of water, enter into the composition of each particle of that liquid, no matter how minute it be.

1055. *Experiments which confirm the preceding analysis of light.* —The validity of the preceding analysis of light is confirmed by the following observations and experiments.

If the several rays composing a spectrum be allowed to pass separately through a small hole in a screen, and be received by another prism similar to A B C placed behind the screen, with the same angle of incidence as that with which P Q is incident upon A B, each ray will be refracted by the second prism, and its angle of deflection will be found to be exactly equal to the angle of deflection produced by the first prism A B C upon it. In this refraction of the second prism, the ray will not be dilated as the original ray of solar light was by the first prism, and no second spectrum will be formed; the ray will

be merely turned from its direction by the refraction of the prism, but will undergo no other change.

Let a band of white paper, A L, *fig.* 346., be divided into seven spaces, and let those spaces be coloured red, orange, yellow, green, light blue, indigo, and violet, severally, each colour being of uniform tint, and as closely resembling the seven colours of the spectrum as possible. Let this band be placed upon a black ground, and viewed through a prism whose refracting angle is presented upwards, with its edge horizontal and parallel to the band A L. The images of the several coloured spaces seen through the prism will be in positions more or less elevated above A L, according to the greater or less refrangibility of the different colored lights. The image of the red space R will be seen at R′, that of the orange space O at O′, that of the yellow space Y at Y′, and so on. The image O′ will be a little above R′, the image Y′ a little above O′, and so on, as represented in the figure, the image of the violet space V′ being in the highest position. This phenomenon is obviously the result of the relative refrangibilities of the different colours deposited on the spaces of the paper band

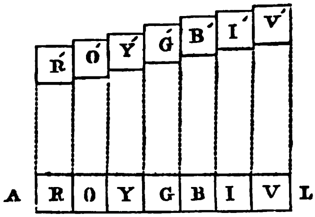

Fig 346.

Instead of artificial colours, let the spectrum itself be thrown on a sheet of white paper in a horizontal position A L, *fig.* 347., which may be done by placing the prism which produces the spectrum with the edge of its refracting angle vertical. Let this spectrum A L be viewed through a prism having the edge of its refracting angle horizontal, and presented upwards. The image of the spectrum seen through the prism will have the position A′ L′, oblique to A L, the violet end being more raised than the red end. The coloured space of the image will not form, as in *fig.* 346., a series of ascending steps, but will be included in one uniform line. This is explained by the fact already stated, that the light composing each of the coloured spaces R, O, Y, &c. of the spectrum is not uniformly refrangible.

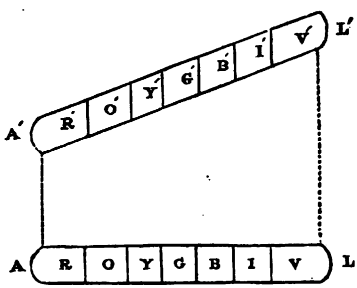

Fig. 347.

The rays which illuminate the red space R increase gradually in refrangibility from the extremity A to the boundary of the orange

space; and in like manner, the rays which illuminate the orange space O increase gradually in refrangibility to the boundary of the yellow space; and so on.

Hence it is that the boundary of the image of the spectrum is a line uniformly inclined to A L. The divisions of the coloured spaces in the image correspond, however, with those of the spectrum, each colour in the image being vertically above the corresponding colour in the spectrum.

1056. *Experimental proof by recomposition.* — As the solar light is resolved by the prism into the various coloured lights exhibited in the spectrum, it might be expected that, these coloured lights being mixed together in the proportion in which they are found in the spectrum, white light would be reproduced. This is accordingly found to be the case. If the spectrum formed by the prism A B C, *fig.* 348., instead of being thrown upon a screen, be received upon a concave

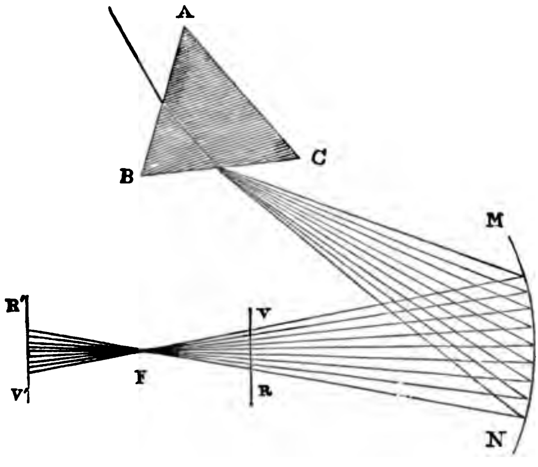

Fig. 348.

reflector M N, the rays which diverged from the prism and formed the spectrum will be reflected converging to the focus F, and after intersecting each other at that point, they will again diverge, the ray R F passing in the direction F R', and V F in the direction F V'.

Now, if a screen be held between F and the reflector, the spectrum will be seen upon the screen. If the screen be then moved from the reflector towards the focus F, the spectrum upon the screen will gradually diminish in length, the extreme colours R and V approaching each other. When it comes so near to F that the extreme limits of the red and violet touch each other, the central point of the spec-

trum will become white; and when the screen arrives at the point F, the coloured rays being all mingled together, the spectrum will be reduced to a white colourless spot.

Just before the screen arrives at F, it will present the appearance of a white spot, fringed at the top with the colours forming the upper end of the spectrum, violet, blue, and green, and at the bottom with those forming the lower end of the spectrum, red, orange and yellow. This effect is explained by the fact, that until the screen is brought to the focus F, the extreme rays at the other end of the spectrum are not combined with the other colours.

If the screen be removed beyond F, the same succession of appearances will be produced upon it as were exhibited in its approach to F, but the colours will be shown in a reversed position.

As the screen leaves F, the white spot upon it is fringed as before, but the upper fringe is composed of red, orange, and yellow, while the lower is composed of violet, blue, and green; and when the screen is removed so far from the focus F as to prevent the superposition of the colours, the spectrum will be produced upon it, with the red at the top, and the violet at the bottom, the position being inverted with respect to that which the screen exhibited at the other side of the focus. These circumstances are all explained by the fact that the rays converging to F intersect each other there.

Similar effects may be produced by receiving the spectrum upon a double convex lens, as represented in *fig.* 349. The rays are made as before to converge to a focus F, where a white spot would be produced upon the screen. Before the screen arrives at F, and after it passes it, the same effects will be produced as with the concave reflector.

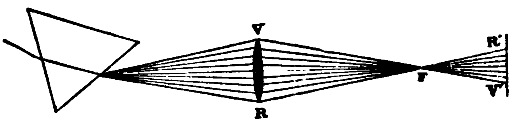

Fig. 349.

The proposition, that the combination of colours exhibited in the prismatic spectrum produces whiteness, may be further verified by the following experiment:—

Let a circular card be framed with a blackened circle, and its centre surrounded by a white circular band, and a black external border, as represented in *fig.* 350.

Let the white circular band be divided into seven spaces proportional in magnitude to the spaces occupied by the seven colours in the prismatic spectrum, these spaces being R, O, Y, G, B, I, and V. Let these spaces be respectively coloured with artificial colours resembling

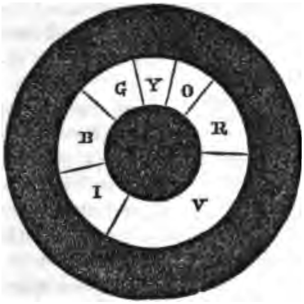

Fig. 350.

as near as practicable in their tints the colours of the spectrum. If the centre of this card be placed upon a spindle, and a very rapid motion of rotation be imparted to it, the ring on which the seven colours are painted will present the appearance of a greyish white. In this case, if all the colours except one were covered with black, the revolving card would present the appearance of a continuous ring of that colour; and, consequently, if all the coloured spaces be uncovered, seven continuous rings of the several colours would be produced; but these rings being superposed and mingled together will produce the same effect on the sight as if all the seven colours were mixed together in the proportion which they occupy on the card. If the colours were as intense and as pure as they are in the spectrum, the revolving card would exhibit a perfectly white ring; but as the colours of natural bodies are never perfectly pure, the colour produced in this case is greyish.

This experiment may be further varied by having uncovered any two, three, or more combinations of the colours depicted on the card. In such case the rotation of the card produces the appearance of a ring of that colour which would result from the mixture of the colours left uncovered: thus, if the red and yellow spaces remain uncovered, the card will produce the appearance of an orange ring; if the yellow and blue remain uncovered, it will produce the appearance of a green ring; and so on.

1057. *Lights of the same colour may have different refrangibilities.* — Although the phenomena attending the prismatic spectrum prove that rays of light which differ in refrangibility also differ in colour, the converse of this proposition must not be inferred; for it is easy to show that two lights which are of precisely the same colour, may suffer very different effects when transmitted through a prism.

Let us suppose two holes made in the screen on which the spectrum is thrown in the middle of the space occupied by the blue and yellow colours, so that rays of these colours may be transmitted hrough the holes. Let these rays be received upon a double convex lens, and brought to a focus at G,′ *fig.* 351., upon a sheet of white paper, so as to illuminate the spot G′. The colour that it produces then will be a green. Let another spectrum be now thrown by a prism upon the screen, and let a hole be made in the screen at that part of the green space where the tint is precisely similar to the colour produced at G′ on the white paper, and let the light which passes through this hole fall upon the spot G beyond G′.

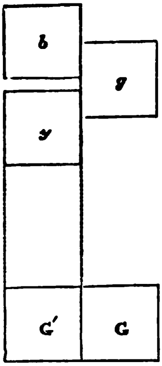

Fig. 351.

The spaces G and G′ will then be illuminated by lights of precisely the same colour; but it will be easy to show that these lights are not similarly refrangible.

Let them be viewed through a prism having its refracting angle presented upwards. The image of the illuminated space G will be seen in a more elevated position at *g*; but two images will be produced of the space G′, one yellow, and the other blue, at *y* and *b*, the yellow image *y* being a little below *g*, and the blue image *b* a little above it. Thus it is evident that the green light on the space G′ is a compound of yellow and blue, and is separable into its constituents by refraction, while the similar green light on the space G is incapable of decomposition by refraction.

1058. *Colours produced by combining different rays of the spectrum.*—An endless variety of tints may be produced by combining in various ways the colours composing the prismatic spectrum; indeed, there is no colour whatever which may not be produced by some combination of these tints. Thus, all the shades of red may be produced by combining some proportion of the yellow and orange with the prismatic red; all the shades of orange may be produced by combining more or less of the red and yellow with each other and with the orange; all the shades of yellow may be produced by varying the proportion of green, yellow, and orange; and so on.

1059. *Complementary colours.*—If two tints T and T′ be produced, the former T by combining a certain number of prismatic colours, and the latter T′ by combining the remainder together, these two tints T and T′ are called *complementary*, because each of these contains just those colours which the other wants to produce complete whiteness; and, consequently, if the two be mixed together, whiteness will be the result. Thus, a colour produced by the combination of the red, orange, yellow, and green of the spectrum in their just proportions, will be complementary to another colour produced by the blue, indigo, and violet in their just proportions, and these two colours, if mixed together, would produce whiteness.

1060. *Colours of natural bodies generally compound.*—Almost all colours, natural or artificial, except those of the prismatic spectrum itself, are more or less compounded, and their combined character belongs to them equally when they have tints identical with the coloured spaces of the spectrum. Thus, a natural object whose colour is indistinguishable from the yellow space of the spectrum, will be found, when subjected to the action of the prism, to refract light in which there is more or less of green or orange; and an object which appears blue will be found to have in its colour more or less of green and violet.

1061. *Method of observing the spectrum by direct vision.* — Instead of receiving the spectrum on a screen, it may be viewed directly by placing the eye behind the prism A B C, *fig.* 352., at L, so as to receive the light as it emerges. This mode of observing the prismatic

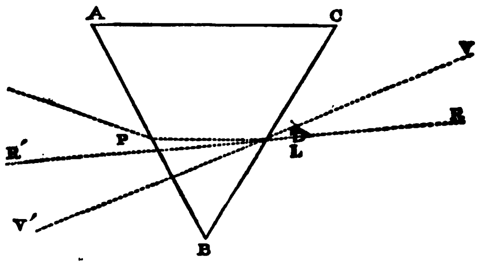

Fig. 352.

effects is in many cases more convenient than by means of the screen, colours being thus rendered observable which would be too feeble to be visible after reflection from the surface of the screen. It is necessary, however, to consider that in this manner of viewing the prismatic phenomena, the colours will be seen in an order the reverse of that which they would hold on the screen; for if the eye be placed at L, it will receive the violet ray which enters in the direction L V as if such ray had proceeded from V′, and it will receive the red ray which enters it in the direction R as if it had proceeded from R′; the red will therefore appear at the top, and the violet at the bottom of the spectrum, when the refracting angle B of the prism is turned downwards.

But if the refracting angle B be turned upwards, as represented in *fig.* 353., then the red will appear at the bottom, and the violet at the top of the spectrum, as will be perceived from the figure.

1062. *Why objects seen through prisms are fringed with colours.*— In general, when objects are viewed through a prism they appear with their proper colours, except at their boundaries, where they are fringed with the prismatic tints in directions parallel to the edge of the reflecting angle of the prism.

Let A A M M, *fig.* 354., be a small rectangular object seen upon a black ground, the sides A M being vertical, and A A and M M horizontal. Let us first suppose that this object has the colour of a pure homogeneous red. If this object be viewed through a prism whose refracting angle is directed upwards with its edge horizontal, it will be seen in a more elevated position, such as *a a m m*, as already explained.

Let us next suppose that the object A A M M has the colour of a pure homogeneous orange. When viewed through the prism it will, as already explained, appear in a position *b b n n*, a little above *a a m m*.

If we next suppose the object A A M M to be coloured with homo-

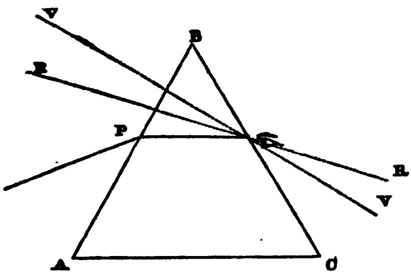

Fig. 353.

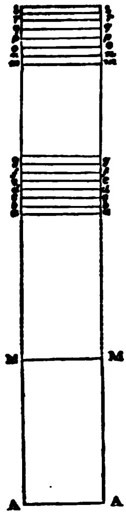

Fig. 354.

geneous yellow, it will be raised by the prism to *c c o o*, a little above the orange image.

If it be next supposed to have the colour of a prismatic green, it will be seen at *d d p p*, a little above the yellow image; and if it be coloured light blue, its image will be seen at *e e q q*, above the green image; if it be dark blue or indigo, its image will be in the position *f f r r*; if it be violet, its image will be in the position *g g s s*.

Now, if we suppose the object A A M M to be white, that is to say, to have a colour which combines all the prismatic colours together, then all these several images will be seen at once through the prism in the respective positions already described. They will therefore be more or less superposed one upon the other, and the image will exhibit in its different parts those tints which correspond to the mixture of the colours thus superposed.

Hence it appears that the space between *a a* and *b b* from which all colour except the red is excluded, will appear red; in the space between *b b* and *c c*, in which the orange image is superposed upon the red image, a colour will be exhibited corresponding to the mixture of these two colours; in the space between *c c* and *d d*, the three images red, orange, and yellow are superposed, and a colour corresponding to the combination of these will be produced. In fine, the colours which are superposed between every successive division of the upper and lower edges of the combined images are as follows, where the

prismatic colours are designated by the capital letters, and their mixture or superposition by the sign +:—

Between	*a a* and	*b b*	R
"	*b b* "	*c c*	R + O
"	*c c* "	*d d*	R + O + Y
"	*d d* "	*e e*	R + O + Y + G
"	*e e* "	*f f*	R + O + Y + G + B
"	*f f* "	*g g*	R + O + Y + G + B + I
"	*g g* "	*m m*	R + O + Y + G + B + I + V = W.

Thus it appears that the spaces *g g* of the violet image and the top *m m* of the red image are coloured with a white light, because in this space all the seven images are superposed.

In the space between *g g*, the bottom of the violet image, and *f f*, the bottom of the dark blue image, there is a space which is illuminated by all the prismatic colours except the violet, and this space consequently approaches so near a white as to be scarcely distinguishable from it. The space between *f f*, the bottom of the dark blue image, and *e e*, the bottom of the light blue image, is illuminated by all the colours except the dark blue and indigo, and it consequently has a yellowish tint. The succeeding divisions downwards towards *a a* become more and more red until they attain the pure prismatic red of the lowest division. The colours of the upper extremity of the image may in like manner be shown to be as follows:—

Between	*s s* and	*r r*	V
"	*r r* "	*q q*	V + I
"	*q q* "	*p p*	V + I + B
"	*p p* "	*o o*	V + I + B + G
"	*o o* "	*n n*	V + I + B + G + Y
"	*n n* "	*m m*	V + I + B + G + Y + O
"	*m m* "	*g g*	V + I + B + G + Y + O + R = W.

Thus it appears that the highest fringe at the upper edge is violet, that those which succeed it are formed by the mixture of violet and blue, to which green and yellow are successively added, until the colours become so completely combined that the fringe is scarcely distinguishable from a pure white. It is evident, therefore, that at the lower extremity the reds, and at the upper the blues, prevail.

If the object A A M M viewed through the prism be not white, then the preceding conclusions must be modified according to the analysis of its colour. Thus, if its colour be a green, it may be either a pure homogeneous green, or one formed by the combination of blue and yellow or other prismatic tints. In the former case, the prism will exhibit the object without fringes, but in the latter it will be fringed according to the composition of its colour, determined by the same principles as those which have been applied to the object A A M M.

1063. *Law of refraction applied to compound solar light.*—The analysis of light, which has been here explained and illustrated, will enable us to generalize and extend the law of refraction explained in 979.

Let A M B, *fig.* 355., be a transparent medium having a semi-cylindrical form, C being its centre.

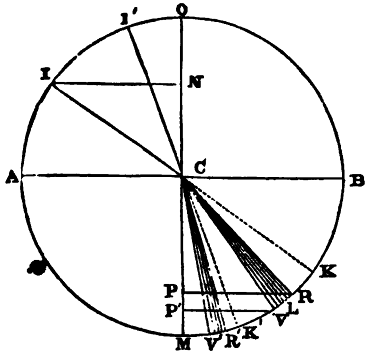

Fig. 355.

Let I C be a ray of solar light incident at C, the angle of incidence being I C O. This ray, on entering the transparent medium, will, according to what has been already explained, be resolved into an infinite number of other rays differently refracted, that which is least refracted being C R, and that which is most refracted being C V. The ray C R is red, and the ray C V is violet; the rays of intermediate colours and intermediate refrangibilities being included between them. The angle R C M is the angle of refraction of the extreme red ray corresponding to the angle of incidence I C O, and the angle V C M is the angle of refraction of the extreme violet ray corresponding to the same angle of incidence.

The index of refraction of the former will be found by dividing I N by R P; and the index of refraction of the latter will be found by dividing I N by V P'.

It is evident that the indices of refraction for the intermediate rays will be included between these two, being greater than the index of the extreme red, and less than the index of the extreme violet.

If the angle of incidence I C O be diminished, the angles of refraction R C M and V C M will be both diminished, since their sines will still bear the same ratio to the sine of the angle of incidence. Thus, if I' C, *fig.* 355., be the incident ray, and I' C O the angle of incidence, then C R' will be the extreme red, and C V' the extreme violet refracted rays, and the intermediate rays, into which the incident ray is resolved, will lie between these as before.

In this case, the angle R' C V' which measures the divergence of the extreme rays into which the incident ray is resolved, will be less than the angle R C V, which measures their divergence with the greater angle of incidence I C O. Thus it appears that the divergence of the decomposed rays is diminished as the angle of incidence is diminished, and increased as the angle of incidence is increased; but with the same angle of incidence this divergence is always the same in the same transparent medium.

The angle K C R, formed by the direction of any ray, such as C R, with the direction C K, which it would have followed had it not been refracted, is called the *refraction* of that ray.

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Now it is necessary to distinguish carefully this term from the *angle of refraction* already defined.

Thus it appears that the refraction of the different rays into which the ray C I is resolved is different; that of the extreme red being K C R, and that of the extreme violet being K C V.

1064. *Dispersion of light.*—The difference between the refraction of these extreme rays, or the angle of divergence R C V of the rays into which the original solar ray I C has been resolved by refraction, is called the *dispersion* produced upon the solar ray I C by the process of refraction.

It follows from what has been just explained, that this dispersion in the same medium diminishes and increases as the angle of incidence or the angle of refraction, or, in fine, as the refraction itself, diminishes or increases.

1065. *Mean refraction.*—But the term refraction, to have a definite meaning, in this case, must be applied to some one of the rays into which the solar ray is resolved, since each of these rays has a different refraction, varying from K C R to K C V. The middle ray, therefore, C L, of the rays diverging from C, is adopted for this purpose; and, accordingly, the ray C L is called the mean ray, and the angle K C L the mean refraction.

The refraction produced by any transparent medium upon a given ray at a given angle of incidence, is the measure of the refracting power of the medium on such ray; but as this refraction is always the difference between the angles of incidence and refraction, and as this difference may be taken to be proportional to the difference between their sines, we shall have the refractive power of the medium expressed thus:

$$\frac{\sin. \text{I} - \sin. \text{R}}{\sin. \text{R}} = n - 1;$$

where n expresses the index of refraction.

The measure, therefore, of the refracting powers of different media, is the number found by subtracting 1 from their index of refraction.

It follows, from what has been explained, that in the same medium the dispersion increases and diminishes as the mean refraction increases or diminishes.

1066. *Dispersive power.*—When different media are compared together, it is found, that with the same mean refraction there will be different dispersions,—a fact which supplies a characteristic of different media, which has been called their *dispersive power;* one medium being said to have a greater or less dispersive power than another medium, according as the dispersion it produces with the same mean refraction is greater or less than that produced by the other medium.

The dispersion, therefore, produced by any medium being expressed by the difference of the indices of refraction n'' and n' of the extreme rays, and the refracting power being expressed by $n - 1$, the absolute dispersive power is the quotient obtained by dividing the dispersion by the refracting power, and will be

$$\text{D} = \frac{n'' - n'}{n - 1}.$$

In the tables of refraction which have been given in page 65, the indices of refraction must be understood to refer to the mean ray of the spectrum, produced by the various media indicated in the tables.

To illustrate the application of this formula, let us take the case of crown-glass and diamond. The index of refraction of the extreme and mean rays of crown glass are as follows:—

$$n'' = 1{\cdot}5466,\ n' = 1{\cdot}5258,\ n = 1{\cdot}5330;$$

consequently we shall have for crown-glass,

$$\text{D}' = \frac{208}{5330} = 0{\cdot}0390.$$

In like manner, the indices for diamond are

$$n'' = 2{\cdot}4670,\ n' = 2{\cdot}4110,\ n = 2{\cdot}4390;$$

therefore, we shall have

$$\text{D}' = \frac{56}{1439} = 0{\cdot}0389.$$

From whence it appears that although the refracting powers of the diamond and crown-glass are as 3 to 1, their dispersive powers are the same.

This identity of their dispersive powers may be proved experimentally by taking two prisms, one of diamond and the other of crown glass, and producing with them two spectra in the manner represented in *fig*, 345., so that the mean ray L F of each shall be equally inclined to the direction P P′ of the incident ray. It will be found that the two spectra thus produced will have equal lengths, and consequently that the dispersions which correspond to equal refractions are equal.

Transparent media differ from each other, not only in the dispersive powers which they have on solar light, but also in the dispersive powers with which they act on the different elements which compose such light. Thus, for instance, it will happen that although two media, such as the diamond and crown glass, may have equal dispersive powers in relation to the compound light of day, they will have very different dispersive powers upon the several coloured lights of which each compound light is made up.

This may be rendered experimentally apparent by producing two spectra of equal lengths, with prisms of different materials.

If these two spectra be placed in juxtaposition, so that their extre-

mities shall coincide, although their coloured spaces will succeed each other invariably in the order already described, yet the boundaries which separate these coloured spaces will not coincide. The red in the one will be more or less extensive than in the other, and the same will be true of the other colours.

Let two spectra, A B and C D, *fig.* 356., be produced in this manner, equal in length, by two hollow prisms, one filled with the oil of cassia, and the other with sulphuric acid. In the spectrum A B, produced by the oil of cassia, the red, orange, and yellow spaces are less than in the spectrum C D, produced by the sulphuric acid, while in the latter the blue, indigo, and violet spaces are less than in the former.

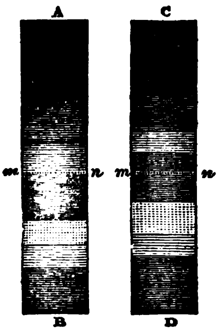

Fig. 356.

The middle ray *m n* in the spectrum A B passes through the blue space, while it passes through the green space in the spectrum C D.

It appears, therefore, that in this case sulphuric acid has a greater dispersive power upon the less refrangible rays, and a less dispersive power on the more refrangible rays, than the oil of cassia.

These effects are consequences of the fact, that although the indices of refraction of the extreme rays for any two media may be equal, the index of refraction of the intermediate rays may be unequal, and a difference of position of the corresponding colours in the spectrum will be the necessary consequence.

In the following table, the indices of refraction corresponding to the mean rays of the seven coloured spaces of the spectrum are given according to the experiments of Frauenhofer.

1067. *Table of the indices of refraction of the mean rays of each of the prismatic colours for certain media.*

Refracting Substances.	n_1.	n_2.	n_3.	n_4.	n_5.	n_6.	n_7.
Flint glass, No. 13.	1·627749	1·629681	1·635036	1·642024	1·648260	1·660285	1·671062
Crown glass	1·525832	1·526849	1·529587	1·533005	1·536052	1·541657	1·546566
Water	1·330935	1·331712	1·333577	1·335851	1·337818	1·341293	1·344177
Water	1·330977	1·331709	1·333577	1·335849	1·337788	1·341261	1·344162
Potash	1·399629	1·400515	1·402805	1·405632	1·408082	1·412579	1·416368
Oil of turpentine	1·470496	1·471530	1·474434	1·478353	1·481736	1·488198	1·493874
Flint glass, No. 3	1·602042	1·603800	1·608494	1·614532	1·620042	1·630772	1·640373
Flint glass, No. 30	1·623570	1·625477	1·630585	1·637356	1·643466	1·655406	1·666072
Crown glass, No. 13	1·524312	1·525299	1·527982	1·531372	1·534337	1·579908	1·544684
Crown glass, Litt. M.	1·554774	1·555933	1·559075	1·563150	1·566741	1·583535	1·579470
Flint glass, No. 23. and prism of 60°	1·626596	1·628469	1·633667	1·640495	1·646756	1·658848	1.669686
Flint glass, No. 23. and prism of 45°	1·626564	1·628451	1·633666	1·640544	1·646780	1·658849	1·669680

1068. *Dispersion of each component colour, how found.*—The dispersion proper to each successive colour will be found by taking the difference of the two adjacent indices, and the total dispersion produced by each medium by taking the difference between the extreme indices. Thus the total dispersion produced by each medium given in the above table will be as follows:

Flint glass, No. 13.	0·043313
Crown glass, No. 9.	0·020734
Water	0·013242
Water	0·013185
Potash	0·016739
Turpentine	0·023378
Flint glass, No. 3.	0·038331
Flint glass, No. 30.	0·042502
Crown glass, No. 13.	0·020372
Crown glass, Lett. M.	0·024696
Flint glass, No. 23., prism 60°	0·043090
Flint glass, No. 23., prism 45°	0·043116

1069. *Light uniformly refrangible not necessarily simple.*—In all that precedes, it has been assumed that the light composing each part of the prismatic spectrum is simple and homogeneous. This conclusion, deduced by Newton, and adopted generally by all physical investigators since his time, is based on the assumption, that light which, being refracted by transparent media, cannot be resolved into parts differently refrangible, is simple and homogeneous.

Sir David Brewster has, however, published the results of a series of observations, from which it would follow, that a pencil of light which does not consist of parts differently refrangible, may, nevertheless, be resolved into parts which have different colours; in other words, that the light of certain parts of the spectrum, such, for example, as orange and green, although simple so far as respects refraction, is compound so far as respects colour. Thus, the orange light may be resolved into two lights equally refrangible, but differing in colour, one being red and the other yellow; and the green light may in like manner be resolved into two equally refrangible, one being yellow and the other blue.

1070. *Sir D. Brewster's analysis of the spectrum.*—In a word, the observations and experiments of Sir David Brewster have led him to the conclusion that the prismatic spectrum consists in reality of three spectra of nearly equal length, each of uniform colour, superposed one upon another; and that the colours which the actual spectrum exhibits arise from the mixture of the uniform colours of these three spectra superposed. The colours of these three elementary spectra, according to Sir David Brewster, are red, yellow, and blue. He shows that by the combination of these three, not only all the colours exhibited in the prismatic spectrum may be reproduced, but that their combination also produces white light. He contends, there-

fore, that the white light of the sun consists not of seven, but of three constituent lights, red, yellow, and blue.

This conclusion is established by showing that there is another method by which light may be resolved into its components, besides the method of refraction by prisms. In passing through certain coloured media, it is admitted that a portion of the light incident is intercepted at the surface upon which it is incident and in its passage through the medium; a part only is transmitted.

Now, this property of colours is taken by Sir David Brewster as another method, independently of refraction, of decomposing colours. He assumes that such a medium resolves the light incident upon it into two parts: first, the part which it transmits; and secondly, the part which it intercepts. He concludes that these two parts are complementary, that is to say, that each contains what the other wants to make up white solar light; or, more generally, that the incident light, whatever be its nature, must be assumed to be a compound, consisting of the light transmitted and the light intercepted.

This being assumed, let a coloured medium, such as a plate of blue glass, be held between the eye and the spectrum. Certain colours of the spectrum will be transmitted and others intercepted. If the colours of the spectrum be simple and homogeneous light, such as they are assumed to be in the Newtonian theory of the decomposition of light, then the consequence would be that the appearance of the spectrum seen through the coloured medium would consist of dark and coloured spots; those simple lights intercepted by the glass appearing dark, and those transmitted by the glass having their proper colour. But if each colour of the prism be, as is assumed in the chromatic theory, simple, then the plate of glass can make no change in its colour by transmission.

It must therefore be wholly transmitted, partly transmitted, or wholly intercepted. If it be wholly transmitted, no change will be made, therefore, in its colour or intensity; if it be partly transmitted, its colour will remain the same; but its intensity will be diminished; if it be wholly intercepted, the space it occupied on the spectrum will be black. But these are not the effects, as Sir David Brewster states, which are observed. He finds, on the other hand, that the coloured spaces on the spectrum are not merely diminished in intensity, but actually changed in colour. Now, if any space of the spectrum be changed in colour, it follows from what has been stated, that the light transmitted must be a constituent of the colour of that space, to which the light intercepted being added, would reproduce the colour of the spectrum. By such an experiment as this, Sir David Brewster found that the parts of the spectrum occupied by the orange and green lights produced yellow, from which he inferred that the glass intercepted the red, which combined with the yellow produced orange, and the blue, which combined with the yellow produced green. But

if the glass have the power of thus intercepting the red and blue light, it might be expected that the red and the blue spaces of the spectrum would appear dark. He accordingly found that the light of the middle of the red space was almost entirely absorbed, as was also a considerable part of the blue space.

From experiments like these, which he made in great number, and under various conditions, Sir David Brewster deduced the conclusion to which we have adverted above.

He inferred that at every point of the spectrum, red, yellow, and blue light are combined in various proportions, the colour of each part being determined by the proportional intensities of these three colours in the mixture. In the red space, the proportions of blue and yellow are exactly those necessary to produce white light, but the red is in excess; a portion of it combined with the blue and yellow produces a white light, which is reddened by the surplusage of red. In the same manner, in the yellow space the proportion of blue and red is that which is proper to white light, but there is a greater than the just proportion of yellow.

A part of this combining with the blue and red produces white light, which is rendered yellow by the surplus. In the same manner exactly, the blue space is shown to consist of a surplusage of blue, combined with the proportion of red and yellow, and the remainder of the blue necessary for whiteness. The other colours of the spectrum, according to Sir David Brewster, are secondary, or the result of combinations of red, yellow, and blue.

The means by which these three primary colours produce the tints of the spectrum may be more clearly understood by reference to *fig.* 357., wherein M N represents the prismatic spectrum with its usual tints. The curve M R N represents the varying intensity of the red spectrum, M Y N that of the yellow, and M B N that of the blue spectrum. The distance of each part of these curves respectively from M N is understood to be proportional to the intensity of the colour of that part, and the relative lengths of the perpendicular included within each curve represents the proportion of the intensities of the combined colours. Thus, at the point P, the three colours are mixed in the proportion of the lengths of the perpendiculars $p\,n$, $p'\,n$, $p''\,n$, the first representing the proportion of yellow, the second red, and the third blue; the red and yellow predominating, the colour at this point will be orange.

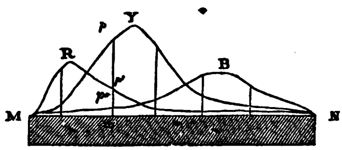

Fig. 357.

These observations and experiments, and the conclusions deduced from them by Sir David Brewster, have been now before the scientific world for more than twenty years. The experiments do not appear

to have been repeated, nor the chromatic doctrine inferred from them to have been yet generally assented to or adopted. The chromatic analysis of Newton is the only theory advanced by physical authors.

CHAP. XII.

SPECTRAL LINES—PHOTOMETRIC, THERMAL, AND CHEMICAL PROPERTIES OF THE SPECTRUM.

1071. *Number of spectral lines.*—If the prismatic spectrum produced under certain conditions be examined by the aid of a telescope, it will be found to be crossed throughout its entire length by dark lines of various breadths. The total number of these lines is nearly seven hundred, and they are distributed over the spectrum without any apparent relation to the limits of its coloured spaces.

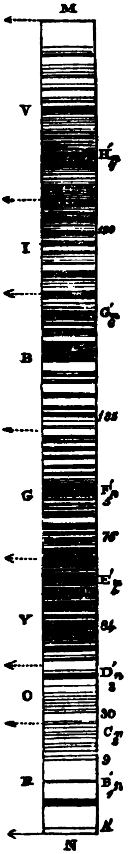

Fig. 358.

In *fig.* 358., M N represents a spectrum, M being its violet, and N its red extremity. The arrows to the left of the diagram represent the boundaries between the coloured spaces, these spaces being indicated by the letters R, O, Y, G, B, I, and V.

The general distribution of the spectral lines is exhibited in the diagram.

It will be observed, that in the distribution of these remarkable phenomena, there is no apparent regularity, either in their arrangement or in their intensity. In some places they are thickly crowded together, while in others they are separated by white spaces, more or less considerable. In some, the lines are extremely fine and scarcely visible; in others they are of distinct breadth.

Among these numerous lines, seven were selected by their discoverer, Frauenhofer, as standards of reference or fixed points by which the position of the others could be designated. These seven are those marked on the right by the letters B′, C′, D′, E′, F′, G′, H′.

The first of these, B′, is in the middle of the red space; the second, third, and fourth, C′, D′, and E′, are nearer the boundaries which separate the red and orange, the orange and yellow, and the yellow and green; the fifth, F′, is near the middle of the green space, and the seventh near the middle of the violet space; while the sixth is near the boundary which separates the blue and indigo.

The numbers which appear in the diagram between each pair of these lines indicate the number of spectral lines which have been ascertained to exist between them.

Thus, between B′ and C′ there are 9, between C′ and D′ 30, and so on; the entire number of lines between the first, B′, and the seventh, H′, being 574. The remainder of the spectral lines between the extreme red and B′, and between the extreme violet and H′, amount to about 100; but they are more difficult of observation, and have not been so precisely ascertained.

A little above the extreme red, there is a well-defined dark line A′; and about half way between that line and the line B′, there is a dark band composed of seven or eight lines.

It was ascertained by Frauenhofer, that these lines are altogether independent either of the magnitude of the refracting angle, or of the matter of the prism; and that their number, order, and intensity are absolutely invariable, no matter what prism be used, provided only the light come through, directly or indirectly, from the sun. Thus it is found that the spectra produced by moonlight and by the light of the planets give exactly the same lines.

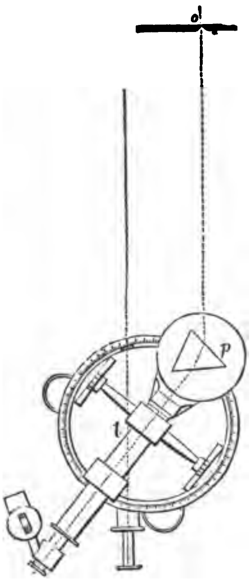

Fig. 359.

1072. *Manner of observing the spectral lines.* — The best method of observing these interesting phenomena is by means of telescopes and a prism, represented in *fig.* 359. Let a narrow slit be made in a window-shutter or a screen, so as to admit a broad thin beam of the sun's light. This slit is represented in section at right angles to its length at *o*. The beam of light is received on a prism of the finest and purest flint glass at *p*. After being refracted by the prism, it is received by a small telescope, which plays upon a graduated arc, on which is a second telescope to indicate the original direction of the ray *o p*. The angle under the two telescopes will indicate the refraction which the ray has suffered by the prism. The prisms used in these observations have been made of the purest and finest flint glass, perfectly free from threads and striæ. The prism ought to be placed at a distance of fifteen or twenty feet from the telescope.

By turning, therefore, the telescope or the prism, the successive rays of the spectrum are made to pass through the telescope, so that the spectrum may be viewed successively from one extremity to the

other. The telescopes suited to these observations should magnify from eight to ten times.

1073. *Spectral lines of artificial lights and of the moon, planets, and stars.* — By these means the spectra produced not only by solar light, but also by various artificial lights, as well as electric light, have been observed. The electric light gives the spectral lines bright instead of dark, one of the most remarkable for its brilliancy passing through the green space. The flame of a lamp, whether produced by gas, oil, or spirits, also gives the spectral lines bright. Two of these are especially distinguishable in the red and orange spaces.

The moon and planets have the same dark lines as the sun, but less easily distinguishable, especially near the extremities of the spectrum. The spectra produced by the light of the fixed stars are marked with dark lines, but little different in their number, intensity, and disposition from those exhibited in the solar spectrum. It is remarkable that the spectra produced by different fixed stars differ from each other.

1074. *Use of spectral lines as standards of refrangibility.* — The invariable position which Frauenhofer's lines are found to have in the solar spectrum has rendered them eminently useful for establishing standards of refrangibility of the component parts of solar light. From what has been stated respecting the gradual variation of the tints composing the solar spectrum, it may be easily understood that much uncertainty will attend any methods of defining a particular ray to which a certain index of refraction is imputed. Thus the middle of the red or the middle of the green space is necessarily an indefinite term, so long as the limits of these spaces admit of no exact definition.

The seven lines B′, C′, D′, &c., which have been already noticed, have been accordingly adopted as points invariable in their position, of which the indices of refraction once determined may always serve as standards of reference. The indices accordingly which have been given in table, p. 125., are those which belong to these points, n_1 being the index of refraction at B′, n_2 that of the rays at C′, n_3 at D′, and so on.

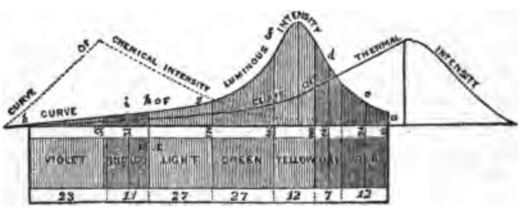

Fig. 360.

1075. *Relative intensity of light in different parts of the spectrum.* — Frauenhofer also ascertained by photometric observations the relative intensity of the light in different parts of the spectrum.

The result of these observations is denoted by the curve marked "Luminous intensity," in *fig.* 360.; the perpendicular distance of each point of this curve from the edge of the spectrum being proportional to the brilliancy of the light produced by a flint glass prism. It appears from this that the most intense illumination corresponds to a point about the middle of the yellow space.

In the following table are given the numerical intensities of the other points, the light of the point of greatest intensity being expressed by 1000.

At the red extremity	000
At B′	32
At C′	94
At D′	640
At E′	480
At F′	170
At G′	31
At H′	5·6
At violet extremity	000

1076. *Relative calorific intensity of the spectral rays.* — The heating power of the light composing the different parts of the spectrum was examined first by the late Sir William Herschell, and later by M. Berard, Sir H. Davy, MM. Seebeck, Wunsch, and, in fine, by M. Melloni, who has supplied a vast body of interesting experiments on this subject. The general result of these observations, the details of which would be inadmissible here, are as follows: —

The heating power, being nothing at the violet extremity, augments gradually as the thermometer is moved to the red extremity.

At this point, or near it, the heating power is a maximum; but the presence of thermal rays beyond the red extremity is manifested by the thermometer, which, though it declines on being moved beyond this extremity, continues to show a temperature greater than that of the surrounding air, to a considerable distance from the spectrum.

We are therefore compelled to admit the existence of invisible rays in the sun's light, which have the power of producing heat, and which have a less degree of refrangibility than red light.

The curve marked "Thermal intensity," in *fig.* 360., indicates the variation of the heating power of the rays of the spectrum in the same manner as the former curve represented the luminous intensity. The point of maximum thermal intensity is according to some at the red extremity, and according to others a little below it, but it is found that this depends in some degree upon the material composing the prism.

1077. *Relative chemical intensity of the spectral rays.*—The action of light in changing the colour of certain substances has long been known; but one of the most remarkable of this class of objects has lately acquired increased interest from its application in the art called Daguerreotype.

If the chemical substance called muriate of silver be exposed to solar light, it will be blackened. Now, in order to ascertain whether this effect is due collectively to all the rays composing solar light, or is caused by the action of some rather than other rays, it is only necessary to expose it successively to all the rays composing the prismatic spectrum.

If this be done, it will be found that the least refrangible rays near the red extremity do not produce this effect in any sensible degree, while the more refrangible rays at the violet extremity produce it in a very great degree; in a word, by ascertaining and indicating the intensity of this chemical action in the same manner as the intensities of the illuminating and heating power as already expressed, we shall be enabled to determine the curve of chemical intensity indicated in *fig.* 360., from which it appears that this action is at its maximum near the boundaries between the violet and the indigo.

CHAP. XIII.

CHROMATIC ABERRATION.—ACHROMATISM.

1078. *Chromatic aberration of lenses.*—It appears from what has been established in Chap. X., that the power of a lens whether it be convergent or divergent, and therefore also its focal length, depends not only on the curvature of its surface, but on the index of refraction of the substance composing it.

But, from what has been explained in the last chapter, it appears that the index of refraction for the same transparent medium is different for the different component elements of light. Thus, the index of refraction for flint glass, which corresponds to violet light, is greater than the index of refraction for red light, the former being more refrangible than the latter. The focal length, therefore, of a lens for red light, will be different from the focal length of the same lens for the violet light. This circumstance produces important consequences, which we shall now proceed to explain.

Let A B C, *fig.* 361., be a converging lens, which we will here suppose to be double convex. Its focal length F will, according to what has been explained in Chapter X., be

$$F = \frac{r \times r'}{(n-1)(r+r')},$$

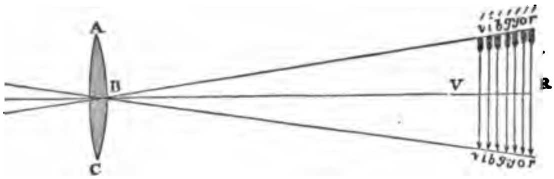

Fig. 361.

where r and r' express the length of the radii of the lens, and n the index of refraction. Now, since the index of refraction which corresponds to the extreme violet rays is greater than the index of refraction which corresponds to the extreme red rays, the value of F will be less for the former than for the latter; and, consequently, the focus of the extreme violet rays will be nearer the lens than the focus of the extreme red rays; and, in like manner, it follows, that the focus of the rays of intermediate refrangibilities will lie between these two points.

If V and R, therefore, be the foci of the extreme violet and extreme red rays respectively, the foci of all the rays of intermediate refrangibilities will be distributed between V and R.

Let us suppose any object which transmits the extreme violet light to be placed before the lens at such a distance that the pencil of rays proceeding from each point upon it to the lens may be considered as consisting of parallel rays; an inverted image of such object will be formed at $v\,v'$, at a distance B V from the lens determined by the preceding formula, n having in it the value which corresponds to the index of the extreme violet rays.

If, now, a similar and equal object be similarly placed before the lens, but emitting the extreme red light instead of the extreme violet light, an inverted image of this object will be formed at $r\,r'$, at a distance B R from the lens, determined in like manner by the above formula, in which the value assigned to n shall be the index of refraction corresponding to the extreme red light.

If, in like manner, the object placed before the lens be supposed to be successively illuminated by all the varying tints of the spectrum, a succession of inverted images corresponding in colour to these tints will be formed at $o\,o'$, $y\,y'$, $g\,g'$, $b\,b'$, and $i\,i'$, between R and V.

Now, if the object placed before the lens, instead of being successively illuminated by these various homogeneous lights, be illuminated with the white light of the sun, or if such object be the sun itself, then the various component parts of the light which it transmits will be brought by the lens to different foci corresponding to their various degrees of refrangibility, and the lens will accordingly produce, not one white image, but an infinite number of coloured images included between the extreme positions V and R. Each ray will form

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an image, having a position and colour corresponding to its degree of refrangibility, and the space included between V and R will be a truncated cone filled with images, which increase in magnitude from V to R, and which, beginning with a violet colour at V, pass through all the tints of the spectrum; the last image at R having a red colour corresponding to the red of the extreme light of the spectrum.

A white screen held at R would exhibit a well-defined red image of the object, if it did not also receive upon it the pencils of rays forming all the other images between R and V, such pencils diverging from the various points of such images. Thus, a pencil which is brought to an exact focus upon the image *o o'*, would form upon a a screen placed at *r r'*, not a point, but a small spot of orange light. In like manner, a pencil whose focus lies upon the image *y y'* would form upon a screen placed at R a small spot of yellow light, greater in magnitude than the spot of orange light, because of the greater distance of its focus from the screen. In like manner, the points upon the image *g g'*, *b b'*, *i i'*, and *v v'*, would produce upon the screen at *r* luminous spots of green, blue, indigo, and violet light, increasing in magnitude in proportion to their respective distances from the screen.

The image, therefore, formed upon the screen, arising from this combination of pencils of variously coloured lights, will exhibit a confused representation of the object; the colours diffused over the internal parts of its area being those which combined together form white light, the general area of the image will not be coloured; but the coloured pencils thus mingled together, being none of them brought to their foci on the screen, except those of the extreme red light, a confusion will ensue. At the edges there will be coloured fringes, because at the edges the pencils diverging from the edges of the series of images do not overlay each other as they do at the central pencils; and, consequently, the colours necessary for the production of white light are not mingled in these pencils.

The consequence of all this is, that there will be formed upon the screen an image of the object, everywhere indistinct, and fringed with prismatic colours at its edges.

The degree of indistinctness and the breadth of the fringes will depend upon the length of the space V R; that is to say, upon the *dispersion* produced by the lens, and also upon the difference between the magnitudes of the extreme images *r r'* and *v v'*, which latter depends upon the opening of the lens *r* B *r'*, and on the dispersion V R conjointly.

The consequence of this is, the indistinctness of the image and the coloured fringes arising from this cause increase as the focal length of the lens diminishes, as its opening increases, and as the dispersive power of the material of which it is composed increases.

These effects are called the *chromatic aberration* of lenses.

1079. *Aberration of a diverging lens.*—We have assumed in the preceding examples that the lens is a converging lens; and, consequently, that the image of a distant object is real, and may be exhibited on a screen.

If, however, the lens be a diverging lens, the effects of aberration will be the same, but the image being imaginary cannot be exhibited in the same manner. A diverging lens A B C is represented in *fig.* 362.

Let the object, as before, be placed at such a distance from it that the pencils proceeding from it may be considered as parallel. After passing through the lens they will diverge, as if they had proceeded from an object placed at a distance before the lens, equal to its focal length. Thus, if the object emit red light, the rays after passing through the lens will diverge as if they had proceeded from *r r′* at the distance B R, equal to the principal focal length corresponding to

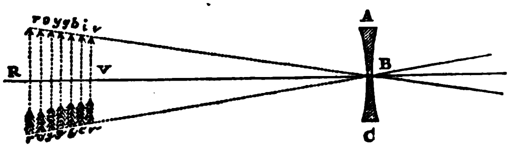

Fig. 362.

the index of refraction of red rays, and in like manner, if the object transmit violet rays, the light, after passing through the lens will diverge as if it had proceeded from points in an object placed at *v v′*, and for the intermediate colours it would diverge as if it had proceeded from intermediate points between R and V.

Thus, if, as before, the object be supposed to emit white solar light, the rays after passing the lens would diverge from points between R and V, varying according to their refrangibilities in the manner already expressed.

1080. *Images formed by single lenses must always be coloured.*—It appears, therefore, from what has been here explained, that no single lens can produce a distinct image of an object free from coloured fringes, since to accomplish this it would be necessary that each lens should possess the same power of convergence over all the component rays proceeding from all points of such object.

But since the converging power of the lens depends upon the index of refraction of the light, and since the index of refraction varies with the colour and refrangibility of the light, it follows that unless the object transmit light of a single refrangibility, that is to say, homogeneous light, the lens cannot cause the pencils which proceed from it to converge to the same focus, and, consequently cannot produce a distinct image. This object, however, which cannot be accomplished by a single lens, may be attained by a combination of lenses

composed of transparent substances, which differ from each other in their dispersive powers.

1081. *Conditions under which combined lenses may be rendered achromatic.*—To put the question first under its most simple form, let it be required to find what form must be given to two lenses composed of media having different refracting powers, so as to render the focal length of the compound lens for light of any one refrangibility, equal to its focal length for light of any other refrangibility.

Let F' and F'' be the focal lengths of the two lenses for light, of which the indices of refraction are n' and n'' for the media composing the lenses respectively.

Let f' and f'' be their focal lengths for light of which the indices of refraction are m' and m''.

Let F be the focal length of the compound lens.

The converging power of the compound lens on each kind of light will be equal to the sum of the converging powers of the two lenses separately on the same kind of light. The converging power of the compound lens, therefore, on the light whose indices of refraction are n' and n'', will be

$$\frac{1}{F'} + \frac{1}{F''};$$

and in like manner its converging powers on the light whose indices of refraction are m' and m'', is

$$\frac{1}{f'} + \frac{1}{f''}.$$

But since, by the supposition, these two converging powers must be rendered equal, we shall have

$$\frac{1}{F'} + \frac{1}{F''} = \frac{1}{f'} + \frac{1}{f''}.$$

The question is, then, to assign such magnitudes to the radii of the surfaces of the lenses as will make them fulfil this condition.

Let R_1 and R_2 be the radii of the surfaces of the first, and r_1 and r_2 those of the surfaces of the second lens. We shall then have, by the formulæ given in 1031. and 1032.,

$$\frac{1}{F'} = \frac{(n'-1)(R_1 - R_2)}{R_1 \times R_2}, \quad \frac{1}{F''} = \frac{(n''-1)(r_1 - r_2)}{r_1 \times r_2};$$

$$\frac{1}{f'} = \frac{(m'-1)(R_1 - R_2)}{R_1 \times R_2}, \quad \frac{1}{f''} = \frac{(m''-1)(r_1 - r_2)}{r_1 \times r_2}.$$

But since

$$\frac{1}{F'} + \frac{1}{F''} = \frac{1}{f'} + \frac{1}{f''},$$

we shall have

$$\frac{1}{F'} - \frac{1}{f'} = \frac{1}{f''} - \frac{1}{F''};$$

therefore

$$\frac{(n' - m')\,(R_1 - R_2)}{R_1 \times R_2} = - \frac{(n'' - m'')\,(r_1 - r_2)}{r_1 \times r_2};$$

and consequently

$$\frac{n' - m'}{n'' - m''} = - \frac{(r_1 - r_2) \times R_1 \times R_2}{(R_1 - R_2) \times r_1 \times r_2}.$$

The numbers expressed by $n' - m'$ and $n'' - m''$ are the differences between the indices of the two lights having different refrangibilities, which are supposed to be transmitted through the lenses. These are the dispersive powers of the media composing the lenses for each of the two lights. If, then, the radii of the two lenses be so selected as to render the fraction expressed by the second member of the preceding equation equal to the ratio of the dispersive powers of the material of the lenses for the two sorts of light, they will be brought to the same focus by the compound lens.

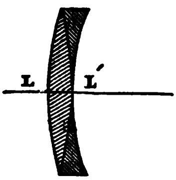

Fig. 363.

To simplify this, let us divest the preceding formula of its generality, and suppose that the first is a double convex lens L, *fig.* 363., with equal radii, and that the second is a double concave lens L′, the surface of which, in contact with the first, has the same curvature with it, and consequently the same radius. Observing that when the convexities are turned in contrary directions, the radii have contrary signs, the preceding formulæ will now be reduced to

$$\frac{n' - m'}{n'' - m''} = \frac{r_2 - P}{2\,r_2}.$$

Now it is always possible so to select the radii as to fulfil this condition; and therefore a compound lens, composed of two lenses of different refracting media, can always be constructed which will bring to the same focus two lights of different refrangibilities.

Let us suppose that the double convex lens is composed of crown glass, for which

$$n' = 1{\cdot}546566,\ m' = 1{\cdot}525832,$$

and the double concave of flint glass, for which

$$n'' = 1{\cdot}671062\ \ m'' = 1{\cdot}627749\cdot$$

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we shall therefore have

$$\frac{n' - m'}{n'' - m''} = \frac{20734}{43313} = \frac{r_2 - R_1}{2\, r_2};$$

from which we find that

$$r_2 = 23{\cdot}47 \times R_1.$$

The radius of the second surface of the double concave lens must in this case, therefore, be 23½ times the radius of the double convex.

It is easy to show that if the two lenses were composed of glass having equal dispersions, the result would not supply a solution of the problem; for in that case we should have the radius of the second surface of the double concave lens equal to the radius of the double convex, and consequently the refraction of the two lenses would neutralize each other, and parallel rays would emerge parallel. It is therefore essential to the solution of the problem that the two lenses should be composed of glass or other transparent media having different dispersive powers.

If the dispersive powers of the two lenses for every part of the light composing the spectrum were in the same ratio, which would be the case if the colours filled proportional spaces in the two spectra, the lenses, then, which would bring two coloured rays to the same focus would bring all the colours to that focus, and they would be absolutely achromatic. But it has been already explained that different transparent media not only produce spectra of different lengths, but divide them into coloured spaces in different proportions. It follows, therefore, that although the radii of the lenses be in the necessary proportion to their dispersive powers over lights of two particular colours, they will not be in the proportion necessary to bring the lights of other colours to the same focus. In this case, nevertheless, by bringing together the extreme images $r\,r'$ and $v\,v'$, the longitudinal chromatic aberration R V is considerably diminished; so much so, that in most cases the indistinctness of the image and the coloured fringes are not perceptible with a triple lens, so adapted as to achromatize rays of three refrangibilities, such as the extreme and mean rays of the spectrum: there is thus an annihilation of chromatic aberration for all practical purposes, so that achromatism may be conceived to be realized.

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CHAP. XIV.

THE EYE.

1082. *Sense of sight an extensive source of knowledge.*—Among the organs of sense there is none from which we derive so great a

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share of knowledge of external nature as the eye. Although, strictly speaking, this organ is cognizant only of light and colours, yet from the effects of them we are enabled by habit and reflection to infer with great promptitude and precision the forms, magnitudes, motions, distances, and positions, not only of the objects around us, but of the great bodies of the universe. Indeed, it is to the information derived from the eye alone that we are indebted for all the knowledge we possess of the material universe beyond the immediate precincts of the world we inhabit.

1083. *Knowledge of the structure of the eye necessary to comprehend optical instruments.* — The eye, therefore, is a subject of interesting inquiry, were it only for the importance of the information it conveys to us; but it is also necessary to understand its structure and functions before we can comprehend the use and application of those optical instruments which have been adapted with such marvellous success to enlarge the range of vision.

It is necessary first to investigate the powers of the organ of sight, and to determine the conditions which limit these powers, before we can appreciate the instruments by which these limits are extended.

1084. *Structure of the eye.* — The eyes, as they exist in the human species, have the form, as is well known, of two spheres, each about an inch in diameter, which are surrounded and protected by strong bony sockets placed on each side of the upper part of the nose. The external coating of these spheres is lubricated by a fluid secreted

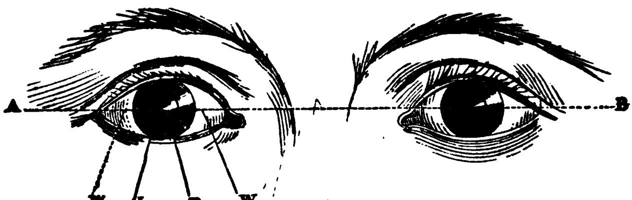

Fig. 364.

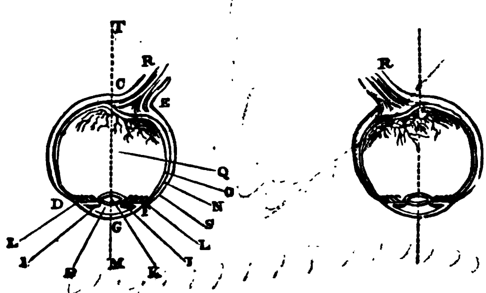

Fig. 365.

in adjacent glands, and spread upon them from time to time by the action of the eyelids in winking.

The eye-balls are moved by muscles connected with them within the socket which move them upon the principle known in mechanics as the ball and socket joint.

A front view of the eyes and surrounding parts is represented in *fig.* 364.; and a section of them made by a horizontal plane through the line A B, which passes through the centre of the point of the eye-balls, is represented in *fig.* 365.

1085. *The sclerotica and cornea.* — The external coating C D F E consists of a strong and tough membrane, called the *sclerotica*, or sclerotic coat. A part of this membrane is visible when the eye-lids are open at W, *fig.* 364., and is called the *white of the eye.* In this part of the eye-ball there is a circular opening formed in this sclerotic coat, which is covered by a thin and perfectly transparent shell D G F, called the *cornea.* This cornea is more convex than the general surface of the eye-ball, and may be compared to a watch-glass. It is connected round its edge with the sclerotica, which differs from it, however, both in colour and opacity, the sclerotica being white and opaque, while the cornea is perfectly colourless and transparent. The thickness of this cornea is everywhere the same.

The cornea covers that part of the point of the eye which is coloured, and is terminated round the coloured part at the commencement of the white of the eye.

1086. *The aqueous humour — the iris — the pupil.* — Within the cornea is a small chamber filled with a transparent liquid, called the *aqueous humour.* This chamber is partially divided by a thin annular partition I, called the iris, in the centre of which there is a circular aperture P, called the *pupil.* The *iris* is a membranous substance varying in colour in different individuals. It is this which gives the peculiar colour to the eye. It is the pupil which presents the appearance of a black spot in the centre of the coloured part of the eye. A front view of the iris and pupil is given at I and P, in *fig.* 364., and a section of them is indicated by the same letters in *fig.* 365.

1087. *The crystalline humour — ciliary processes.* — The chamber containing the aqueous humour is terminated at its posterior part by a substance in the form of a double convex lens, which contains another transparent liquid, called the *crystalline humour.* This lens K is somewhat greater in diameter than the pupil, and it is supported by a ring of muscles, called the *ciliary processes,* represented at L, in such a position that its axis passes through the centre of the pupil.

Thus the crystalline and the ciliary processes, with the cornea, include the membrane containing the aqueous humour.

1088. *The choroid.* — Within the sclerotica is a second coat N,

called the *choroid*. This is a vascular membrane which lines the internal surface of the sclerotic coat, and which terminates in front in the ciliary processes, by which the crystalline lens is set in it in the same manner as the cornea is set in the sclerotic coat.

Some anatomists maintain that the iris is only a continuation of the choroid, and that the cornea is a continuation of the sclerotic coat, which there becomes transparent. The inner surface of this choroid coat is covered with a slimy pigment of an intensely black colour, by which the reflection of the light entering the eye is prevented.

1089. *The retina — the vitreous humour.* — A third coating, represented at O, called the *retina*, from the resemblance of its structure to network, lines this black coating.

The internal membrane Q of the eye-ball contains another transparent liquor, called the *vitreous humour*, which is included in a membranous capsule, called the hyaloid.

Thus between the cornea and the posterior surface of the eye there are three successive humours; the aqueous, contained by the cornea; the crystalline, contained by the crystalline lens; and the vitreous, which fills the inner and larger chamber of the eye-ball.

1090. *The optic axis — the optic nerve.* — A straight line M T passing through the centre of the cornea, coinciding with the axis of the crystalline lens, and passing through the centre of the eye-ball, is called the *optical axis*, or the *axis of the eye*.

At a point of the posterior surface of the eye-ball between the optical axis M T and the nose, the sclerotic coat is formed into a tube, which leads backwards and upwards towards the brain. This tube contains within it the *optic nerve*, which at the point C E, where it enters the eye-ball, spreads out over the inner surface of the choroid and forms the retina, and immediately includes the *hyaloid capsule* containing the vitreous humour.

The retina must therefore be regarded as nothing more than the continuation and diffusion of the optic nerve.

The retina, which in dissection admits of being easily separated from the choroid, is absolutely transparent, so that the light or colours which enter the inner chamber of the eye are not intersected by it, but penetrate it as they would any other thin and perfectly transparent substance, and are only arrested by the black coating spread upon the choroid.

1091. *Numerical data connected with the human eye.* — The following are the average numerical data connected with the eye:—

	100ths of Inch.
Radius of sclerotic coating	39 to 43
Radius of cornea	28 — 32
External diameter of iris	43 — 47
Diameter of pupil	12 — 28
Thickness of cornea	4
Distance of pupil from centre of cornea	8
Distance of pupil from centre of crystalline	4
Radius of anterior surface of crystalline	28 — 39
Radius of posterior surface of crystalline	20 — 24
Diameter of crystalline	39
Thickness of do	30
Length of optic axis	87 — 95
Index of refraction from air into aqueous humour	1·3366
Index of refraction from air into vitreous humour	1·3394
Index of refraction from air into crystalline humour:—	
At the surface	1·3767
At the centre	1·3990
At the mean	1·3839
Index of refraction from aqueous humour to crystalline humour:—	
At the surface	1·0466
At the mean	1·0353
Index of refraction from vitreous humour to crystalline humour:—	
At the surface	1·0445
At the mean	1·0332

According to Sir D. Brewster, who has supplied the preceding indices of refraction, the focal length of the crystalline is 1·73 inches.

1092. *Limits of the play of the eye.* — The limits of the play of the eye-ball are as follows: — The optic axis can turn in the horizontal plane through an angle of 60° towards the nose, and 90° outwards, giving an entire horizontal play of 150°. In the vertical direction it is capable of turning through an angle of 50° upwards and 70° downwards, giving a total vertical play of 120°.

1093. *The eye not perfectly achromatic.* — Sir David Brewster is of opinion that the eye is not perfectly achromatic, but that the chromatic aberration is so small as to produce no indistinctness of vision. He says, if we shut up all the pupil, except a part of its edge, or look past the finger held near the eye, until the finger almost hides a narrow line of white light, we shall see a distinct prismatic spectrum of this line containing all the usual colours, — an effect which could not take place if the eye were perfectly achromatic.

1094. *But must be very nearly so.* — Nevertheless, it is certain that if the achromatism of the eye be not perfect, it is very nearly so. In the analogy observable between the forms and relative densities of the transparent humours which compose this organ, the achromatic combination of lenses is too striking to be casual; and we are irresistibly impressed with the conviction that the combination is made to be nearly achromatic. The two meniscuses formed by the aqueous and vitreous humours, having the double convex crystalline placed between them of greater density than either, and the two former differ-

ing from each other in density, appear to fulfil the conditions of achromatism in a striking manner; and it is doubtless to this combination that is due the apparent freedom from colour in the image depicted on the retina.

1095. *Spherical aberration of the eye corrected.* — Sir David Brewster is also of opinion that the spherical aberration of the eye is corrected by the varying density of the crystalline lens, which, having a greater refractive power near its centre, refracts the central rays in each pencil to the same point as its external rays.

The optic nerves R, which proceed from the two eyes, *decussate*, that is, cross each other like the letter X, before they reach the brain.

1096. *Effect of an illuminated object placed before the eye.* — The structure of the eye being thus understood, it will be easy to explain the effect produced within it by luminous or illuminated objects placed before it.

Let us suppose a pencil of light proceeding from any luminous object, such as the sun, incident upon that part of the eye-ball which is left uncovered by the open eye-lids.

That part of the pencil which falls upon the white of the eye W, *fig.* 364., is irregularly reflected, and renders visible that part of the eye-ball. Those rays of the pencil which fall upon the cornea pass through it. The exterior rays fall upon the iris, by which they are irregularly reflected, and render it visible. The internal rays pass through the pupil, are incident upon the crystalline, which, being transparent, is also penetrated by them, from which they pass through the vitreous humour, and finally reach the posterior surface of the inner part of the eye, where they penetrate the transparent retina, and are received by the black surface of the choroid, upon which they produce an illuminated spot.

The aqueous humour being more dense than the external air, and the surface of the cornea, which includes it, being convex, rays passing from the air into it will be rendered more convergent or less divergent. In like manner, the anterior surface of the crystalline lens being convex, and that humour being more dense than the aqueous, a further convergent effect will be produced.

Again, the posterior surface of the crystalline being convex towards the vitreous humour, and this latter humour being less dense than the crystalline, another convergent effect will take place. These rays passing successively through these three humours, are rendered at each surface more and more convergent.

1097. *Image formed within the eye.* — If an object be placed before the eye, pencils of rays will proceed from it, and penetrate the successive humours; and if these pencils be brought to a focus at the posterior surface of the eye, an inverted image of the object will be formed there, exactly as it would be formed by lenses composed of

any transparent medium whose refracting powers would correspond with each of the humours of the eye.

1098. *Experimental proof of its existence.* — That this phenomenon is actually produced in the interior of the eye may be rendered experimentally manifest by taking the eye-ball of an ox recently killed, and dissecting the posterior part, so as to lay bare the choroid. If the eye thus prepared be fixed in an aperture in a screen, and a candle be placed before it at a distance of eighteen or twenty inches, an inverted image of the candle will be seen through the retina, as if it were produced upon ground glass or oiled paper.

1099. *Immediate cause of vision.* — It appears, then, that the immediate cause of vision, and the immediate object of perception in the sensorium when we see, is the image thus depicted on the retina by means of the refracting powers of the humours of the eye.

1100. *Conditions of perfect vision.* — In order, therefore, to perfect vision, the following conditions must be fulfilled: —

1°. The image on the retina must be perfectly distinct.

2°. It must have sufficient magnitude.

3°. It must be sufficiently illuminated.

4°. It must continue on the retina for a sufficient length of time.

Let us examine the circumstances which affect these conditions.

1101. 1°. DISTINCTNESS OF THE IMAGE.

The image formed on the retina will be distinct or not, according as the pencils of rays proceeding from each point of the object placed before the eye, are brought to an exact focus on the retina or not. If they be not brought to an exact focus on the retina, their focus will be a point, therefore, beyond the retina, or within it.

In either case, the rays proceeding from any part of the object, instead of forming a corresponding point on the retina, will form a spot of more or less magnitude, according to the distance of the focus of the pencil from the retina, and the assemblage of such luminous spots will form a confused picture of the object. This deviation of the foci of the pencils from the retina is caused by the refracting powers of the eye being either too feeble or too strong. If the refracting power be too feeble, the rays are intercepted by the retina before they are brought to a focus; if the refracting power be too strong, they are brought to a focus before they arrive at the retina.

1102. *Effects of distant and near objects.* — The objects of vision may be distributed into two classes, in relation to the refracting powers of the eye: 1st, those which are at so great a distance from the eye, that the pencils proceeding from them may be regarded as consisting of parallel rays; 2dly, those which are so near that their rays have sensible divergence.

It has been stated that the diameter of the pupil varies from $\frac{1}{8}$ to $\frac{1}{4}$ an inch in magnitude, the variation depending upon a power of dilatation and contraction with which the iris is endued. Taking the

diameter of the pupil at its greatest magnitude of a quarter of an inch, pencils proceeding from an object placed at the distance of three feet from the eye would have an extreme divergence amounting to less than half a degree; and if the pupil be in its most contracted state when its diameter is only the one-eighth of an inch, then the divergence of the pencils proceeding from such an object would amount to about fifteen minutes of a degree. It may therefore be concluded, that pencils proceeding from all objects more distant from the eye than two or three feet, may be regarded as consisting of parallel rays.

The pencils of rays, therefore, proceeding from all such objects will be made to converge to the principal focus of the eye.

1103. *Position of the optical centre of the eye.* — Sir David Brewster concludes from observations made by him that the optical centre of the eye, that is to say, the point at which the axes of secondary pencils intersect the optic axis, is situate in the geometric centre of the eye-ball, and consequently must be a little within the crystalline. If, therefore, round this centre we imagine a spherical surface described, whose radius is equal to the focal distance of the combination of the humours of the eye, the image of all objects more distant from the eye than two or three feet will be found on such a surface. Now, since the retina is spread over the surface of the choroid, and since the form of the eye is spherical, and its diameter but an inch, it follows that the retina is a spherical surface, whose centre coincides with the optical centre of the eye, and which is at a distance from that centre of about half an inch. If the distance of the retina from this centre be exactly equal to the focal distance of the humours, then the foci of all pencils of parallel rays entering the eye will be formed upon it, and consequently it will receive distinct images of all objects whose distance from the eye exceeds two or three feet. But if the focal distance of the humours be less or greater than that, then, as already stated, the image on the retina will be indistinct.

1104. *Optical remedies for defects in the refracting powers of the eye.* — The remedy for such a defect in vision is supplied by the properties of convergent and divergent lenses, already explained.

If the eye possess too little convergent power, a convergent lens is placed before it, which, receiving the parallel pencils, renders them convergent when they enter the pupil, and this enables the eye to bring them to a focus on the retina, provided the power of the lens be equal to the deficient convergence of the eye.

If, on the other hand, the convergent power of the eye be too great, so that the parallel rays are brought to a focus before arriving at the retina, a divergent lens is placed before the eye, by means of which parallel pencils are rendered divergent before they enter the pupil; and the power of the lens is so adapted to the convergent power of the eye, that the rays shall be brought to a focus on the retina.

The two opposite defects of vision here indicated are generally called, the one *weak-sightedness* or *far-sightedness*, and the other *near-sightedness*.

If the objects of vision be placed so near the eye that the rays composing the pencils which proceed from them have sensible divergence, then the foci of these rays within the eye will be at a distance from the optical centre greater than the principal focus. If, therefore, in this case, the principal focus fall upon the retina, the focus of rays proceeding from such near objects would fall beyond it, and consequently the image on the retina would be indistinct.

1105. *Power of the eye to adapt itself to objects differently distant.* — It follows, therefore, that eyes which see distant objects at the greater class of distances would see indistinctly all objects at less distances, unless there were in the eye some means of self-adjustment, by which its convergent power may be augmented. Such means of self-adjustment are provided, which operate within certain limits, and by which we are enabled so to accommodate the eye to the divergence of the pencils proceeding from near objects, that the same eyes which are capable of seeing distinctly objects sensibly so distant as to render the rays of the pencils sensibly parallel, are also capable of seeing with equal distinctness objects at distances varying from ten to twelve inches and upwards.

1106. *Experimental proof of this power.* — By what means the convergent power of the humours is thus varied is not certainly known, but that such means of self-adjustment exist may be proved by the following experiment.

Let a small black spot be made upon a thin transparent plate of glass, and let it be placed at a distance of about twelve inches from the eye. If the eye be directed to it, the spot will be seen as well as distant objects visible through the glass. Let the attention be earnestly directed to the black spot, so that a distinct perception of its form may be produced. The objects visible at a distance will then be found to become indistinct.

But if the attention be directed more to the distant objects, so as to obtain a distinct perception of them, the perception of the black spot on the glass will then become indistinct. It is evident, therefore, that when the eye accommodates itself so as to form upon the retina a distinct image of an object at twelve inches' distance, the image produced by objects at great distances will become indistinct; and that, on the other hand, when the eye so accommodates itself as to render the image produced on the retina by distant objects distinct, the image produced by an object at twelve inches distance will become indistinct.

1107. *Hypotheses which explain this power.* — It is evident, therefore, that the power of the eye to refract the pencils of light incident upon it, is to a certain extent under the control of the will;

but by what means this change in the refracting power of the organ is made is not so apparent. Various hypotheses have been advanced to explain it. According to some, the form of the eye-ball, by a muscular action, is changed in such a manner as to increase the length of the optic axis, and thus to remove the posterior surface of the retina to a greater distance from the crystalline, when it is necessary to obtain a distinct view of near objects; and, on the contrary, to elongate the transverse diameter of the eye, and shorten the optic axis so as to bring the retina closer to the crystalline, when it is desired to obtain a distinct view of distant objects.

According to others, this change of form is only effected in the cornea, which being rendered more or less convex by a muscular action, gives a greater or less convergent power to the aqueous humour.

According to others, the eye accommodates itself to different distances by the action of the crystalline, which is moved by the ciliary processes either towards or from the cornea, thus transferring the focus of rays proceeding from it within a certain limit of distance to and from the retina; or, by a similar action of the ciliary process, the crystalline lens may be supposed to be rendered more or less convex, and thus to increase or diminish its convergent power.

None of these hypotheses have, however, found general acceptation. It is denied as a matter of fact, that the eye-ball is elongated, or that the curvature of the cornea is changed; and it is doubted, to say the least of it, that the crystalline is capable either of displacement or change of convexity.

1108. *Explanation proposed by M. Pouillet.* — M. Pouillet maintains (and affirms that his opinion is founded on the dissection of a great number of crystalline lenses) that this humour is composed of layers or strata one within another, differing in curvature and density, so that its section would exhibit a series of concentrical ellipses having varying eccentricities. It would follow from this, that the internal strata being more curved and more dense than the external strata, the rays which pass from the latter will converge to a more distant point than those which pass from the former. The crystalline, therefore, according to M. Pouillet, has not one but many different foci.

When a pencil of rays falls upon it, those rays which are near the axis of the pencil, and therefore near the centre of the crystalline, are brought to a shorter focus than those which are near the borders. According to the hypothesis advanced by M. Pouillet, the eye sees near objects, therefore, by means of the central rays, and distant objects by means of those rays which fall near the borders of the crystalline.

The pencils which proceed from near objects being more divergent than those which proceed from distant objects, are refracted by the central part of the crystalline, so as to be brought to a focus on the retina, while those rays of the same pencil which would fall upon the

borders of the crystalline would be brought to a focus beyond the retina.

When pencils, however, proceed from objects so distant that the rays composing them may be regarded as parallel, the central rays of such pencils would be brought to a focus before arriving at the retina, while the rays falling near the borders of the crystalline would be brought to a focus upon the retina. Now, according to these conditions, it would follow, that a certain confusion of vision would ensue in both cases; for near objects, the image produced by the central rays would be rendered confused by the rays passing near the borders of the crystalline, which meet the retina before they are brought to a focus; and in the case of distant objects, the image formed by the rays passing near the borders of the crystalline would be rendered confused by those which pass near the centre of the crystalline, and which are brought to a focus before they arrive at the retina.

M. Pouillet meets these difficulties by the following considerations. He supposes that when the eye views near objects the pupil contracts itself, so as to intercept to a greater or less extent the external rays of the pencils, and to admit only to the crystalline those which fall immediately under its axis. In this way the confusion which would be produced by the external rays of the pencils is prevented. But the same expedient would not prevent the confusion produced in the image of distant objects by the central rays of the pencils brought to a focus before arriving at the retina. This difficulty M. Pouillet answers, by stating that the comparative number of the central rays is so small that their action upon the retina is inconsiderable compared with that of the external rays, and that consequently their effect is not sensible.

M. Pouillet appeals to observation to establish the fact that the pupil always contracts when the eye views near objects.

1109. *Limits of the power of adaptation to varying distance.*— Whatever be the provisions made in the organization of the eye, by which it is enabled to adapt itself to the reception of divergent pencils proceeding from near objects, the power with which it is thus endued has a certain limit. Thus, eyes which see distinctly distant objects, and which therefore bring parallel rays to a focus on the retina in their ordinary state, are not capable of seeing distinctly objects brought nearer to them than ten or twelve inches. The power of accommodating the vision to different rays is therefore limited to a divergence not exceeding that which is determined by the diameter of the pupil compared with a distance of ten or twelve inches. Now, as the diameter of the pupil is most contracted when the organ is directed to such near objects, we may assume it at its smallest magnitude at one-eighth of an inch, and therefore the divergence of a pencil proceeding from a distance of twelve inches would be $\frac{1}{96}$th, and the angle of divergence would therefore be very nearly half a degree.

It may, therefore, be assumed that eyes adapted to the vision of distant objects are in general incapable of seeing distinctly objects from which pencils have greater divergence than this, or, which is the same, objects applied at less than ten or twelve inches from the eye.

1110. *Case of eyes having feeble convergent power.* — In the case of eyes whose convergent power is too feeble to bring pencils proceeding from distant objects to a focus on the retina, they will be in a still greater degree inadequate to bring pencils to a focus which diverge from near objects; and consequently such eyes will require to be aided, for near as well as distant objects, by the interposition of convergent lenses. It would, however, be necessary to provide lenses of different convergent powers for distant and near objects, the latter requiring a greater convergent power than the former; and in general the nearer the objects viewed, the greater the convergent power required from the lens.

1111. *Case of eyes having strong convergent power.* — In the case of eyes whose convergent power is so great as to bring pencils proceeding from distant objects to a focus short of the retina, and which therefore, for such distant objects, require the intervention of divergent lenses, distinct vision will be attained without the interposition of any lens, provided the object be placed at such a distance that the divergence of the pencils proceeding from it shall be such that the convergent power of the eye bring them to a focus on the retina.

Hence it is that eyes of this sort are called *short-sighted*, because they can see distinctly such objects only as are placed at the distance which gives the pencils proceeding from them such a divergence, that the convergent power of the eye would bring them to a focus on the retina.

1112. *Method of ascertaining the power of the lens required by defective eyes.* — If it be desired to ascertain the focal length of the divergent lens which such an eye would require to see distant objects distinctly, it is only necessary to ascertain at what distance it is enabled to see distinctly the same class of objects without the aid of a lens. A lens having a focal length equal to this distance will enable the eye to see distant objects distinctly, because such a lens would give the parallel rays a divergence equal to the divergence of pencils proceeding from a distance equal to its focal length.

1113. *Power of adaptation to varying distance in short-sighted eyes.* — Persons are said to be more or less near-sighted, according to the distance at which they are enabled to see objects with perfect distinctness, and they accordingly require, to enable them to see distant objects distinctly, diverging lenses of greater or less focal length.

As persons who are enabled to see distant objects distinctly have the power of accommodating the eye so as to see objects at ten or

twelve inches' distance, so short-sighted persons have a similar power of accommodation, but within proportionally smaller limits. Thus a short-sighted person will be enabled to see distinctly objects placed at distances from the eye varying from two or three inches upwards, according to the degree of short-sightedness with which he is affected.

1114. *Causes of short sight and long sight.*—The two opposite defects of vision which have been mentioned, arising from too great or too little convergent power in the eye, may arise, either from a defect in the quality of the humours or in the form of the eye. Thus near-sightedness may arise from too great convexity in the cornea or in the crystalline, or it may arise from too great a difference of density between the aqueous humour and the crystalline, or between the crystalline humour and the vitreous, or both of them; or, in fine, it may arise from defects both of the form and of the relative densities of the humours.

1115. *Defective sight arising from imperfect transparency of the humours.*—In a certain class of maladies incidental to the sight, the humours of the eye lose in a greater or less degree their transparency, and the crystalline humour is more especially liable to this. In such cases vision is sometimes recovered by means of the removal of the crystalline humour, in which case the vision is reduced to two humours, the aqueous and the vitreous; but as the eye owes in a greater degree to the crystalline than to the other humours the convergent power, it is necessary in this case to supply the place of the crystalline by a very strong convergent lens placed before the eye.

1116. 2°. MAGNITUDE OF THE IMAGE ON THE RETINA.

In order to obtain a perception of any visible object, it is not enough that the image on the retina be distinct, it must also have a certain magnitude.

Let us suppose that a white circular disk, one foot diameter, is placed before the eye at a distance of 57½ feet.

The axes of the pencils of rays proceeding from such disk to the eye will be included within a cone, whose base is the disk, and whose vertex is in the centre of the eye.

These axes, after intersecting at the centre of the eye, will form another cone, whose base will be the image of the disk formed upon the retina. The common angle of the two cones will in this case be 1°.

Let A B, *fig.* 366., be the diameter of the disk. Let C be the centre of the eye, and let *b a* be the diameter of the image on the

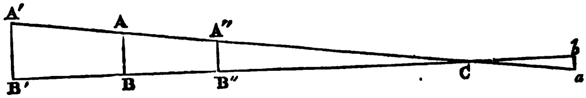

Fig. 366.

retina. It is clear, from the perfect similarity of the triangles A C B and a C b, that the diameter of the image $b\,a$ will have to the diameter of the object B A the same proportion as the distance a C of the retina from the centre C has to the distance A C of the object from the same centre. Therefore in this case, since one-half the diameter of the eye is but half an inch, and the distance A C is in this case supposed to be 57½ feet, the magnitude of the diameter $b\,a$ of the image on the retina will be found by the following proportion:—

$$a\,b : \text{A B} :: \tfrac{1}{2} : 57\tfrac{1}{2} \times 12 = 690.$$

Therefore we have

$$a\,b = \frac{\frac{1}{2} \times \text{A B}}{690} = \frac{6}{690} = \frac{1}{115}.$$

The total magnitude, therefore, of the diameter of the image on the retina would in this case be the $\frac{1}{115}$th part of an inch; yet such is the exquisite sensibility of the organ, that the object is in this case distinctly visible.

If the disk were removed to twice the distance here supposed, the angle of the cone C would be reduced to half a degree, and the diameter of the image on the retina would be reduced to one-half its former magnitude, that is to say, to the $\frac{1}{230}$th part of an inch. If, on the other hand, the disk were moved towards the eye, and placed at half its original distance, then the angle C of the cone would be 2°, and the diameter of the picture on the retina would be double its first magnitude, that is to say, the $\frac{2}{115}$th of an inch.

In general, it may therefore be inferred that the magnitude of the diameter of the picture on the retina is increased or diminished in exactly the same proportion as the angle of the cone C, formed at the centre of the eye, is increased or diminished.

1117. *The visual angle or apparent magnitude.*—This angle is called the visual angle or apparent magnitude of the object; and when it is said that a certain object subtends at the eye a certain angle, it is meant that lines drawn from the extremities of such object to the centre of the eye form such angle.

The *apparent magnitude* of an object must not be confounded with its apparent superficial magnitude, the term being invariably applied to its *linear magnitude*. The apparent superficial magnitude varies in proportion to the square of the apparent magnitude.

Thus, for example, when the disk A B is removed to double its original distance from the eye, the apparent magnitude, or the angle C, is diminished one-half, and consequently the diameter $a\,b$ of the picture on the retina is also diminished one-half; and since the diameter is diminished in the ratio of 2 to 1, the superficial magnitude of the image, or its area, will be diminished in the proportion of 4 to 1.

1118. *Apparent magnitude increases in proportion as the distance diminishes, and vice versâ.*—It is clear from what has been stated

also, that when the same object is moved from or towards the eye, its apparent magnitude varies inversely as its distance; that is, its apparent magnitude is increased in the same proportion as its distance is diminished, and *vice versâ*.

It is easy to perceive that the objects which are seen under the same visual angle will have the same apparent magnitude. Thus let A′ B′, *fig*. 366., be an object more distant than A B, and of such a magnitude that its highest point A′ shall be in the continuation of the line C A, and its lowest point B′ in the continuation of the line C B. The apparent magnitude of A′ B′ will then be measured by the angle at C. This angle will therefore at the same time represent the apparent magnitude of the object A B and of the object A′ B′. It is evident that an eye placed at C will see every point of the object A B upon the corresponding points of the object A′ B′; so that if the object A B were opaque, and of a form similar to the object A′ B′, every point of the one would be seen upon a corresponding point of the other. In like manner, if an object A″ B″ were placed nearer the eye than A B, so that its highest point may lie upon the line C A, and its lowest point upon the line C B, the object, being similar in form to A B, would appear to be of the same magnitude. Now it is evident that the real magnitudes of the three objects A″ B″, A B, and A′ B′, are in proportion of their respective distances from the eye; A′ B′ is just so much greater than A B, and A B than A″ B″, as C B′ is greater than C B, and as C B greater than C B″.

Thus it appears that if several objects be placed before the eye in the same direction at different distances, and that the real linear magnitudes of these objects are in the proportion of their distances, they will have the same apparent magnitude.

1119. *Case of the sun and moon illustrates this.* — A striking example of this principle is presented by the case of the sun and moon. These objects appear in the heavens equal in size, the full moon being equal in apparent magnitude to the sun. Now it is proved by astronomical observation that the real diameter of the sun is, in round numbers, four hundred times that of the moon; but it is also proved that the distance of the sun from the earth is also, in round numbers, four hundred times greater than that of the moon. The distance, therefore, of these two objects being in the same proportion as their real diameter, their visual or apparent magnitudes are equal.

1120. *Apparent magnitude corresponds with the real magnitude of the picture on the retina.* — It is evident from what has been explained, that objects which have equal apparent magnitudes, and are therefore seen under equal visual angles, will have pictures of equal magnitude on the retina, a fact which proves that the visual angle is the measure of the apparent magnitude.

1121. *The apparent magnitude of an object diminished by removing it from the eye.* — If the same object be moved successively to

increasing distances, its apparent magnitude will be diminished in the same proportion, exactly as its distance from the eye is increased. Thus, if L M, *fig.* 367., be such an object, its distance E M being ex-

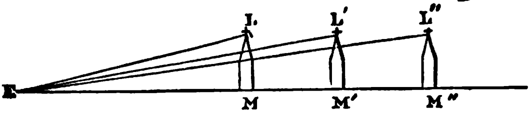

Fig. 367.

pressed by D, and its height L M by H, the visual angle L E M, which measures its apparent magnitude, will be expressed, according to what what has been formerly explained, by $\frac{H}{D}$. If the object be now removed to double its former distance, such as E M′, the visual angle or apparent magnitude L′ E M′ will be expressed by $\frac{H}{2D}$, which is just one-half $\frac{H}{D}$, the former visual angle; and, in like manner, if the object be removed to three times its first distance, such as E M″, its visual angle or apparent magnitude will be $\frac{H}{3D}$, which is one-third of its original apparent magnitude.

1122. *Apparent superficial magnitude.* — The apparent superficial magnitude of a body is determined by a section of the body made by a plane at right angles to the lines containing the visual angle. Thus, the apparent superficial magnitude of the sun or moon is determined by a section of those bodies passing through the points where lines drawn from the eye touching them would meet them, which, in consequence of the great distance of these bodies, would be a circular section through their centres, and at right angles to a line drawn from the centre to the eye.

1123. *Section of vision.* — This circle, in the case of the sun or moon or other celestial object, is called the *circle of vision;* and a corresponding section of any other object drawn at right angles to the sides of the visual angle would be called the *section of vision.*

For all distant objects, this section is a plane at right angles to the direction in which the object is seen.

1124. *The smallest magnitudes which can be distinctly seen.* — If the circular disk A B, *fig.* 366., which we have supposed to be presented before the eye at a distance of fifty-seven and a half times its own diameter, and which therefore subtends at the centre of the eye a visual angle of 1°, be removed to a distance sixty times greater, or to a distance equal to 3,450 times its own diameter, it will subtend an angle at C proportionally less, which will therefore be, in this case, an angle of one minute; and if it be removed to double the latter dis-

tance, or 6,900 times its own diameter, it will subtend a visual angle of thirty seconds. Now it is found that if such an object be directly illuminated by the sun, it will be barely visible. This limit, however, depends as well on the colour of the object as on the degree of its illumination. Plateau affirms that a white disk, such as we have here supposed to be presented to the eye, if the light of the sun shone fully upon it, will be visible when seen under a visual angle of twelve se conds, or the one-fifth part of a minute. The disk would subtend thi. angle at the eye if placed at a distance equal to 17,250 times its diameter.

He says also that if the disk, under the same circumstances, were red, it would be distinctly seen until its apparent magnitude were reduced to twenty-three seconds; and that if it were blue, the limit would be twenty-six seconds; but that, if instead of being illuminated by the direct solar light, it were illuminated by the light of day reflected from the clouds, these limiting angles would be half as large again.

1125. *Distinctness of vision compared with the magnitude of the pictures on the retina.*—Nothing can be more calculated to excite our wonder and admiration than the distinctness of our perception of visible objects, compared with the magnitude of the picture on the retina, from which immediately we receive such perception.

1126. *Example of the picture of the full moon on the retina.*—If we look at the full moon on a clear night, we perceive with considerable distinctness by the naked eye the lineaments of light and shade which characterize its disk.

Now let us consider only for a moment what are the dimensions of the picture of the moon formed on the retina, from which alone we derive this distinct perception.

The disk of the moon subtends a visual angle of half a degree, and consequently, according to what has been explained, the diameter of its picture on the retina will be $\frac{1}{230}$th part of an inch, and the entire superficial magnitude of the image from which we derive this distinct perception is less than the $\frac{1}{52900}$th part of a square inch; yet within this minute space, we are able to distinguish a multiplicity of still more minute details. We perceive, for example, forms of light and shade, whose linear dimensions do not exceed one-tenth part of the apparent diameter of the moon, and which therefore occupy upon the retina a space whose diameter does not exceed the $\frac{1}{5290000}$th part of a square inch.

1127. *Example of the human figure.*—To take another example, the figure of a man 70 inches high, seen at a distance of 40 feet, produces an image upon the retina the height of which is about one-fourteenth part of an inch. The face of such an image is included in a circle whose diameter is about one-twelfth of the height, and therefore occupies on the retina a circle whose diameter is about the

$\frac{1}{178}$th part of an inch; nevertheless, within this circle, the eyes, nose, and lineaments are distinctly seen. The diameter of the eye is about one-twelfth of that of the face, and therefore, though distinctly seen, does not occupy upon the retina a space exceeding the $\frac{1}{4000000}$th of a square inch.

If the retina be the canvas on which this exquisite miniature is delineated, how infinitely delicate must be its structure, to receive and transmit details so minute with such marvellous precision; and if, according to the opinion of some, the perception of these details be obtained by the retina *feeling* the image formed upon the choroid, how exquisitely sensitive must be its touch!

1128. 3°. Sufficiency of illumination.

It is not enough for distinct vision that a well-defined picture of the object shall be formed on the retina. This picture must be sufficiently illuminated to affect the senses, and at the same time not be so intensely illuminated as to overpower the organ.

Thus it is possible to conceive a picture on the retina so extremely faint as to be insufficient to produce sensation, or, on the other hand, so intensely brilliant as to dazzle the eye, to destroy the distinctness of sense, and to produce pain.

When we direct the eye to the sun, near the meridian, in an unclouded sky, we have no distinct perception of his disk, because the splendour is so great as to overpower the sense of vision, just as sounds are sometimes so intense as to be deafening.

That it is the intense splendour alone which prevents a distinct perception of the solar disk in this case is rendered manifest by the fact that if a portion of the solar rays be intercepted by a coloured glass, or by a thin cloud, a distinct image of the sun will be seen.

When we direct the eye to the firmament on a clear night, there are innumerable stars which transmit light to the eye, and which therefore must produce some image on the retina, but of which we are altogether insensible, owing to the faintness of the illumination. That the light, however, does enter the eye and arrive at the retina is proved by the fact that if a telescope be directed to the stars in question, so as to collect a greater quantity of their light upon the retina, they will become visible.

1129. *The eye has power of accommodation to different degrees of illumination.*—The eye possesses a certain limited power of accommodating itself to various degrees of illumination. Circumstances which are familiar to every one render the exercise of this power evident.

If a person, after remaining a certain time in a dark room, pass suddenly into another room strongly illuminated, the eye suffers instantly a degree of inconvenience, and even pain, which causes the eyelids to close; and it is not until after the lapse of a certain time that they can be opened without inconvenience.

The cause of this is easily explained. While the observer remains in the darkened or less illuminated room, the pupil is dilated so as to admit into the eye as great a quantity of light as the structure of the organ allows of. When he passes suddenly into the strongly illuminated room, the flood of light arriving through the widely dilated pupil acts with such violence on the retina as to produce pain, which necessarily calls for the relief and protection of the organ. The iris, then, by an action peculiar to it, contracts the dimensions of the pupil so as to admit proportionally less light, and the eye is opened with impunity.

Effects the reverse of these are observed when a person passes from a strongly illuminated room into one comparatively dark, or into the open air at night. For a certain time he sees nothing, because the contraction of the pupil, which was adapted to the strong light to which it had previously been exposed, admits so little light to the retina that no sensation is produced. The pupil, however, after awhile dilates, and, admitting more light, objects are perceived which were before invisible.*

1130. *Relative brilliancy of equidistant luminaries.—Brightness of the picture on the retina.*— If two points from which light radiates be placed at the same distance from the eye, the brightness of their image on the retina will be in proportion to their absolute brilliancy. But if either point be removed to a greater distance, the number of rays passing from it which enter the pupil will be diminished in the same proportion as the square of its distance is increased, and *vice versâ.* It consequently follows that the brightness of each point of the image to an object formed upon the retina will be in direct proportion to the absolute brilliancy of such point, and in the inverse proportion of the square of its distance from the eye.

Thus, if I express the intensity of the light of the point upon the object, and D its distance from the eye, then the brightness of the image of such point upon the retina will be expressed by $\frac{I}{D^2}$.

It is therefore clear that the brightness of the image of each point of an object will be diminished as the square of the distance of the object from the eye is increased.

1131. *Apparent brightness the same at all distances.*— It is sometimes inferred from this, though erroneously, that the apparent splen-

* "Thus, when the lamp that lighted
The traveller at first, goes out,
He feels awhile benighted,
And wanders on in fear and doubt;
But soon the prospect clearing,
In cloudless starlight on he treads,
And finds no lamp so cheering
As that light which Heaven sheds."—MOORE.

dour of the image of a visible object decreases as the square of the distance increases. This would be the case in the strictest sense, if, while the object were withdrawn from the eye to an increased distance, its image on the retina continued to have the same magnitude; for, in this case, the absolute brightness of each point composing such image would diminish as the square of the distance increases, and the area of the retina over which such points are diffused would remain the same; but it must be considered, that as the object retires from the eye the superficial magnitude of the image on the retina is diminished in the same proportion as the square of the distance of the object from the eye is increased. It therefore follows that while the points composing the image on the retina are diminished in the intensity of their illumination, they are collected into a smaller space, so that what each point of the image on the retina loses in splendour, the entire image gains by concentration.

1132. *If the distance of the sun were increased or diminished, its apparent magnitude would be changed, but its apparent brightness would remain the same.* — If the sun were brought as close to the earth as the moon, its apparent diameter would be 400 times greater, and the area of its apparent disk 160,000 times greater than at present, but the apparent brightness of its surface would not be in any degree increased. In the same manner, if the sun were removed to ten times its present distance, it would appear under a visual angle ten times less than at present, as in fact it would to an observer on the planet Saturn, and its visible area would be a hundred times less than it is, but the splendour of its diminished area would be exactly the same as the present splendour of the sun's disk.

These consequences, which are of considerable physical importance, obviously follow from the principles explained above.

The sun seen from the planet Saturn has an apparent diameter ten times less than it has when seen from the earth.

The appearance from Saturn will then be the same as would be the appearance of a portion of the disk of the sun seen from the earth through a circular aperture in an opaque plate, which would exhibit a portion of the disk whose diameter is one-tenth of the whole.

1133. *An object may be visible even though it have no sensible visual magnitude.* — *The fixed stars examples of this.* — When the light which radiates from a luminous object has a certain intensity, it will continue to affect the retina in a sensible manner, even when the object is removed to such a distance that the visual angle shall cease to have any perceivable magnitude. The fixed stars present innumerable examples of this effect. None of these objects, even the most brilliant of them, subtend any sensible angle to the eye. When viewed through the most perfect telescopes they appear merely as brilliant points. In this case, therefore, the eye is affected by the light alone, and not by the magnitude of the object seen.

1134. *By increase of distance, however, such objects may cease to affect the retina sensibly.* — Nevertheless, the distance of such an object may be increased to such an extent that the light, intense as it is, will cease to produce a sensible effect upon the retina.

It will be explained in the second volume of this series, that seven classes of the fixed stars, diminishing gradually in brightness,* produce an effect on the retina such as to render them visible to a naked eye. This diminution of splendour is produced by their increased distance. The telescope, however, as has been already stated, brings into view innumerable other stars, whose intrinsic splendour is as great as the brightest among those which we see, but which do not transmit to the retina, without the aid of the telescope, enough of light to produce any sensible effect. Nevertheless it is demonstrable that, even without the telescope, they do transmit a certain definite quantity of light to the retina; the quantity of light which they thus transmit, and which is insufficient to produce a sensible effect, having to the quantity obtained by the telescope a ratio depending upon the proportion of the magnitude of the object-glass of the telescope to the magnitude of the pupil.

1135. *The intensity of illumination necessary to produce sensation also depends on the relative splendour of other objects present before the eye.* — The quantity and intensity of the light transmitted by an external object to the retina, which is sufficient to produce a perception of such object, depends also upon the light received at the same time by the retina from other objects present before the eye. The proof of this is, that the same objects which are visible at one time are not visible at another, though equally before the eye, and transmitting equal quantities of light of the same intensity to the retina. Thus, the stars are present in the heavens by day as well as by night, and transmit the same quantity of light to the retina, yet they are not visible in the presence of the sun, because the light proceeding from that luminary, directly and indirectly reflected and refracted by the air and innumerable other objects, is so much greater in quantity and intensity as to overpower the inferior and much less intense light of the stars. This case is altogether analogous to that of the ear, which, when under the impression of loud and intense sounds, is incapable of perceiving sounds of less intensity, which nevertheless affect the organ in the same manner as they do when, in the absence of louder sounds, they are distinctly heard.

Even when an object is perceived, the intensity of the perception is relative, and determined by other perceptions produced at the same time. Thus, the moon seen at night is incomparably more splendid than the same moon seen by day or in the twilight, although in each

* The term magnitude is used in astronomy, as applied to the fixed stars, to express their apparent brightness; no fixed star, however splendid, subtends any sensible angle.

case the moon transmits precisely the same quantity of light, of precisely the same intensity, to the eye; but in the one case the eye is overpowered by the superior splendour of the light of day, which dims the comparatively less intense light proceeding from the moon.

1136. 4°. THE IMAGE MUST CONTINUE A SUFFICIENT TIME UPON THE RETINA TO ENABLE THAT MEMBRANE TO PRODUCE A PERCEPTION OF IT.

It will be proved hereafter that the velocity with which light is propagated through space is at the rate of about 200,000 miles per second. Its transmission, therefore, from all objects at ordinary distances to the eye may be considered as instantaneous. The moment, therefore, any object is placed before the eye an image of it is formed on the retina, and this image continues there until the object is removed. Now it is easy to show experimentally that an object may be placed before the eye for a certain definite interval of time, and that a picture may be painted upon the retina during that interval without producing any perception or any consciousness of the presence of the object.

To illustrate this, let a circular disk A B C D, *fig.* 368, about 20 inches in diameter, be formed in card or tin, and let a circle A′ B′ C′ D′ be described upon it, about 2 inches less in radius than the disk, so as to leave between the circle and the disk a zone about two inches wide. Let the entire zone be blackened, except the space A M M′ A′, forming about the one-twentieth of it. Let the disk thus prepared be attached to the back of a blackened screen, so as to be capable of revolving behind it, and let a hole one inch in diameter be made in the screen at any point, behind which the zone A B C D is placed. If the disk be now made to revolve behind the screen, the hole will appear as a circular white spot so long as the white space A M passes behind it, and will disappear, leaving the same black colour as the screen during the remainder of the revolution of the disk. The hole will therefore be seen as a white circular spot upon the black screen during one-twentieth of each revolution of the disk. If the disk be now put in motion at a slow rate, the white hole will be seen on the screen during one-twentieth of each revolution. If the velocity of rotation imparted to the disk be gradually increased, the white spot will ulti-

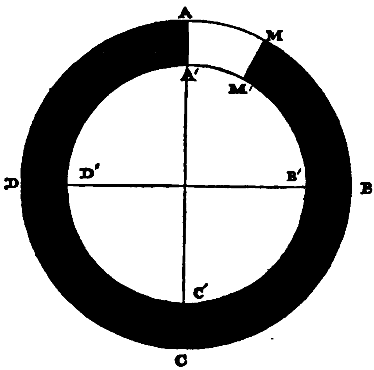

Fig. 368

mately *disappear*, and the screen appear of a uniform black colour, although it be certain that during the twentieth part of each revolution, whatever be the rate of rotation, a picture of the white spot is formed on the retina.

1137. *To determine experimentally the time a picture must continue on the retina to produce sensation.* — The length of time necessary in this case for the action of light upon the retina to produce sensation may be determined by ascertaining the most rapid motion of the disk which is capable of producing a distinct perception of the white spot. This interval will be found to vary with the degree of illumination. If the spot be strongly illuminated, a less interval will be sufficient to produce a perception of it; if it be more feebly illuminated, a longer interval will be required. The experiment may be made by varying the colour of the space A M of the zone, and it will be found that the interval necessary to produce sensation will vary with the colour as well as with the degree of illumination.

1138. *The perception of a visible object is continued for a certain time after the object is removed from before the eye.* — Numerous observations on the most familiar effects of vision, and various experiments expressly contrived for the purpose, show that the retina, when once impressed with the picture of an object placed before the eye, retains this impression, sometimes with its full intensity and sometimes more faintly, just as the ear retains for a time the sensation of a sound after the cause which has put the tympanum in vibration has ceased to act. The duration of this impression on the retina, after the removal of the visible object which produced it, varies according to the degree of illumination and the colour of the object. The more intense the illumination, and the brighter the colour, the longer will be the interval during which the retina will retain their effects.

1139. *Experimental illustration of this.* — To illustrate this experimentally, let a circular disk formed of blackened card or tin, of 12 or 14 inches in diameter, be pierced with 8 holes round its circumference, at equal distances, each hole being about half an inch in diameter, as represented in *fig.* 369.

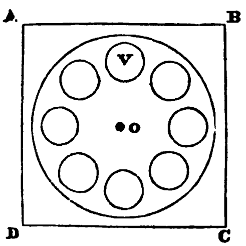

Fig. 369.

Let this disk be attached upon a pivot or pin at its centre O to a board A B C D, which is blackened everywhere, except upon a circular spot at V, corresponding in magnitude to the holes made in the circular plate.

Let this spot be first supposed to be white. Let the circular disk be made to revolve upon the point O, so as to bring the circular holes successively before the white spot at V. The retina

will thus be impressed at intervals with the image of this circular white spot. In the intervals between the transits of the holes over it, the entire board will appear black, and the retina will receive no impression. If the disk be made to revolve with a very slow motion, the eye will see the white spot at intervals, but if the velocity of rotation be gradually increased, it will be found that the eye will perceive the white spot permanently represented at V, as if the disk had been placed with one of its holes opposite to it without moving. It is evident, therefore, that in this case the impression produced upon the retina, when any hole is opposite the white spot, remains until the succeeding hole comes opposite to it, and thus a continued perception of the white spot is produced.

If the white spot be illuminated in various degrees, or if it be differently coloured, the velocity of the disk necessary to produce a continuous perception of it will differ. The brighter the colour and the stronger the illumination, the less will be the velocity of rotation of the disk which is necessary to produce a continuous perception of the spot.

These effects show that the stronger the illumination and the brighter the colour, the longer is the interval during which the impression is retained by the retina.

1140. *Why we are not sensible of darkness when we wink.*— This continuance of the impression of external objects on the retina, after the light from the objects ceases to act, is also manifested by the fact, that the continual winking of the eyes for the purpose of lubricating the eye-ball by the eye-lid does not intercept our vision. If we look at any external objects, they never cease for a moment to be visible to us, notwithstanding the frequent intermissions which take place in the action of light upon the retina, in consequence of its being thus intercepted by the eye-lid.

1141. *Experimental illustration suggested by Sir D. Brewster.* — According to Sir David Brewster, the most instructive experiment on this subject, which, however, requires a great deal of practice to be made with success, is to look for a short time at a window at the end of a long room, and then suddenly to turn the eye to a dark wall. In general, a common observer will in this case see a representation of the window on the wall, in which, however, the dark bars of the sash will appear white and the panes of glass dark.

A practised observer, however, who makes the observation with great promptness, will see at the moment his eyes are turned to the wall a correct representation of the window. This representation will almost immediately be succeeded by the reversed picture just mentioned, in which the bars are bright and the panes dark.

1142. *Why a lighted stick revolving produces apparently a luminous ring.* — If a lighted stick be turned round in a circle in a dark room, the appearance to the eye will be a continuous circle of light;

for in this case the impression produced upon the retina by the light, when the stick is at any point of the circle, is retained until the stick returns to that point.

1143. *Flash of lightning.* — In the same manner, a flash of lightning appears to the eye as a continuous line of light, because the light emitted at any point of the line remains upon the retina until the cause of the light passes over the succeeding points.

In the same manner, any objects moving before the eye with such a velocity that the retina shall retain the impression produced at one point in the line of its motion until it passes through the other points, will appear as a continuous line of light or colour.

1144. *Why an object moving with a great speed becomes invisible.* — But to produce this effect, it is not enough that the body change its position so rapidly that the impression produced at one point of its path continue until its arrival at another point; it is necessary, also, that its motion should not be so rapid as to make it pass from any of the positions which it successively assumes before it has time to impress the eye with a perception of it; for it must be remembered, as has been already explained, that the perception of a visible object presented to the eye, though rapid, is not instantaneous.

The object must remain present before the organ of vision a certain definite time, and its position must continue upon the retina during such time, before any perception of it is obtained. Now, if the body move from its position before the lapse of this time, it necessarily follows that no perception of its presence, therefore, will be obtained. If, then, we suppose a body moving so rapidly before the eye that it remains in no position long enough to produce a perception of it, such object will not be seen.

1145. *Example of a cannon-ball.* — Hence it is that the ball discharged from a cannon passing transversely to the line of vision is not seen; but if the eye be placed in the direction in which the ball moves, so that the angular motion of the ball round the eye as a centre will be slow notwithstanding its great velocity, it will be visible, because however rapid its real motion through space, its angular motion with respect to the eye (and consequently of the image of its picture on the retina) will be sufficiently slow to give the necessary time for the production of a perception of it.

1146. *Quickness of vision depends on colour, brightness, and magnitude.*—The time thus necessary to obtain the perception of a visible object varies with the degree of illumination, the colour, and the apparent magnitude of the object. The more intense the illumination, the more vivid the colour, and the greater the apparent magnitude, the less will be the time necessary to produce a perception of the object.

1147. *Conditions which determine apparent motion.*—In applying this principle to the phenomena of vision, it must be carefully remem-

bered that the question is affected, not by the real but by the apparent motion of the object, that is to say, not by the velocity with which the object really moves through space, but by the angle which the line drawn from the eye to the object describes per second. Now this angle is affected by two conditions, which it is important to attend to: 1°. the direction of the motion of the object compared with the line of vision; and 2°. by the velocity of the motion compared with the distance of the object. If the object were to move exactly in the direction of the line of vision, it would appear to the eye to be absolutely stationary, since the line drawn to it would have no angular motion; and if it were to move in a direction forming an oblique angle to the line of vision, its apparent motion might be indefinitely slow, however great its real velocity might be.

For example, let it be supposed that the eye being at E, *fig.* 370, an object O moves in the direction O O′, so as to move from O to O′ in

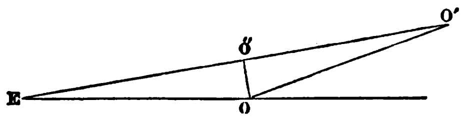

Fig. 370.

one second. Taking E as a centre, and E O as a radius, let a circular arc O O″ be described. The apparent motion of the object will then be the same as if, instead of moving from O to O′ in one second, it moved from O to O″ in one second.

The more nearly, therefore, at right angles to the line of vision the direction of the motion is, the greater will be the apparent motion produced by any real motion of an object.

1148. *How apparent motion is affected by distance.* — A motion which is visible at one distance may be invisible at another, inasmuch as the angular velocity will be increased as the distance is diminished.

Thus if an object at a distance of 57½ feet from the eye move at the rate of a foot per second, it will appear to move at the rate of one degree per second, inasmuch as a line one foot long at 57½ feet distance subtends an angle of one degree. Now if the eye be removed from such an object to a distance of 115 feet, the apparent motion will be half a degree, or thirty minutes per second; and if it be removed to thirty times that distance, the apparent motion will be thirty times slower. Or if, on the other hand, the eye be brought nearer to the object, the apparent motion will be accelerated in exactly the same proportion as the distance of the eye is diminished.

1149. *Example of a cannon-ball and the moon.* — A cannon-ball moving at 1000 miles an hour transversely to the line of vision, and at a distance of 50 yards from the eye, will be invisible, since it will not remain a sufficient time in any one position to produce perception.

The moon, however, moving with more than double the velocity of the cannon-ball, being at a distance of 240,000 miles, has an apparent motion, so slow as to be imperceptible.

1150. *What motions are imperceptible.* — The angular motion of the line of vision may be so diminished as to become imperceptible; and the body thus moved will in this case appear stationary. It is found by experience that unless a body moves in such a manner that the line of vision shall describe at least one degree in each minute of time, its motion will not be perceptible.

1151. *Why the diurnal motion of the heavens is not immediately perceptible.* — Thus it is that we are not conscious of the diurnal motion of the firmament. If we look at the moon and stars on a clear night, they appear to the eye to be quiescent; but if we observe them after the lapse of some hours, we find that their positions are changed, those which were near the horizon being nearer the meridian, and those which were in the meridian having descended towards the horizon. Since we are conscious that this change did not take place suddenly, we infer that the entire firmament must have been in continual motion round us, but that this motion is so slow as to be imperceptible.

Since the heavens appear to make a complete revolution in twenty-four hours, each object on the firmament must move at the rate of 15° an hour; or at the rate of one quarter of a degree a minute. But since no motion is perceptible to the eye which has a less apparent velocity than 1° per minute, this motion of the firmament is unperceived. If, however, the earth revolved on its axis in six hours instead of twenty-four hours, then the sun, moon, stars, and other celestial objects, would have a motion at the rate of 60° an hour, or 1° per minute. The sun would appear to move over a space equal to twice its own diameter each minute, and this motion would be distinctly perceived.

The fact that the motion of the hands of a clock is not perceived is explained in the same manner.

1152. *Why objects in extremely rapid motion are not perceivable.* — But if the object which thus moves be not sufficiently illuminated, or be not of a sufficiently bright colour to impress the retina sensibly, it will then, instead of appearing as a continuous line of colour, cease to be visible altogether; for it does not remain in any one position long enough to produce a sensible effect upon the retina. It is for this reason that a ball projected from a cannon or a musket, though passing before the eye, cannot be seen. If two railway trains pass each other with a certain velocity, a person looking out of the window of one of them will be unable to see the other. If the velocity be very moderate, and the light of the day sufficiently strong, the appearance of the passing train will be that of a flash of colour formed by the mixture of the prevailing colours of the vehicles composing it.

An expedient has already been described to show experimentally that the mixture of the seven prismatic colours, in their proper proportions, produce white light, depending on this principle. The colours are laid upon a circular disk surrounding its edge, which they divide into parts proportional to the spaces they occupy in the spectrum. When the disk is made to revolve, each colour produces, like the lighted stick, the impression of a continuous ring, and consequently the eye is sensible of seven rings of the several colours superposed one upon the other, which thus produce the effect of their combination, and appear as white or a whitish grey colour, as already explained.

1153. *The duration of the impression on the retina varies with the brightness of the object.*—The duration of the impression upon the retina, after the object producing it is removed, varies according to the vividness of the light proceeding from the object, being longer according as the light is more intense. It was found that the light proceeding from a piece of coal in combustion moved in a circle at a distance of 165 feet, produced the impression of a continuous circle of light when it revolved at the rate of seven times per second. The inference from this would be that in that particular case the impression upon the retina was continued during the seventh part of a second after the removal of the object.

It is from the cause here indicated that forked lightning presents the appearance of a continuous line of light.

1154. *And also with its colour.*—The duration of the impression on the retina varies also with the colour of the light, that produced by a white object being most visible, and yellow and red being most in degree of durability; the least durable being those tints which belong to the most refrangible lights.

1155. *And with the brightness of the surrounding space.*—The duration of the impression also depends on the state of illumination of the surrounding space; thus the impression produced by a luminous object when in a dark room is more durable than that which would be produced by the same object seen in an illuminated room. This may be ascribed to the greater sensitiveness of the retina when in a state of repose than when its entire surface is excited by surrounding lights. Thus it is found that while the varying duration of the impression of the illuminated object in a dark room was one-third of a second, its duration in a lighted room was only one-sixth of a second.

1156. *Optical toys—thaumatropes, phantascopes, &c.*—Innumerable optical toys and pyrotechnic apparatus owe their effect to this continuance of the impression upon the retina when the object has changed its position.

Amusing toys, called thaumatropes, phenakisticopes, phantascopes, &c., are explained upon this principle. A moving object, which assumes a succession of different positions in performing any action, is

[illegible] playing see saw.

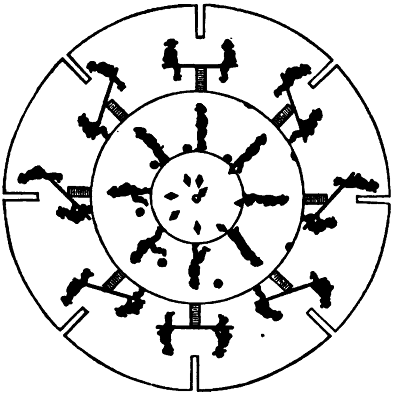

Fig. 371.

represented in the successive divisions of the circumference of a circle, as in *fig* 371., in the successive positions it assumes. These pictures, by causing the disk to revolve, are brought in rapid succession before an aperture, through which the eye is directed, so that the pictures representing the successive attitudes are brought one after another before the eye at intervals; the impression of one remaining until the impression of the next is produced. In this manner the eye never ceases to see the figure, but sees it in such a succession of attitudes as it would assume if it revolved. The effect is, that the figure actually appears to pirouette before the eye. The effects of catherine-wheels and rockets are explained in the same manner.

1157. *The direction in which objects are seen.*—The direction in which any part of an object is seen is that of the line drawn from such point through the optical centre of the eye. This line being carried back to the retina determines the place on the retina where the image of such point is found. If the optical centre of the eye were not at the centre of the eye-ball, the diretion of this line would be changed with every movement of the eye-ball in its socket; every such movement would cause the optical centre to revolve round the centre of the eye-ball, and consequently would cause the line drawn from the optical centre to the object to change its direction. The effect of this would be that every movement of the eye-ball would cause an apparent movement of all visible objects. Now, since there is no apparent motion of this kind, and since the apparent position of external objects remains the same, however the eye may be moved in its socket, it follows that its optical centre must be at the centre of the eye-ball.

1158. *Why the motion of the eye-ball does not produce any apparent motion in the object seen.*—Since lines drawn from the various points of a visible object through the centre of the eye remain unchanged, however the eye-ball may move in its socket, and since the corresponding points of the image placed upon these lines must also remain unchanged, it follows that the position of the image formed on the eye remains fixed, even though the eye-ball revolve in the socket. It appears, therefore, that when the eye-ball is moved in the socket,

the picture of an external object remains fixed, while the retina moves under it just as the picture thrown by a magic lantern on a screen would remain fixed, however the screen itself might be moved.

Thus, if we direct the axis of the eye to the centre O, *fig.* 372., of any object, such as A B, the image of the point O will be formed at *o* on the retina, where the optical axis D C meets it. The axis of the pencil of rays which proceed from the point O will pass through the centre of the cornea D, through the axis of the crystalline, and through the centre C of the eye-ball, and the image of O will be formed at *o*.

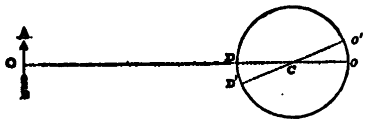

Fig. 372.

Now, if we suppose the eye to be turned a little to the left, so that the optical axis will be inclined to the line O C at the angle D' C O, the image of the point O will still hold the same absolute position *o* as before; but the point of the retina on which it was previously formed will be removed to *o'*. The direction of the point O will be the same as before; but the point of the retina on which its image will be formed will be, not at *o*, at the extremity of the optic axis, but at *o'*, at a distance *o o'* from it, which subtends at the centre C of the eye an angle equal to that through which the optical axis has been turned.

It is evident, therefore, that although the eye in this case be moved round its centre, the point O is still seen in the same direction as before.

But if the optical centre of the eye were different from the centre of the eye-ball, the direction in which the point O would be seen would be changed by a change of position of the eye.

To render this more clear, let C, *fig.* 373., be the centre of the eye-ball, and C' the optical centre of the eye. Let the optical axis C D, as before, be first presented to the point O of the object. The image of this point will, as before, be formed at *o*, the point where the optical axis D C meets the retina. Let us now suppose the axis of the eye to be turned aside through the angle D C D', the optical centre will then be removed from C' to C'', and the image of O will now be formed at the point *o''*, where the line O C'' meets the retina. The direction, therefore, in which O will now be seen, will be that of the line C'' O, whereas the direction in which it was before seen was that of the line C O.

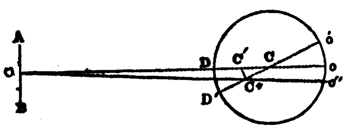

Fig. 373.

The point of the retina at which the image *o* was originally formed is removed to o', while the image is removed to o''. Thus there is a displacement not only of the retina behind the image, but also an absolute displacement of the image, and an absolute change in the apparent direction of the object. Since no such change in the apparent direction is consequent upon the movement of the eye in its socket, it follows that the optical centre C' of the eye must coincide with its geometrical centre C.

1159. *Ocular spectra and accidental colours.*—The continuance of the effect produced by the image of a visible object on the retina after such object has been removed from before the eye, combined with the effect of the image of another object placed before the eye, during such continuance of the effect of that which was removed, produces a class of phenomena called *ocular spectra* and *accidental colours.*

The effect produced by a strongly illuminated image formed on the retina does not appear to be merely the continuance of the same perception after the image is removed, but also a certain diminution or deadening of the sensibility of the membrane to other impressions. If the organ were merely affected by the continuance of the perception of the object for a certain time after its removal, the effect of the immediate perception of another object on the retina would be the perception of the mixture of two colours. Thus, if the eye, after contemplating a bright yellow object, were suddenly directed to a similar object of a light red colour, the effect ought to be the perception of an orange colour; and this perception would continue until the effect of the yellow object on the retina would cease, after which the red object alone would be perceived.

Thus, for example, a disk of white paper being placed upon a black ground, and over it a red wafer which will exactly cover it being laid, if, closing one eye, and gazing intently with the other for a few seconds on the red wafer, the red wafer be suddenly removed so as to expose the white surface under it to the eye, the effect ought to be the combination of the perception of red which continues after the removal of the red wafer, with the perception of white which the uncovered surface produces; and we should consequently expect to see a diluted red disk, similar to that which would be produced by the mixture of red with white.

This, however, is not the case. If the experiment be performed a here described, the eye will, on the removal of the red wafer, per ceive, not a reddish, but a greenish-blue disk.

In like manner, if the wafer, instead of being red, were of a bright greenish-blue, when removed the impression on the eye would be that of a reddish disk.

These and like phenomena are explained as follows:—

When the eye is directed with an intensity of gaze for some time

at the red surface, that part of the retina upon which the image of the red wafer is produced becomes fatigued with the action of the red light, and loses to some extent its sensibility to that light, exactly as the ear is deafened for a moment by an overpowering sound. When the red wafer is removed, the white disk beneath it transmits to the eye the white light, which is composed of all the colours of the spectrum. But the eye, from the previous action of the red light, is comparatively insensible to those tints which form the red end of the spectrum, such as red and orange, but comparatively sensitive to the blues and greens, which occupy the other end. It is therefore that the eye perceives the white disk as if it were a greenish-blue, and continues to perceive it until the retina recovers its sensibility for red light.

1160. *Experiments of Sir D. Brewster on ocular spectra.* — The experiment above described may be varied by using wafers of various colours; and it will in each case be found that on the removal of the wafer the accidental colour or ocular spectrum produced will be that which is given in the second column of the following table, supplied by the observations of Sir David Brewster:—

Colour of the Wafer.	Accidental Colour, or Colour of the Ocular Spectra.
Red	Bluish-green.
Orange	Blue.
Yellow	Indigo.
Green	Violet, reddish.
Blue	Orange red.
Indigo	Orange yellow.
Violet	Yellow green.
Black	White.
White	Black.

It follows, therefore, from the results in the above table, that the primitive and accidental colours are so related to each other, that if the former be reduced to the same degree of intensity as the latter, one will be the complementary colour of the other, or, which is the same, they will be so related that if mingled together they will produce white light.

The experiment may be varied in the following manner:—

If a small particle of red fire be burned in a dark room, so as to illuminate all the surrounding objects with an intense red light, and it be suddenly extinguished, the eye will for a time see a green flame; and this green flame will be visible whether the eye be open or closed.

If, on the other hand, a green fire be burned, it will be succeeded by the perception of a reddish light.

If the eye be directed intently upon the disk of the sun at rising or setting, when it is red, on closing the eyelids a green solar disk will be perceived.

1161. *Why visible objects do not appear inverted.*—A difficulty has been presented in the explanation of the functions of the eye to which, as it appears to me, undue weight has been given. It has been already explained, that the images of external objects which are depicted on the retina are inverted; and it has accordingly been asked why visible objects do not appear upside down. The answer to this appears to be extremely simple. Inversion is a relative term, which it is impossible to explain or even to conceive without reference to something which is not inverted. If we say that any object is inverted, the phrase ceases to have meaning unless some other object or objects are implied which are erect. If all objects whatever hold the same relative position, none can be properly said to be inverted; as the world turns upon its axis once in twenty-four hours, it is certain that the positions which all objects hold at any moment is inverted with respect to that which they held twelve hours before, and to that which they will hold twelve hours later; but the objects as they are contemplated are always erect. In fine, since all the images produced upon the retina hold with relation to each other the same position, none are inverted with respect to others; and as such images alone can be the objects of vision, no one object of vision can be inverted with respect to any other object of vision; and consequently, all being seen in the same position, that position is called the erect position.

1162. *The seat of vision.*—Physiologists are not agreed as to the manner in which the perception of a visible object is obtained from the image formed in the interior of the eye. It is certain, however, that this image is the cause of vision, or that the means whereby it is produced are also instrumental in producing the perception of sight. It may also be considered as established that the perception of a visible object is more or less distinct, according to the greater or less distinctness of the image. But it would be a great error to assume that this image on the retina is itself seen, for that would involve the supposition of a second eye, beyond the first, or within it, by which such image on the retina would be viewed. Now, no means of communicating between the image on the retina and the sensorium exist except the usual conduits of all sensation, the nerves.

It has been already explained that the optic nerve, after entering the eye at a point near the nose, spreads itself over the interior of the globe of the eye behind the vitreous humour, and that this retina or network is perfectly transparent, the coloured image being formed not properly upon it, but upon the black surface of the choroid coat

behind it. Now, it has been maintained, that the functions of vision are performed by this nervous membrane in a manner analogous to that by which the sense of touch is affected by external objects. The membrane of the retina, it is supposed, touching the coloured image, and being in the highest degree sensitive to it, just as the hand is sensitive to an object which it touches, receives from the coloured image an action which, being continued to the brain, produces perception there in accordance with the form and colour of the image upon the choroid. According to this view of the functions of vision, the retina *feels*, as it were, the image on the choroid, and transmits to the sensorium the impression of its colour and figure in the same manner as the hand of a blind person would transmit to the sensorium the form of an object which it touches.

1163. *Light and colour acting directly on the retina produce no sensation.* — If this hypothesis be admitted, it would follow that the retina itself would be incapable of exciting the sense of sight by the mere action of light and colours upon it. This is verified by the fact that when the image produced within the eye is formed upon a point of the optic nerve which has not the choroid behind it, no perception is produced.

In order to prove this, let three wafers be applied in a horizontal line upon a vertical screen, each separated from the other by a distance of two feet. Let the screen be placed before the observer at a distance of about 15 feet, the wafers being on a level with the eye; and let the centre wafer be so placed that a line drawn from the right eye to it shall be perpendicular to the screen. Let the left eye be now closed, and let the right eye be directed to the extreme wafer on the left, but so that all three wafers may still be perceived. Let another person now slowly move the screen, so as to bring it nearer to the observer, maintaining, however, the middle wafer in the direction of the eye at C. It will be found that the screen being so moved to a distance of 10 feet from the eye, the middle wafer will appear to be suddenly extinguished, and the extreme wafers on the right and left will be seen.

1164. *The optic nerve is insensible where it does not cover the choroid.* — This remarkable phenomenon is explained by showing that in this particular position of the eye and the screen, the image of the middle wafer falls upon the base of the optic nerve where the choroid coat is not under it.

This will be rendered more intelligible by reference to *fig.* 374., where B is the middle wafer, A the left-hand, and C the right-hand wafer. The image of A is formed at *a*, to the right of the optic nerve; and the image of C is formed at *c*, to the left of that nerve. In both these positions the choroid coat is behind the retina. But the image of B is formed at *b*, directly upon the point where the optic

nerve issues from the eye-ball, and where the choroid does not extend behind it.

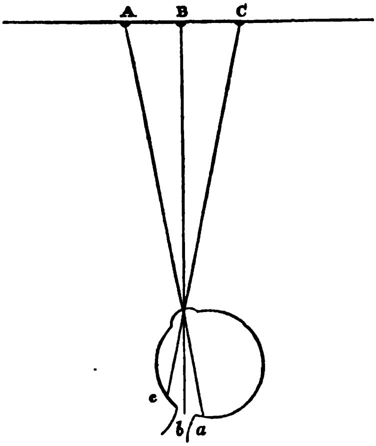

Fig. 374.

1165. *Experiment of Sir D. Brewster to confirm this.* — Sir David Brewster gives the following experiment as a further argument in support of this hypothesis. In the eye of the *Sepia loligo*, or cuttle-fish, an opaque membranous pigment is interposed between the retina and the vitreous humour, so that if the retina were essential to vision, the impression of the image on this black membrane must be conveyed to it by the vibration of this membrane in front of it. Sir David Brewster also mentions that in young persons the choroid coat, instead of being covered with a black pigment, reflects a brilliant crimson, like that of dogs and some other animals; and imagines that if the retina were affected by the rays which pass through it, this crimson light ought to excite a corresponding sensation, which is not the case.

1166. *Why objects are not seen double.* — The question why, having two eyes on which independent impressions are made by external objects, and on the retina of each of which an independent picture of a visible object is formed, we do not see two distinct objects corresponding to each individual external object which impresses the organ, is often asked.

The first reflection which arises on the proposition of this question, is why the same question has not been similarly proposed with reference to the sense of hearing. Why has it not been asked why we do not hear double? why each individual sound produced by a bell or a string is not heard as two distinct sounds, since it must impress independently and separately the two organs of hearing?

It cannot be denied, that, whatever reason there be for demanding a solution of the question, why we do not see double? is equally applicable to the solution of the analogous question, why we do not hear double? Like many disputed questions, this will be stripped of much of its difficulty and obscurity by a strict attention to the meaning of the terms used in the question, and in the discussion consequent upon it. If by seeing double it be meant that the two eyes receive separate and independent impressions from each external object, then it is true that we see double. But if it be meant that the mind

receives two distinct and independent impressions of the same external object, then a qualified answer only can be given.

If the two eyes convey to the mind precisely the same impression of the same external object, differing in no respect whatever, then they will produce in the mind precisely the same perception of the object; and as it is impossible to imagine two perceptions to exist in the mind of the same external object which are precisely the same in all respects, it would involve a contradiction in terms to suppose that, in such case, we perceive the object double.

If to perceive the object double mean anything, it means that the mind has two perceptions of the same object, distinct and different from each other in some respect. Now, if this distinctness or difference exist in the mind, a corresponding distinctness and difference must exist in the impression produced of the external object on the organs. It will presently appear, that cases do occur in which the organs are, in fact, differently impressed by the same external object; and it will also appear, that in such cases precisely we *do see double*, meaning by these terms, that we have two perceptions of the same object, as distinct from each other as are our perceptions of two different objects.

To render this point more clear, let us consider in what respects it is possible for the impressions made upon the two eyes by the same object to differ from each other.

A visible object impresses the eye with a sense of a certain apparent form, of a certain apparent magnitude, of certain colours, of a certain intensity of illumination, and of a certain visible direction. Now, if the impression produced by the same object upon the two eyes agree in all these respects, it is impossible to imagine that the mind can receive two distinct perceptions of the object, for it is not possible that the two perceptions could differ from each other in any respect, except in some of those just mentioned. Let us suppose the two eyes to look at the moon, and that such object impresses them with an image of precisely the same apparent form and magnitude, of precisely the same colours and lineaments, of precisely the same intensity of illumination, and, in fine, in precisely the same direction. Now, the impressions conveyed to the mind by each of the eyes corresponding in all these respects, the object must be perceived in virtue of both impressions precisely in the same manner, that is to say, it must be seen in precisely the same direction, of precisely the same magnitude, of precisely the same form, with precisely the same lineaments of light and shade, and with precisely the same brightness or intensity of illumination. It is therefore, in such a case, clearly impossible to have a double perception of the object.

It will be observed, that the same reasoning exactly will be applicable to the sense of hearing. If the same string or the same pipe affect the membrane of each ear-drum in precisely the same manner,

so as to produce a perception of a sound of the same pitch, the same loudness, and the same quality, it is impossible to conceive that two different perceptions can be produced by the two ears, for there is no respect in which it is possible for two such perceptions to differ, inasmuch as by the very supposition they agree in all the qualities which belong to sound.

But, if we would conceive by any organic derangement that the same musical string would produce in one ear the note *ut*, and produce in the other ear the note *sol*, then the same effect would be produced as if these two sounds had been simultaneously heard by the two ears properly organized, and we should have a sense of harmony of the *fifth*.

In like manner, if the two eyes, by any defect of organization, produced different pictures on the retina, we should then have two perceptions of the same object having a corresponding difference

It has been already shown, that the apparent visual magnitude of an object, and also that its apparent brilliancy, depend on its distance from the eye.

Now, assuming, as we shall do, unless the contrary be expressed, that the two eyes are similarly constituted, it will follow, that an object whose distance from the two eyes is equal will be seen under the same visual angle, and will therefore have the same apparent magnitude; it will also have the same colour and intensity of illumination, and, in fine, if the distance between the eyes bear an insignificant proportion to the distance of the object from them, the lines drawn from the centre of the eyes to any point on the object will be practically parallel; and since these lines, as has been already explained, determine the direction in which the object is seen, such object will then be seen in the same direction. Now, since the apparent form, the apparent magnitude, the apparent colour, the apparent intensity of illumination, and, in fine, the apparent direction are the same for both eyes, it is clear that the same impression must be produced upon the senses, and the same perceptions conveyed to the mind; consequently it follows, demonstratively, that all objects which are placed at a distance compared with which the distance between the eyes is insignificant, will convey a single perception to the mind, and will consequently not be seen double.

1167. *Exceptional cases in which objects are seen double.*—But we have now to consider a different case, which will present peculiar conditions, and consequences of peculiar interest.

Let us suppose an object placed so near the eyes that its distance shall not bear a considerable proportion to the length of the line which separates the centres of the eyes. In this case, the images produced on the retina of the two eyes may differ in magnitude, and intensity of illumination, and even in form, and, in fine, it is clear that the

apparent direction of any point on the object as seen by the two eyes will be sensibly different.

In this case, therefore, the two eyes convey to the mind a different impression of the same object; and we may therefore expect that we should see it double, and in fact we do so.

But the observation of this particular phenomenon requires much attention, inasmuch as the perception of which we are conscious i affected not merely by the impression made upon the organ of sense but by the degree of attention which the mind gives to it. Thus, if the two eyes be differently impressed either by the same or by different objects placed before them, the mind may give its attention so exclusively to either impression, as to lose all consciousness of the other.

Thus, if two stars be at the same time in the field of view of a telescope, as frequently happens, and be viewed together by the eye, we shall be conscious of a perception of both, so long as the attention is not exclusively directed to either; but if we gaze intently on one of them so as to observe its colour, or any other peculiarity attending it, we shall cease to be conscious of the presence of the other. The application of this observation to the question before us will be presently apparent.

Let R L, *fig.* 375., be the line separating the centres of the two eyes, R representing the centre of the right, and L that of the left eye.

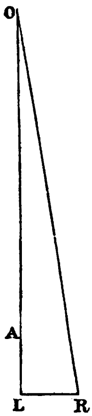

Fig. 375.

Let O be an object, such as the flame of a candle or lamp, seen at the distance of about 40 feet, so that the lines of direction L O and R O converging upon it from the centres of the eyes may be regarded as practically parallel, the distance being about 200 times greater than the distance L R from the eyes. The object O will therefore be seen in the same direction by both eyes, and being at a distance from the two eyes practically equal, will have the same apparent magnitude, form, colour, and intensity of illumination, and, consequently, will be seen single.

Let a small white rod be held at the distance A, of about 8 inches from the left eye L, and in the line L O, so as to intercept the view of the object O from the left eye. The left eye will then see the rod at A, and not the object O; but the right eye will still see the object O, as before. Now, if the attention be earnestly directed to the object O, the object A will not be perceived; but if the attention be directed to the object A, it will be perceived distinctly, but the object O will be seen through it as if it were transparent.

Now, since the object O cannot be seen by the left eye under the circumstances here supposed, the perception we have of it must be derived from the right eye; nevertheless it is seen in the line L A O,

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immediately beyond the intercepting wand, and in the same direction, and in the same manner precisely as it would be seen by the left eye L if the intercepting wand were removed. It follows, therefore, that the perception we obtain of the object O by the right eye is precisely the same as that which we should obtain by the left eye if the right were closed, and the intercepting wand A removed. This may therefore be taken as an experimental proof of what, indeed, may seem sufficiently evident, *à priori*, that an object, such as O, placed at a distance so great that lines drawn to it from the centre of the eyes would be practically parallel, produces precisely the same perception through the vision of both eyes.

But when the distance of an object from the eyes is so small that the line which separates the eyes bears a considerable proportion to it, the directions in which such an object is seen by the two eyes are different, and it is easy to show that in this case such an object would be seen double.

Let L and R, *fig.* 376., as before, be the centres of the two eyes, and let A B be a white screen placed vertically at a distance of 12 or 14 feet, having upon it a horizontal line on a level with the eye, upon which is marked a divided scale, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Let a black wand be held vertically at O, opposite the middle of the line L R. This wand will be seen by the left eye in the direction of the division 8, and by the right eye in the direction of the division 4, on the screen, and two images of the wand will accordingly be perceived; but, according as the attention is directed to the one or to the other, a consciousness of them will be produced. Thus, by an act of the will we may contemplate only the objects as seen with the left eye, in which case the wand will be seen projected on the screen perpendicular to the line A B, at the 8th division; and by a like act of the will, the attention being directed to the impression produced by the right eye, the wand will be seen projected on the screen at the 4th division of the scale. If the attention be withdrawn from either of these, and the wand be viewed indifferently, we shall be conscious of the two images, but not with the same distinctness as that with which we should perceive two wands placed at the 4th and 8th divisions of the scale. It will follow from this, that when we look with both eyes at any object, such as the printed page of a book, at the distance of

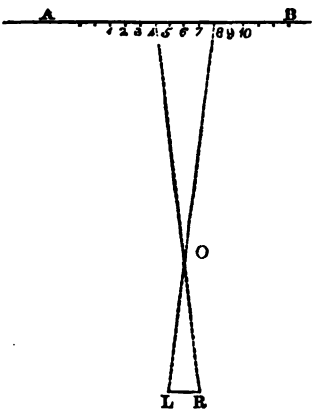

Fig. 376.

8 or 10 inches from the eyes, we have two images of the different parts of the page placed before the eyes, which are seen in different directions, and ought therefore to produce double vision; but this is prevented by habitually directing our attention to one of the two, and neglecting the other.

That the perception of an object will be double if the directions in which it is seen by the two eyes are different, may also be demonstrated in the following manner:—

It has been already shown that the optical centres of the eyes cannot change their position by the mere action of the muscles which move the eye-balls in their sockets, and that the direction in which any distant object is seen by both eyes is the same, and hence it is perceived single; but if a slight pressure be applied to the eye with the finger, the optical centre of the eye may be moved from its position, so that the direction of the same object seen by it and by the other eye will not be the same. A distant object will in this case be seen double, being perceived in one direction by the eye which retains its natural position, and in another by that whose position is deranged by pressure.

1168. *The eye supplies no direct perception of magnitude, figure, or distance.*—It has been already explained that two similar objects whose distances from the eye are to each other in the same proportion as their linear dimensions will have the same apparent magnitude.

In like manner, if an object, such as, for example, a balloon, moves from the eye in a direct line, we have no distinct consciousness of its motion, for the line of direction in which it is seen is still the same. It is true that we may infer its motion through the air by the increase or diminution of its apparent magnitude; for, if we have reason to know that its real magnitude remains unchanged, we ascribe almost intuitively the change of its apparent magnitude to the change of its distance; and we consequently *infer* that it is in motion either towards or from us, according as we perceive its apparent magnitude to be increased or diminished. This information, however, as to the motion of a body in a direct line to or from the centre of the eye, is not a perception obtained directly from vision, but an inference of the reason deduced from certain phenomena. It may therefore be stated generally, that the eye affords no perception of direct distance, and consequently none of direct motion, the term direct being understood here to express a motion in a straight line to or from the optical centre of the eye.

1169. *Manner of estimating the real distance.*—The distance of a visible object is often estimated by comparing it with the apparent magnitude and apparent distance of known objects which intervene between it and the eye.

Thus, the steeple of a church whose real height is unknown cannot by mere vision be estimated either as to distance or magnitude, since

the apparent height would be the same, provided its magnitude were greater or less in proportion to its supposed distance. But, if between the steeple and the eye there intervene buildings, trees, or other objects, whose average magnitudes may be estimated, a proximate estimate of the magnitude and distance of the steeple may be obtained.

For example, if the height of the most distant building between the eye and the steeple be known, the distance of that building may be estimated by its apparent magnitude, and the distance of the steeple will be inferred to be greater than this.

1170. *Appearance of the sun and moon when rising or setting.*— A remarkably deceptive impression, depending on this principle, is deserving of mention here. When the disk of the sun or moon at rising or setting nearly touches the horizon, it appears of enormous magnitude compared with its apparent size when high in the firmament. Now, if the visual angle which it subtends be actually measured in this case, it will be found to be of the same magnitude. How, then, it may be asked, does it happen that the apparent magnitude of the sun at setting and at noon are by measure the same, when they are by estimation, and by the irresistible evidence of sense, so extremely different? This is explained, not by an error of the sense, for there is none, but by an erroneous application of those means of judging or estimating distance which in ordinary cases supply true and just conclusions.

When the disk of the sun is near the horizon, a number of intervening objects of known magnitude and known relative distances supply the means of spacing and measuring a part at least of the distance between the eye and the sun; but when the sun is in the meridian, no such objects intervene. The mind, therefore, assigns a greater magnitude to the distance, a part of which it has the means of measuring, than to the distance no part of which it can measure; and accordingly an impression is produced, that the sun at setting is at a much greater real distance than the sun in the meridian; and since its apparent magnitude in both cases is the same, its real magnitude must be just so much greater as its estimated distance is greater. The judgment, therefore, and not the eye, assigns this erroneous magnitude to the disk of the sun.

It is true that we are not conscious of this mental operation. But this unconsciousness is explained by the effect of habit, which causes innumerable other operations of the reason to pass unobserved.

1171. *Method of estimating by sight the magnitude of distan objects.*— As the eye forms no immediate perception of distance, neither does it of form or of magnitude, since, as has been already proved, objects of very different real magnitudes have the same apparent magnitude to the eye, of which a striking example is afforded in the case of the sun and moon. Nevertheless, although

the eye supplies no immediate perception of the real magnitude of objects, habit and experience enable us to form estimates more or less exact of these magnitudes by the comparison of different effects produced by sight and touch.

Thus, for example, if two objects be seen at the same distance from the eye, the real magnitude of one of which is known, that of the other can be immediately inferred, since, in this case, the apparent magnitudes will be proportional to the real magnitudes. Thus, for example, if we see the figure of a man standing beside a tree, we form an estimate of the height of the latter, that of the former being known or assumed. Ascribing to the individual seen near the tree the average height of the human figure, and comparing the apparent height of the tree with his apparent height, we form an estimate of the height of the tree.

1172. *Singular illusion produced in St. Peter's at Rome.* It is by this kind of inference that buildings constructed upon a scale greatly exceeding common dimensions are estimated, and rendered apparent in pictorial representations of them.

On entering, for example, the aisle of St. Peter's at Rome, or St. Paul's at London, we are not immediately conscious of the vastness of the scale of these structures; but if we happen to see at a distant part of the building a human figure, we immediately become conscious of the scale of the structure, for the known dimensions of this figure supply a modulus which the mind instantly applies to measure the dimensions of the whole. For this reason artists, when they represent these structures, never fail to introduce human figures in or near them.

1173. *Real magnitude may sometimes be inferred from apparent magnitude.* — It has been explained that the apparent magnitude of objects depends conjointly on their real magnitude and their distance. Although, therefore, the eye does not afford any direct perception either of real magnitude or distance, we are by habit enabled to infer one of these from the other.

Thus, if we happen to know the real magnitude of a visible object, we form an estimate of its distance from its apparent magnitude; and, on the other hand, if we happen to know or can ascertain the distance of an object, we immediately form some estimate of its real magnitude.

Thus, for example, the height of a human figure being known, if we observe its apparent visual magnitude to be extremely small, we know that it must be at a distance proportionally great. If we know that at 20 feet the figure of a man will have a certain apparent height, and that we find that his figure seen at a certain distance appears to have only one-fifth of this height, we infer that his distance must be about 100 feet.

In like manner, the real magnitude may be inferred from the appa-

rent magnitude, provided the distance be known or can be ascertained. Thus, for example, in entering Switzerland by its northern frontier, we see in the distance, bounding the horizon, the line of the snowy Alps, and the first impression is that of disappointment, their apparent scale being greatly less than we expected; but when we are informed that their distance is sixty or eighty miles, our estimate is instantly corrected, and we become conscious that the real height of mountains which, seen at so great a distance, is what we observe it, must be proportionally vast.

1174. *The eye not perceiving direct distance can have no perception of any motion but angular motion of which it is the centre.*— When an object moves in any direction which is not in a straight line drawn to or from the centre of the eye, the direction in which it is seen continually changes, and the eye in this case supplies an immediate perception of its motion; but this perception can be easily shown to be one not entirely corresponding to the actual motion of the object, but merely to the continual change of direction which this motion produces in the line drawn from the object to the eye.

Thus, for example, if the eye be at E, *fig.* 377., any object which moves from A to B will cause the line of direction in which it is seen to revolve through the angle A E B, just as though the body which moves were to describe a circular arc, of which E is the centre and E A the radius. But if, instead of moving from A to B, the body were to move from A′ to B′, the impression which its motion would produce upon the sight would be exactly the same. It would still appear to be moving from the direction E A′ A to the direction E B B′.

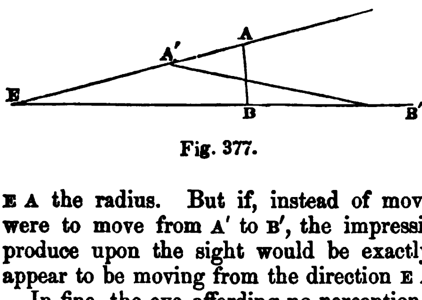

Fig. 377.

In fine, the eye affording no perception of direct distance, supplies no evidence of the extent to which the body may change its distance from the eye during its motion, and the apparent motion will be the same as if the body in motion described a circle of which the eye is the centre.

Hence it is that the only motion of which the eye forms any immediate apprehension is angular motion, that is, a motion which is measured by the angle which a line describes, one extremity of which is at the centre of the eye, and the other at the moving object.

1175. *Real direction of motion may be inferred by comparing apparent motion with apparent magnitude.*—Though the real direction in which a distant object moves cannot be obtained by the direct perception of vision, some estimate of it may be formed by comparing the apparent angular motion of the object with its apparent magnitude.

Thus, for example, if we observe that the apparent magnitude of an object remains constantly the same while it has a certain apparent angular motion, we infer that its distance must necessarily remain the same, and consequently that it revolves in a circle, in the centre of which the observer is placed; or if we find that it has an angular motion, in virtue of which it changes its direction successively around us, so as to make a complete circuit of 360°, and that in making this circuit its apparent magnitude first diminishes to a certain limit, and then augments until it attains a certain major limit from which it again diminishes, we infer that such a body revolves round us at a varying distance, its distance being greatest when the apparent magnitude is least, and least when its apparent magnitude is greatest. An exact observation of the variation of the apparent magnitude would in such a case supply a corresponding estimate of the variation of the real distance, and would thus form the means of ascertaining the path in which the body moves.

1176. *Examples of the sun and moon.* — An example of this is presented in the cases of the sun and moon, whose apparent magnitudes are subject, during their revolution round the earth, to a slight variation, being a minimum at one point and a maximum at the extreme opposite point, the variation being such as to show that their motions are made in an ellipse in the focus of which the earth is placed.

1177. *How the apparent motion of an object is affected by the motion of the observer.* — As the eye perceives the motion of an object only by the change in the direction of the line joining the object with the eye, and as this change of direction may be produced as well by the motion of the observer as by that of the object, we find accordingly that apparent motions are produced sometimes in this manner. Thus, if a person be placed in the cabin of a boat which is moved upon a river or canal with a motion of which the observer is not conscious, the banks and all objects upon them appear to him to move in a contrary direction. In this case the line drawn from the object to the eye is not moved at the end connected with the object, which it would be if the object itself were in motion, but at the end connected with the eye. The change of its direction, however, is the same as if the end connected with the object had a motion in a contrary direction, the end connected with the eye being at rest; consequently, the apparent motion of the objects seen which are really at rest, is in a direction contrary to the real motion of the observer.

1178. *Example of railway trains.*—In some cases the apparent motion of an object is produced by a combination of a real motion in the object and a real motion in the observer. Thus, if a person transported in a railway carriage meet a train coming in the opposite direction, both extremes of the line joining his eye with the train which passes him are in motion in contrary directions; that extremity which is at his eye is moved

by the motion of the train which carries him, and the other extremity is moved by the motion of the train which passes him. The change of direction of the line is accordingly produced by the sum of these motions; and as this change of direction is imputed by the sense to the train which passes, this train appears to move with the sum of the velocities of the two trains. Thus, if one train be moved at twenty miles an hour, while the other is moved at twenty-five miles an hour, the apparent motion of the passing train will be the same as would be the motion of a train moved at forty-five miles an hour passing a train at rest.

1179. *Compounded effects of the motion of the observer and of the object observed.* — If the line joining a visible object with the eye be moved at both its extremities in the same direction, which would be the case if the observer and the object were carried in parallel lines, then the change of direction which the line of motion would undergo would arise from the difference of the velocities of the observer and of the object seen.

If the observer in this case moved slower than the object, the extremity of the line of motion connected with the object would be carried forward faster than the extremity connected with the observer, and the object would appear to move in the direction of the observer's motion, with a velocity equal to the difference; but if, on the contrary, the velocity of the observer were greater than that of the object, the extremity of the line connected with the observer would be carried forward faster than that connected with the object, and the change of direction would be the same as if the object were moved in a contrary direction with the difference of the velocities.

It is easy to perceive that a vast variety of complicated relations which may exist between the directions and motions of the observer and of the object observed, will give rise to very complicated phenomena of apparent motion. Thus, relations may be imagined between the motion of the observer and that of the object perceived, by which, though both are in motion, the object will appear stationary; the motion of the one affecting the line of direction in an equal and contrary manner to that with which it is affected by the other; and, in the same manner, either motion may prevail over the other more or less, so as to give the line of direction a motion in accordance with or contrary to the real motion of the object.

1180. *Examples of the planetary motions.* — All these complicated phenomena of vision are presented in the problems which arise on the deduction of the real motion of the bodies composing the solar system from their apparent motions. The observer placed in the middle of this system is transported upon the earth in virtue of its annual motion round the sun with a prodigious velocity, the direction of his motion changing from day to day according to the curvature of the orbit. The bodies which he observes are also affected with various motions at various distances around the sun, the combi-

nation of which with the motion of the earth gives rise to complicated phenomena, the analysis of which is made upon the principles here explained.

1181. *Angular or visual distances.* — It is usual to express the relative position in which objects are seen by the relative direction of lines drawn to them from the eye; and the angle contained by any two such lines is called the angular or visual distance between the objects. Thus, the angular distance between the objects A and B, *fig.* 377., is expressed by the magnitude of the angle A E B. If this angle be 30°, the objects are said to be 30° asunder. It is evident from this that all objects which lie in the direction of the same lines will be at the same angular distance asunder, however different their real distance from each other may be. Thus, the angular distance between A and B, *fig.* 377., is the same as the angular distance between A′ and B′.

1182. *Vision affords no direct perception of bulk or form.—How such qualities are inferred.*—Sight does not afford any immediate perception either of the volume or shape of an object. The information which we derive from the sense, of the bulk or figure of distant objects, is obtained by the comparison of different impressions made upon the sense of sight by the same object at different times and in different positions. A body of the spherical form seen at a distance appears to the eye as a flat circular disk, and would never be known to have any other form, unless the impression made upon the eye were combined with other knowledge, derived from other impressions through sight or touch, or both these senses, and thus supplied the understanding with data from which the real figure of the object could be inferred. The sun appears to the eye as a flat, circular disk; but, by comparing observations made upon it at different times, it is ascertained that it revolves round one of its diameters in a certain time, presenting itself under aspects infinitely varying to the observer; and this fact, combined with its invariable appearance as a circular disk, proves it to be a sphere; for no body except a sphere, viewed in every direction, would appear circular.

Although we do not obtain from the sense of sight a perception of the shape of a body, we may obtain a perception of the shape of one of its sections. Thus, if a section of the body be made by a plane passing through it at right angles to the line of vision, the sight supplies a distinct perception of the shape of such section. Thus, if an egg were presented to the eye with its length in the direction of the line of vision, it would appear circular, because a section of it made by a plane at right angles to its length is a circle; but if it were presented to the eye with its length at right angles to the line of vision, it would appear oval, that being the shape of a section made by a plane passing through its length.

If a body, therefore, presents itself successively to the eye in seve-

ral different positions, we obtain a knowledge by the sense of sight of so many different sections of it, and the combination of these sections may in many cases supply the reason with data by which the exact figure of the body may be known.

1183. *Visible area.*—As the term "apparent magnitude" is used to express the visual angle under which an object is seen, we shall adopt the term *visible area* to express the apparent magnitude of the section of a visible object made by a plane at right angles to the line of vision, that is to say, to the line drawn from the eye to the centre of the object.

1184. *How the shape is inferred from lights and shades.*—Besides receiving through the sight a perception of the figure of the section of the object which forms its visible area, we also obtain a perception of the lights and shades and the various tints of colour which mark and characterize such area. By comparing the perception derived from the sense of touch with those lights and shades, we are enabled by experience and long habit to judge of the figure of the object from these lights and shades and tints of colour. It is true that we are not conscious of this act of the understanding in inferring shape from colour and from light and shade; but the act is nevertheless performed by the mind. The first experience of inference is the comparison of the impressions of sight with the impressions of touch; and one of the earliest acts of the mind is the inference of the one from the other. It is the character of all mental acts, that their frequent performance produces an unconsciousness of them; and hence it is that when we look at a cube or a sphere of a uniform colour, although the impression upon the sense of sight is that of a flat plane variously shaded, and having a certain outline, the mind instantly substitutes the thing signified for the sign, the cause for the effect; and the conclusion of the judgment, that the object before us is a sphere or a cube of uniform colour, and not, as it appears, a flat plane variously shaded, is so instantaneous, that the act of the mind passes unobserved.

The whole art of the painter consists in an intimate practical knowledge of the relation between these two effects of perception of sight and touch. The more accurately he is able to delineate upon a flat surface those varieties of light and shade which visible objects immediately produce upon the sense, the more exact will be his delineation, and the greater the *vraisemblance* of his picture.

What is called relief in painting, is nothing more than the exact representation on a flat surface of the varieties of light and shade produced by a body of determinate figure upon the eye; and it is accordingly found that the flat surface variously shaded, produced by the art of the painter, has upon the eye exactly the same effect as the object itself, which is in reality so different from the coloured canvass which represents it.

1185. *Power of perceiving and distinguishing colours improved by exercise and experience.* — The immediate impressions received from the sense of sight are those of light and colour. The impressions of distance, magnitude, form, and motion, are the mixed results of the sense of sight and the experience of touch. Even the power of distinguishing colours is not obtained immediately by vision without some cultivation of this sense. The unpractised eye of the new-born infant obtains a general perception of light; and it is certain that the power of distinguishing colours is only found after the organ has been more or less exercised by the varied impressions produced by different lights upon it. It would not be easy to obtain a summary demonstration of this proposition from the experience of infancy, but sufficient evidence to establish it is supplied by the cases in which sight has been suddenly restored to adults blind from their birth. In these cases, the first impression produced by vision is that the objects seen are in immediate contact with the eye. It is not until the hand is stretched forth to ascertain the absence of the objects seen from the space before the eye that this optical fallacy is dissipated.

The eye which has recently gained the power of vision at first cannot distinguish one colour from another, and it is not until time has been given for experience, that either colour or outline is perceived.

1186. *Of certain defects in vision.*—Besides that imperfection incident to the organs of sight arising from the excess or deficiency of their refractive powers, there is another class which appear to depend upon the quality of the humours through which the light, proceeding from visible objects, passes before attaining the retina. It is evident that if these humours be not absolutely transparent and colourless, the image on the retina, though it may correspond in form and outline with the object, will not correspond in colour; for if the humours be not colourless, some constituents of the light proceeding from the object will be intercepted before reaching the retina, and the picture on the retina will accordingly be deprived of the colours thus intercepted. If, for example, the humours of the eye were so constituted as to intercept all the red and orange rays of white light, white paper, or any other white object, such as the sun, for example, would appear of a bluish-green colour; and if, on the other hand, the humours were so constituted as to intercept the blues and violets of white light, all white objects would appear to have a reddish hue. Such defects in the humours of the eye are fortunately rare, but not unprecedented.

1187. *Curious examples of defective eyes.* — Sir David Brewster, who has curiously examined and collected together cases of this kind, gives the following examples of these defects :—

A singular affection of the retina in reference to colour is shown in the inability of some eyes to distinguish certain colours of the spectrum. The persons who experience this defect have their eyes generally in a sound state, and are capable of performing all the most delicate

functions of vision. M. Harris, a shoemaker at Allonby, was unable from his infancy to distinguish the cherries of a cherry-tree from its leaves, in so far as colour was concerned. Two of his brothers were equally defective in this respect, and always mistook orange for grass-green, and light green for yellow. Harris himself could only distinguish black and white. Mr. Scott, who describes his own case in the Philosophical Transactions, mistook pink for a pale blue, and a full red for a full green.

All kinds of yellows and blues, except sky-blue, he could discern with great nicety. His father, his maternal uncle, one of his sisters and her two sons, had all the same defect.

A tailor at Plymouth, whose case is described by Mr. Harvey, regarded the solar spectrum as consisting only of yellow and light blue; and he could distinguish with certainty only yellow, white, and green. He regarded indigo and Prussian blue as black.

M. R. Tucker described the colours of the spectrum as follows:—

Red mistaken for	brown.
Orange "	green.
Yellow sometimes	orange.
Green "	orange.
Blue "	pink.
Indigo "	purple.
Violet "	purple.

A gentleman in the prime of life, whose case I had occasion to examine, saw only two colours in the spectrum, viz. yellow and blue. When the middle of the red space was absorbed by a blue grass, he saw the black space with what he called the yellow on each side of it. This defect in the perception of colour was experienced by the late Mr. Dugald Stewart, who could not perceive any difference in the colour of the scarlet fruit of the Siberian crab, and that of its leaves. Dr. Dalton was unable to distinguish blue from pink by daylight; and in the solar spectrum the red was scarcely visible, the rest of it appearing to consist of two colours. M. Troughton had the same defect, and was capable of fully appreciating only blue and yellow colours; and when he named colours, the names of blue and yellow corresponded to the more and less refrangible rays; all those which belong to the former exciting the sensation of blueness, and those which belong to the latter the sensation of yellowness.

In almost all these cases, the different prismatic colours had the power of exciting the sensation of light, and giving a distinct vision of objects, excepting in the case of Dr. Dalton, who was said to be scarcely able to see the red extremity of the spectrum.

Dr. Dalton endeavoured to explain this peculiarity of vision, by supposing that in his own case the vitreous humour was blue, and therefore absorbed a great portion of the red and other least refran-

gible rays; but this opinion is, we think, not well founded. Sir J. Herschel attributes this state of vision to a defect in the sensorium, by which it is rendered incapable of appreciating exactly those differences between rays on which their colour depends.

CHAP. XV.

OPTICAL INSTRUMENTS.

1188. *Spectacles.* — These are the most simple and most useful class of optical instruments. They consist of two glass lenses mounted in a frame so as to be conveniently supported before the eyes, and to remedy the defects of vision of naturally imperfect eyes.

Whatever be the defects of sight which spectacles may be used to remove, it is evident that the lenses ought to be so mounted that their axes shall be parallel, and that their centres shall coincide with the centres of the pupils when the optical axes are directed perpendicular to the general plane of the face, that is to say, when the eyes look straight forward.

These conditions, though important, are rarely attended to in the choice of spectacles. If spectacles be mounted in extremely light and flexible frames, the lenses almost invariably lose their parallelism, and their axes not only cease to be parallel, but are frequently in different planes. Spectacles ought therefore to be constructed with mounting sufficiently strong to prevent this derangement of the axes of the lenses, and in their original construction care should be taken that the axes of the lenses be truly parallel.

In the adaptation of spectacles it is necessary that the distance between the centres of the lenses should be precisely equal to the distance between the centres of the pupils. The clearest vision being obtained by looking through the centres of the lenses, the eyes have a constant tendency to look in that direction. Now, if the distance between the centres of the lenses be greater than the distance between the centres of the pupils, the eyes having a tendency to look through the centres of the lenses, their axes will cease to be parallel, and will diverge as in the case of an outsquint. On the other hand, if the distance between the centres of the lenses be less than the distance between the centres of the pupils, there will, for a like reason, be a tendency to produce an insquint.

It has been already shown that the pencils most free from aberration are those whose axes coincide with the axis of the lens, and the more the axes of secondary pencils deviate from this, the greater will be the effects of aberration.

It follows from this that the most perfect vision with spectacles is produced when the eye looks in the direction of the axis of the lenses, and that more or less imperfection attends oblique vision through them. Persons who use spectacles, therefore, generally turn the head, when those whose sight does not require such aid merely turn the eye.

1189. *Periscopic spectacles.*—To diminish this inconvenience, the late Dr. Wollaston suggested the use of meniscuses or concavo-convex lenses, instead of double concave or double convex lenses with equal radii, which had been invariably used.

For persons requiring convergent lenses he proposed meniscuses with the concave surface next the eye; and for persons requiring divergent glasses, he proposed the concavo-convex lens, with the concave side next the eye. The effect of this is that the secondary pencils have less aberration than in the case of double convex and double concave lenses; and, consequently, that there is a greater freedom of vision by turning the eye without turning the head, from which property they were named *periscopic spectacles.*

1190. *Weak sight and short sight.*—It has been already explained that the optical defects of the eye which are capable of being corrected by lenses placed before it, are either a deficiency or an excess of their refractive power. Eyes which are deficient in refractive power, and which are called *weak-sighted eyes,* are those which are not capable of converging the pencils proceeding from visible objects at the usual distances to a focus on the retina. Eyes, on the other hand, which have too great refractive power, bring the rays proceeding from visible objects to a focus before they come to the retina, and are called *short-sighted eyes,* because objects which are near them are distinctly visible without the interposition of lenses.

1191. *Spectacles for weak-sighted eyes.*—The convergent power of the lenses necessary for weak-sighted eyes will necessarily be determined by the degree of the deficiency which exists in the refractive power of the eye. If the eyes be capable of affording distinct vision of objects so distant that the rays proceeding from them may be regarded as parallel, they will be capable of refracting parallel rays to an exact focus on the retina; but if they are so feeble in their refractive power as to be incapable of converging rays in the slightest degree divergent to a focus, they will be incapable of seeing distinctly any objects whose distances from the eye are less than from two to three feet, because the rays composing the pencils from such objects have a divergence which, though slight, the eye is incapable of surmounting, and the pencils accordingly, after entering the eye, converge to a focus not on the retina, but behind it.

Hence we find that persons having feebly refracting eyes are obliged to remove a printed or written page to a considerable distance from the eye to be able to read it. The pencils are thus rendered parallel,

and therefore such as the eye may bring to a focus on the retina, but this increase of distance from the eye is attended with the consequence of rendering the light proceeding from the object more feeble, and often too feeble to produce distinct vision. Hence we find that when weak-sighted persons hold a book or newspaper which they desire to read at a considerable distance from the eye, they are obliged at the same time to place a candle or lamp near the page to produce an illumination of the necessary intensity.

Since such eyes are, according to the supposition, adapted to the refraction of parallel rays, the lenses which they require must be such as to render the pencils proceeding from the objects at which they look parallel, and consequently they must be lenses *whose focal length is equal to the distance of the objects looked at.*

Nothing, therefore, can be more simple than the rule to be followed by such persons in the selection of spectacles. They have only to use for their spectacles lenses whose focal length is equal to the distance of the objects which they desire to see distinctly; and if they have occasion to look at objects at different distances, as, for example, at ten and at twenty inches, they ought to be provided with different pairs of spectacles for the purpose, one having a focal length of ten inches, and the other a focal length of twenty inches. When they look at an object at ten inches from the eye with spectacles of ten inches focal length, the rays will enter the eye exactly as they would if the object were at a distance of several feet from them; and those rays, being parallel, will be refracted to a focus on the retina.

It may be asked, in this case, how it happens that, if it be necessary for such persons to use spectacles having a focal length equal to the distance of the object at which they look, they can, nevertheless, see with the same spectacles distinctly objects at distances greater or less, within certain limits, than the focal distance of the spectacles? The answer is, that this arises from the power with which the eye is endued to adapt itself within certain limits to vision at different distances, as has been already explained.

1192. *How to determine the refracting power of weak-sighted eyes.* — If the weakness of the sight be such that the eye is incapable of bringing even parallel rays to a focus on the retina, it will be necessary to use convergent lenses even for the most distant objects. The power of the lenses which are necessary to render the vision of distant objects clear in that case will supply means of calculating the natural convergent power of the eye; for since the convergent power of the lens, together with the natural convergent power of the eye, bring parallel rays to a focus on the retina, the natural convergent power of the eye will be equal to the difference between the convergent power of the lens and the convergent power of an eye capable of bringing parallel rays to a focus on the retina.

To render this more clear let f be the focal length of a lens which

is equivalent to the refracting power of an eye which would bring parallel rays to a focus on the retina. Let f' be the focal length of the lens which is sufficient to enable the defective eye to bring parallel rays to a focus on the retina; and let f'' be the focal length of a lens optically equivalent to the defective eye. We shall then have

$$\frac{1}{f'}+\frac{1}{f''}=\frac{1}{f};$$

consequently we shall have

$$\frac{1}{f''}=\frac{1}{f}-\frac{1}{f'}.$$

From this condition the focal length of the eye can be found, since its reciprocal is equal to the difference between the reciprocals of the focal length of an eye adapted to parallel rays, and the focal length of the lens which produces clear vision in the defective eye.

In the same case, spectacles of different convergent power will be necessary when near objects are viewed; for in this case the pencils, having more divergence, will require a more convergent lens to aid the eye in bringing them to a focus on the retina. Such eyes, therefore, will require spectacles of different powers for distant and near objects; and if the power of the eye in adapting itself to different distances be not great, it may even be advisable to provide different spectacles for near objects which differ in their distance, as already explained in the case of eyes adapted to the refraction of parallel rays.

1193. *Spectacles for near-sighted eyes.* — To determine the focal length of the lens which will enable near-sighted eyes to see distinctly distant objects, it is only necessary to ascertain the distance at which, without an effort, the same eyes can see objects distinctly. This distance determines the degree of divergence of the pencils which the eyes bring to a focus on the retina. If diverging lenses be applied before the eyes whose focal length is equal to this distance, such lenses will give to parallel rays proceeding from distant objects the same degree of divergence as pencils would naturally have proceeding from objects whose distance is equal to their focal length; consequently, according to the supposition, the eye will bring such rays to a focus on the retina. The lenses, therefore, which fulfil this condition, will render the vision of distant objects with such eyes as distinct as would be the vision of objects placed at a distance from the eyes equal to the focal length of the lenses.

If the excess of the refractive power of short-sighted eyes be so great, and the power of adaptation to varying distances so small, that the same divergent lenses which render distant objects distinct will not render objects which are near the eyes, but not near enough for

distinct vision without spectacles, distinct, then lenses of less divergent power must be used to produce a distinct vision of such objects.

Thus, for example, suppose the case of eyes so near-sighted as to see distinctly objects only when they are at five inches distance. To enable these eyes to see an object at ten inches distance distinctly, it will be necessary to use divergent lenses; but these lenses must have less diverging power than those which render the vision of distant objects distinct, because the same lenses which would give the necessary divergence to the parallel rays which proceed from distant objects would give too great a divergence to the pencils which proceed from an object at ten inches distance.

1194. *Case in which the eyes of the same person have different refracting powers.* — In the selection and adaptation of spectacles, it is invariably assumed without question, that the two eyes in the same individual have exactly the same refracting power. That this is the case is evident, from the fact that the lenses provided in the same spectacles have invariably the same focal length.

Now, although it is generally true that the two eyes in the same individual have the same refractive power, it is not invariably so; and if it be not, it is evident that lenses of equal focal length cannot be at once adapted to both eyes.

When the difference of the refractive power of the two eyes is not great (which is generally the case when a difference exists at all), this inequality is not perceived. By an instinctive act of the mind of which we are unsconscious, the perception obtained by the more perfect of the two eyes in case of inequality is that to which our attention is directed, the impression on the more defective eye not being perceived.

It might be expected, however, that the inequality would become apparent, by looking alternately at the same object with each of the eyes, closing the other; but it is so difficult to compare the powers of vision of the two eyes when they are not very unequal by objects contemplated at different times, even though they should be exhibited in immediate succession, that this method fails.

1195. *Apparatus for comparing the power of vision of the two eyes.* — My attention having been recently directed to this question, I have contrived an apparatus which may not inaptly be called an *Opthalmometer*, by which the least difference in the powers of the two eyes may be rendered immediately apparent.

The principle I have adopted for this purpose resembles that which has been otherwise applied with success in photometers. I have so arranged the apparatus, that two similar objects similarly illuminated shall be at the same time visible in immediate juxtaposition, the one by the right eye being invisible to the left eye, and the other by the left eye being invisible to the right eye.

This apparatus consists of a small box A B C D, *fig.* 378., about five

inches in width A D, ten inches in length A B, and six inches in height. Within this there slides another box, A′ B′ C′ D′, made nearly to fit it, but to move freely within it, the interior of this box being blackened, or lined with black velvet. In the end B′ C′ is a rectangular aperture M N, the length of which M N is about an inch, and the height about half an inch; the length, however, being capable of being augmented and diminished by slides. Opposite to the end of the box B C is a white screen, on which is traced a horizontal line parallel and opposite to the opening M N, and marked with a divided scale, the 0 of which is opposite to the centre of the aperture M N, and the divisions upon which are numbered in each direction from 0 by 1, 2, 3, 4, 5, 6.

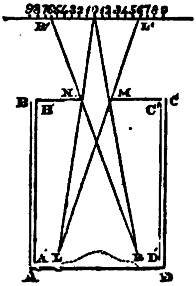

Fig. 378.

Let us suppose the eyes now applied at R and L. Let the sliding interior box B′ C′ be moved until, on closing the left eye, the division 0 of the scale coincides with the edge M of the opening, and at the same time, by closing the right eye, the same division 0 of the scale coincides with the edge N of the opening. It will be always possible to make this adjustment, provided the eyes are placed centrally opposite the opening M N, which may be easily managed by cutting in the edge of the box A D an opening to receive the bridge of the nose. This arrangement being made, it is clear that if we close the left eye we shall see the space upon the scale included by the lines R N and R M continued to the screen R′ L′. Let us suppose this space to include the six divisions of the scale from 0 to 6. If we close the right eye, we shall see with the left eye the six divisions of the scale to the right of 0. Now if we open both eyes and look steadily with them through the aperture M N, giving no more attention to the impression on the one than on the other, we shall see the twelve divisions of the scale, six to the right and six to the left of 0; the six divisions to the left of O being seen only with the right eye, and the six divisions to the right of 0 being seen only with the left eye.

In this way we have two similar objects, similarly illuminated and of equal magnitude, in immediate juxtaposition, the one seen by the right and the other by the left eye; and any difference in their distinctness, quality, brilliancy, or colour, will be as clearly and instantly perceivable as the comparative brilliancy of spaces illuminated by two different lights in the photometer. I have already experimented with this apparatus upon my own eyes, the result of which is, that I find that the sight of the right eye is much better than that of the left, the figures to the left of 0 being always more distinct than those to the right of it; but, what is more remarkable, I find that the transparency of the humours of the right eye is more perfect than that of

the humours of the left eye, for the space to the right of 0 always appears less bright than the space to the left of it.

1196. *Method of adapting spectacles to eyes with unequal powers of vision.*—To apply this instrument for the purpose of adapting spectacle lenses to eyes of unequal powers of vision, it is necessary first to ascertain the existence of the inequality of power in the manner already explained. It would then be necessary to provide two distinct screens on which similar scales might be drawn; so that they might be placed at different distances from the aperture M N. Let their relative distances be then determined, so that the two eyes would see the scales with equal distinctness. These distances will then represent the focal lengths of the divergent lenses which it would be necessary to provide for the eyes, so as to make them see different objects with equal distinctness.

In the case of weak-sighted eyes, this method will not be applicable. In that case let the two screens be placed at equal distances from the aperture M N, and let lenses be selected for each eye separately, closing the other, so as to give a distinct perception of the scales. The two lenses being then simultaneously applied to the eyes, let the scale be viewed with both eyes open. If the lenses be adapted to correct the defect of vision, the two parts of the scale to the right and to the left of 0, seen at the same time by each eye alone, will appear of uniform brilliancy and distinctness.

If defective eyes were tested by this method, I believe it would be found that inequality of vision would be much more common than is generally supposed, and accordingly the adaptation of spectacles would be considerably improved.

1197. *Remarkable case of vision defective in different degrees in different directions.*—Cases occur not only in which the comparative powers of vision of the two eyes differ, but in which the power of vision, even of the same eye, is different when estimated in different directions.

I have known short-sighted persons who were more short-sighted for objects taken in a vertical than in a horizontal direction. Thus with them the height of an object would be more perceptible than its breadth, and in general vertical dimensions more clearly seen than horizonal. This difference arises from the refractive power of the eye taken in vertical planes being different from the refractive power taken in horizontal planes; and the defect is accordingly removed by the use of lenses whose curvatures, measured in their vertical direction, is different from their curvature, measured in their horizontal direction. The lenses, in fact, instead of having *spherical* surfaces, have *elliptical* surfaces, the eccentricities of which corresponded with the variation of the refractive power of the eye.

1198. *Camera lucida.*—This instrument, the invention of the late Dr Wollaston, has proved of great utility in the arts, presenting a

remarkable facility for tracing a drawing of any distant objects, such as a building, a landscape, &c.

A quadrangular prism, *a b c d*, *fig.* 379, having a right angle at *b*, an angle of 135° at *d*, and angles at *a* and *c* equal to 67½°, is supported on a vertical pillar, with one side *a b* of its right angle horizontal, and the other *b c* vertical.

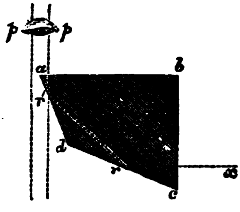

Fig. 379.

If an object be placed at a distance opposite *b c*, the rays proceeding from it will enter the prism in the direction *x r*, and will fall upon the surface *b c* so as to make the angle *x r c* equal to 22½°, and consequently the angle of incidence with the surface *d c* is 67½°. This angle being greater than the limit of transmission from glass into air, the light will be reflected from *r*, making the angle *d r r'* equal to the angle *c r x*; consequently, it will fall upon the surface *d a* at an angle of incidence of 67½°, and will therefore be again reflected from *r'*, making the angle *a r' p* equal to 22½°. The ray will thus fall upon the surface *a b* perpendicularly, and will pass through without further refraction. An eye placed at *p* would therefore see the object from which the original ray *x r* had proceeded in the direction *p r'*, and the same being true of all rays proceeding from the object, an image of the object will be seen by an eye presented downwards over the prism at *a*.

If a sheet of white paper be placed upon the table which supports the prism, an eye placed at *p* will see a picture of the object projected upon the paper; and if the eye be placed so close to the edge *a* of the prism that while it sees the picture projected upon the paper it also sees the paper directly, the observer will be able to trace with a pencil an outline corresponding with the picture, for while the picture is seen through the prism, the point of the pencil is directed upon the paper so seen directly outside the edge of the prism.

The use of this instrument requires some dexterity obtained by practice; but when the necessary skill is acquired, its use is simple and effectual.

1199. *Camera obscura.*—It has been already explained, that if an object be placed before a converging lens, at any distance greater than the focal length, a real image of the object will be formed on the other side of the lens, at the point corresponding to a position of the focus conjugate to the object. If the object, as is generally the case, be so distant that pencils of rays proceeding from it to the lens may be regarded as parallel, the lens will then form a picture of the object at a distance from it equal to its focal length. If a white screen be placed at right angles to the axis of the lens, and at the distance at which the image is formed, the image will be depicted upon it with

its proper form and colours; and if arrangements were provided by which a draughtsman could have access to the image thus formed on the screen, he would be enabled to trace its outline.

To obtain the necessary convenience and facilities for accomplishing this, arrangements are necessary by which all other light should be excluded except that which forms the picture, and that the position of the screen or paper on which the picture is formed should be such as may be convenient for the operator, and, in fine, so that the person of the operator may not intercept the rays forming the picture. These objects are attained by different arrangements, one of the most simple of which is represented in *fig.* 380.

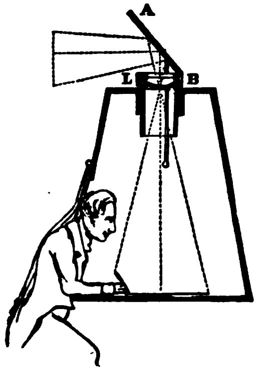

Fig. 380.

The lens L is placed in the centre of the top of a rectangular box, whose height corresponds with the focal length of the lens, whose bottom forms a desk upon which the draughtsman works, and in the side of which is an opening through which he may introduce his head and arms, over which a curtain is suspended, so that while it includes his person it may exclude the light. A plane reflector A B is placed above the lens, and is moveable on hinges. It is capable of being adjusted by a handle, which descends into the box, so that the operator may raise and lower it until the picture is thrown in a proper position. Means are also provided by which the lens L and the mirror A B can be moved round their centre so as to receive any required direction. The lens L is adjusted in a sliding tube, by which the focus can be brought exactly to correspond with the surface of the paper.

The oblique mirror A B and the lens L may be replaced by a prism with curved faces, such as that represented in *fig.* 381. The face of the prism *a c*, at which the rays first enter, is convex, by which the rays are made to converge; they then fall upon the plane side *a b*, by which they are reflected, and pass through the curved side *c b*, by which they are again refracted. The curvatures of the two sides *a c* and *c b* may be related in any required manner, so that their convergent powers may be equivalent to that of a lens of any proposed focal length.

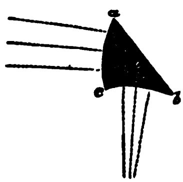

Fig. 381.

Strictly speaking, the picture of distant objects may be formed free from spherical aberration, if the surface of the paper form the surface of a sphere of which the optical centre of the lens is the centre. This

is sometimes accomplished by throwing the picture upon a concave surface formed of plaster of Paris, whose centre corresponds with that of the lens.

The phenomena exhibited by this instrument are rendered especially pleasing, inasmuch as it exhibits not only a picture of the external scenery, but shows all the objects in motion upon it as in the real scene. Thus carriages, horses, and pedestrians, appear with their proper motion, the leaves tremble on the trees, and the smoke curls from the chimney.

The opening at which the observer is placed, ought, of course, to be at that side of the box at which the picture appears erect.

1200. *The magic lantern.*—The magic lantern is an instrument adapted for exhibiting, on an enlarged scale, pictures painted in transparent colours on glass, by means of magnifying lenses, by which the rays proceeding from the picture, after being transmitted, are brought to a focus at a distance from it upon a screen. The position of the screen and that of the picture are conjugate foci, and their linear dimensions are in the proportion of their distances from the lens. In proportion as the picture approaches the lens, the image formed on the screen recedes from it, and consequently becomes more magnified. These instruments vary in their form, according to the circumstances under which they are placed, and the cost expended on their construction.

They are usually arranged as represented in *fig.* 382. A lamp L is included in a dark lantern; behind it is placed a metallic reflector

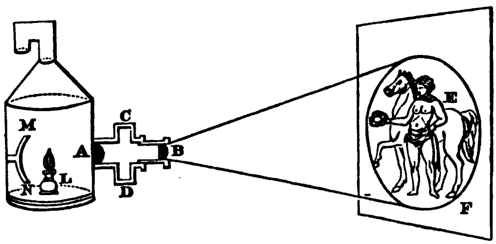

Fig. 382.

M N, and before it a large converging lens A, usually plano-convex. A groove is provided in the nozzle of the lantern, at C D, to receive the pictures, which are called sliders, in consequence of being successively passed in and out of the groove C D.

The colours in which they are painted and prepared are transparent gums, so that the light which passes through them may have corresponding colours. The magnifying lens B, which is also a convergent lens, is set in a tube, which slides in that which forms the fixed nozzle

of the lantern, and can be moved to and from the picture with a motion like that of a telescope or opera-glass. The light proceeding directly from the lamp L, and that which is reflected by M N passing through the lens A, which is called the *illuminating lens*, is made to converge upon the picture placed in the groove C D, so as to produce an equal illumination of every part of it. The light which thus proceeds from the picture being received by the convergent lens B, is brought to a focus on a screen E F, the screen being placed at such a distance from B as to correspond with the focus conjugate to that of the picture. We are, therefore, to consider the slider as placed in the focus of incident rays, and the screen in the focus of refracted rays. If the tube B be pushed in so that the lens be brought nearer to the picture, the screen must be moved to a greater distance, since by what has been already established, according as the distance of the focus of incident rays from the lens B approaches to equality with its focal length, the conjugate focus must recede from it. The magnifying power of the lantern is measured by dividing the height of the picture formed on the screen by the height of the picture formed on the slider.

Thus, if the slide be two inches and a half wide, and the picture formed on the screen be twenty-five inches high, then the magnifying power will be ten. The picture may be either viewed by spectators placed before the screen or behind it. In the former case, the screen must be formed of some material which is not penetrable by the light; in the latter case, it must be semi-transparent.

The best surface for exhibiting such pictures to spectators placed in front of the screen, is white paper or pasteboard. To exhibit them to the spectator placed on the other side of the screen, it is usual to prepare the screen with fine wax, so as to stop all its pores, and prevent the direct transmission of light. In this case, the screen being semi-transparent, the picture formed on the side next the lantern is visible on the other side, just as it would be through a plate of ground glass.

Since, however, at least one half the light which falls on the screen is reflected from it in the direction of the lantern, the pictures thus formed are never so vivid as those which are produced upon a properly prepared opaque screen.

Since the light which forms the picture on the screen is in all cases that which proceeds from the picture on the slider, it is evident that the greater the magnifying power used, the less intense will be the brilliancy of the picture. Whether the picture on the screen be great or small, the same quantity of light will be diffused over it, being the light which proceeds from the picture on the slider. This light, therefore, will be less intense in the same proportion as the magnitude of the picture on the screen is increased. If, therefore, the picture on the screen be ten times the height of the picture on the slider, its

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brilliancy will be a hundred times less than the brilliancy of the picture on the slider.

1201. *Phantasmagoria.* — When the pictures produced by a magic lantern are shown through a transparent screen not visible to the spectator, they may be made to vary in magnitude, gradually increasing and gradually diminishing, by moving the light to or from the screen, and at the same time moving the lens B proportionally from or to the slider. In this case the effect produced on the spectators is that of an object approaching to or receding from them; the change of apparent magnitude which the picture undergoes being imputed to a change of distance in the object. When the picture diminishes, it is supposed to recede from the eye, and when it increases, it is supposed to approach the eye. Various effects of this kind, combined by means of different lanterns used at the same time, are called Phantasmagoria.

1202. *Dissolving views.* — The exhibition called Dissolving Views is produced by placing two lanterns of equal power so as to throw pictures of equal magnitude in the same position on the same screen.

A sliding shutter is placed upon the nozzle of each lantern, and the two shutters are moved simultaneously, in such a manner that when the nozzle of one lantern is open, that of the other is completely closed; and according as the nozzle of the former is gradually closed, that of the latter is gradually opened.

Let us suppose, then, that two slides are placed in the lanterns, one representing a landscape by day, and the other representing precisely the same landscape by night; and let the nozzle of that which contains the landscape by day be open, the other being closed, the picture on the screen will then represent the landscape by day. If the slides be now slowly moved, the nozzle of the lantern which shows the day landscape will begin gradually to close, and that which shows the night landscape will gradually open. The effect will be, that the daylight will gradually decline upon the picture, and the objects represented will assume by slow degrees the appearance of approaching night. This gradual change will go on until the nozzle of the lantern containing the day picture is completely closed and that containing the night picture completely open, when the change from day to night will have been completed.

Various other effects, familiar to those who have witnessed phantasmagoric exhibitions, are produced by combining two or more magic lanterns. Thus, for example, the picture of a castle with a portcullis and drawbridge is exhibited. The portcullis rises, and a knight in armour on horseback issues from it and crosses the drawbridge. The opening of the portcullis is in this case produced by a moveable plate attached to the slider representing the castle, and the figure of the knight is produced by means of a second lantern, so skillfully managed as to throw the image of the knight upon the screen, and to move it so as to cross the drawbridge.

1203. *Simple microscopes.*—If the eye possessed the faculty of increasing its convergent power without limit, it would be capable of rendering itself microscopic, and of perceiving accurately and distinctly, without artificial aid, the most minute objects.

It has been already shown that the apparent magnitude of an object, measured by the angle it subtends at the eye, varies inversely as the distance at which the object is viewed. It therefore follows, that as we approach an object, its apparent magnitude increases in the same proportion as its distance from the eye is diminished. Now if there were no circumstance to prevent the eye from seeing distinctly an object at any distance, however small, we could see it with any apparent magnitude, however great, by merely bringing it close to the eye. Thus, if an insect placed at six inches from the eye be seen with a certain apparent magnitude, it will be seen with ten times that apparent magnitude at a distance of $\frac{6}{10}$ths of an inch, and with a hundred times that apparent magnitude at the distance of $\frac{6}{100}$ths of an inch; and so on.

But while the eye approaches to any object, the divergence of the pencils of light proceeding from each point of this object and passing through the pupil, increases exactly in the same proportion as the distance of the eye from the object is diminished. At half the distance the pencil would have double the divergence, and at one-tenth of the distance it would have ten times the divergence; and so on. Now in order to obtain distinct vision of an object, it is not enough that the picture on the retina be large. It is necessary also that the pencils proceeding from the various points of the object should be severally brought to a focus on the retina. Now the more divergent these pencils are, the greater will be the refracting power necessary to bring them to a focus on the retina; and although the eye possesses the faculty of augmenting its refractive power, the exercise of this faculty has a narrow limit, and there is accordingly a distance from the eye, within which a visible point being placed a pencil proceeding from it cannot be made to converge upon the retina; and though an object placed within such distance may produce a large image on the retina, such image will be so indistinct and confused, owing to the pencils not coming to a focus, as to afford no clear perception of the object.

But if, by any contrivance, we could enable the eye, as it approaches an object, to increase its converging power in the same proportion as its distance from the object is diminished, we should then enable it to see such object distinctly, however diminished its distance from the eye might be; and we should consequently, by the same means, obtain a picture on the retina at once magnified and distinct.

Now this object is attained by the simple microscope, which is nothing more than a convergent lens applied between the eye and the object, the effect of which is to cause the pencils of rays which diverge

from the several points of the object to enter the pupil with a degree of divergence so much diminished as to enable the eye to bring them to a focus on the retina. It must be remembered, that a convergent lens placed at a distance equal to its focal length from the focus of a divergent pencil renders the rays of such pencil parallel, thus destroying altogether their divergence.

1204. *How simple microscopes are adapted to different eyes.* — If such a lens be applied at a still less distance than the focal length from the focus of a divergent pencil, its effect will be not altogether to destroy, but merely to diminish the divergence of the rays of the pencil.

Now some eyes, such as those called *far-sighted*, are adapted to the reception of parallel rays, which they bring without effort to a focus on the retina. Others, called *near-sighted*, are adapted to the reception of rays more or less divergent, which they bring to a focus on the retina. A convergent lens placed between the eye and an object near it may, in either case, be so adapted as to bring the rays diverging from the points of such object to a focus on the retina, and therefore to afford clear vision of it.

If the eye, for example, be *far-sighted*, and therefore adapted to the reception of parallel rays, the lens must be held at a distance from the object equal to its focal length, in which case the pencils diverging from the various points of the object will be parallel after passing through the lens, and will therefore be brought to a focus on the retina.

If the eye be *near-sighted*, so as to be adapted to rays more or less divergent, then the lens must be placed at a distance from the object less than its focal length, and the distance must be regulated, which it always may be by trial, so that the rays of the pencil, after passing through it, shall have just that degree of divergence which will enable the eye to bring them to a focus on the retina.

The more short-sighted the eye is in this case, the greater will be the divergence with which the rays will enter it, and consequently the nearer to the object the lens must be brought. The apparent magnitude, therefore, of an object, when seen through a single converging lens, is equal to the apparent magnitude which it would have if the eye could view it at the same distance without the intervention of any lens; and from what has been just explained it follows that the same lens will give a greater apparent magnitude to an object to a near-sighted than to a far-sighted person, and the more near-sighted the eye is, the greater will be the apparent magnitude of the object seen through the lens.

1205. *Magnifying power explained.* — The term *magnifying power*, as applied to a microscope, is one which, in its ordinary application, is generally vague and uncertain. It has in all cases a relative signification, the object viewed being said to be magnified or increased

in magnitude, in comparison with the apparent magnitude which it would have if viewed without the interposition of a lens. But since the eye is capable of obtaining distinct vision of the same object at different distances, it is capable of seeing the same object with different apparent magnitudes.

To give a distinct meaning, therefore, to the term magnifying power, it is necessary to state what is the standard magnitude with which the effect of the lens is to be compared.

This standard is, or ought to be, the greatest apparent magnitude under which the object is capable of being distinctly seen by an eye without the interposition of a lens. But here a distinction becomes necessary. The greatest apparent magnitude under which a given object can be seen by one person is not the same as the greatest apparent magnitude under which it can be seen by another. The greatest apparent magnitude under which an object appears to a short-sighted eye, which can obtain a clear perception of it when viewed at five inches' distance, is greater than the greatest apparent magnitude under which it can be seen by a long-sighted eye, which is not capable of obtaining a clear perception of it at a less distance than ten inches. It is evident, therefore, that the magnifying power of a given microscope applied to the eye of the one person will be different from its magnifying power applied to the eye of the other.

In general, however, it may be stated that the apparent magnitude of an object seen through a simple microscope, is so many times greater than the apparent magnitude of the same object seen at any distance at which distinct vision can be obtained, as the latter distance is greater than the distance of the object from the microscope.

Thus, if an object which cannot be distinctly seen at a less distance than eight inches be made distinctly visible at the distance of one inch by the interposition of a convergent lens, the magnifying power of such lens to the eye which thus views the object is eight times, inasmuch as the angle subtended by the object at the distance of an inch is eight times that which it would subtend at the distance of eight inches.

1206. *Apparent brightness is diminished as the square of the magnifying power is increased.* — What has been stated respecting the brilliancy of a picture thrown upon a screen by a magic lantern, is equally applicable to the brilliancy of the image of an object formed on the retina either by means of the naked eye or by the interposition of a convergent lens. The light diffused over such a picture can only be that which is transmitted from the object, and it follows that the larger the picture the less, proportionally, will be its brilliancy; and in this case it must be remembered, that the area of the picture is increased in proportion to the square of the magnifying power. Thus, if the magnifying power be four, the height or diameter of the picture on the retina formed by the lens will be four times

the height or diameter of the picture formed on the retina without the lens. But if the height or diameter be increased in a four-fold proportion, the area of the picture will be increased in a sixteen-fold proportion, and the light which was before diffused over the smaller area will, by means of the lens, be spread over an area sixteen times greater, and consequently the brilliancy of the image will be sixteen times less.

If it be desired that the magnified image of an object produced by a microscope should have the same brilliancy as the object itself has when viewed without a microscope, it would be necessary to illuminate the object, when viewed through the microscope, with light of an intensity proportional to the square of the magnifying power. Thus, if a magnifying power of four were used, the image cannot have the same intensity of illumination as the object, unless it be illuminated with light of sixteen times the intensity.

1207. *Compound microscope.* — With the simple microscope the object itself is viewed directly by the eye, but at a less distance than would be compatible with distinctness of vision without the interposition of the lens. It is, however, sometimes necessary to submit to microscopic observation objects so minute that practical difficulties would arise in viewing them with simple lenses of sufficiently small focal length to produce the requisite magnifying effect. In such cases, instead of submitting the object itself to immediate observation by means of the simple microscope, an optical image of it is produced by means of convergent lenses or concave reflectors.

The image thus formed may, according to the principles already established (Chap. X.), be rendered larger in any desired proportion than the object, and may, therefore, be viewed with a simple microscope of proportionally less magnifying power. Thus, for example, if an image of a minute object be produced, having linear dimensions ten times greater than those of the object, and if such image be viewed by a simple microscope whose magnifying power is twenty, the object will, by such a combination, be magnified two hundred times; for the image which forms the immediate object of examination with the simple microscope has ten times the linear magnitude of the object, and is itself magnified twenty times by the simple microscope.

A compound microscope consists, then, of such a combination. If the image, which is the immediate object of observation be formed by lenses, the microscope is called a *compound refracting microscope*, and if it be formed by a concave reflector it is called a *compound reflecting microscope.*

The form, dimensions, and power of compound microscopes are infinitely various, according to the purposes to which they are applied, to the exigencies of the observer for whose use they are intended, and to the taste and ability of their constructors.

The principles common to all refracting microscopes, which are

more generally used than reflectors, may be more clearly understood by reference to *fig*. 383., where *b* represents a small convergent lens of very short focal length, before which a minute object *o*, which it is

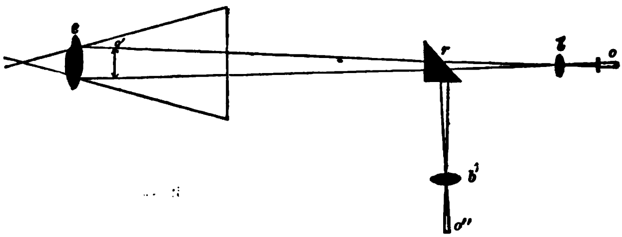

Fig. 383.

desired to magnify, is placed. The distance *b o* will a little exceed the focal length of the lens *b*. An image of *o* will be formed at *o'*, the focus conjugate to *o*. The magnitude of the image at *o'* will be just so much greater than the magnitude of the object, as the distance *o' b* is greater than the distance *o b*. This image *o'* is the immediate object of examination with the simple microscope *e*; and everything which has been explained with respect to the application of simple microscopes to magnify objects will be equally applicable to the microscope *e* as applied to the image *o'*, considered as an object. The lens *b* is called the object-glass, and the lens *e* is called the eye-glass. In some cases, instead of a single lens two or more are used at *b*, so as to form a compound object-glass. The lens *b* is rendered achromatic in the manner already explained, by constructing it of two lenses having different dispersive powers and corresponding curvatures, as already explained.

The magnifying power of the object-glass is generally increased, not by increasing its curvature, but by combining together two or more lenses of equal magnifying power, and of equal opening. In this way the observer is enabled with great facility to vary the power of his microscope at pleasure, according to the magnitude of the object he examines.

It is only necessary to screw upon the end of the microscope next the object one, two, or three lenses, so as to increase the magnifying power.

In like manner the eye-glass *e* may be varied in power by combining different lenses. It is usual in compound microscopes to provide several *eye-pieces*, as they are called, having different powers; the eye-piece being the name given to the sliding tube which contains the eye-glass, and which moves with the motion of a telescope joint, so as to vary at pleasure its distance from the image *o'*. These eye

pieces usually consist of two plano-convex lenses instead of a single double convex lens.

It is sometimes convenient to enable the observer looking in a horizontal direction to see an object which is placed vertically below the end of the instrument. This is accomplished by placing the great tube containing the object-glass at right angles to the tube along which the observer looks, a rectangular prism being placed, as represented at *r*, in the angle formed by the two tubes. This arrangement is represented at *r b'*, where *b'* is the object-glass, and *o''* the object, and *r* the prism by which the pencils proceeding from the object-glass *b'* are reflected at right angles, so that the image is formed at *o'*.

It has been already explained that in all cases where a distinct image is required to be formed by means of a convergent lens, the divergence of the pencils proceeding from the object must not exceed such a limit as would render the diameter of the lens sensibly different from the length of a circular arc described with the extreme pencils as radii, and the point of the object from which the rays diverge as a centre. Consequently it follows that in all cases the diameter of the lens must bear a very small proportion to the distance of the object from it.

Now, since in microscopes of every kind where an eye-piece is used the focal length of the lens itself, if it be a simple microscope, and of the object-glass, if it be a compound microscope, is extremely short, and the distance of the object from the lens very small, it follows that, according to the principle just explained, the diameter of such lenses must be also extremely small, since their diameters must bear an inconsiderable proportion to such distance, and the higher the magnifying power, the smaller must be the diameter of the lens. Thus the diameter of such lenses used for the object-glasses of compound microscopes do not exceed in general a small fraction of an inch. The same observation is applicable to the reflectors or specula used to form the image in reflecting microscopes.

1208. *Compound universal microscope of Charles Chevalier.* — It would not be compatible with the object of this volume to enter into any detailed description of the various forms of compound microscopes. It may be useful, however, to indicate the arrangements adopted in one as an example. For this purpose I shall here briefly describe the form of microscope constructed by Mr. Charles Chevalier, and called from its general utility the Universal Microscope.

This instrument is represented in *fig.* 384. The object-glass is at Y, and the eye-piece at S. The rectangular prism by which the pencils are turned along the horizontal tube is at V. The object-glasses consist of three achromatic lenses, whose focal lengths vary from three-tenths to four-tenths of an inch. These lenses are constructed with screws so as to be successively screwed upon the end Y of the tube

They may be used separately or together, according to the magnifying power required.

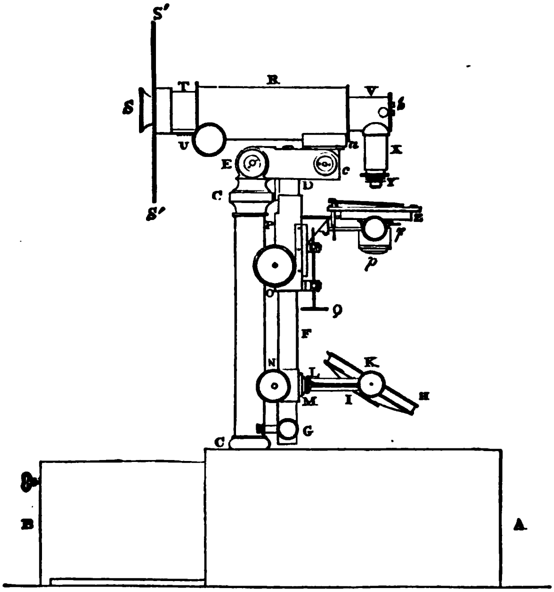

Fig. 384.

The instrument is usually provided with six eye-pieces; the first four are constructed upon the same principle, each being composed of two plano-convex lenses, whose convexities are turned towards the image. The two others are simple converging lenses of short focus. Z is a stage provided to receive the slider on which the object to be examined is placed. This stage can be moved gradually by an adjusting screw O, upwards and downwards on a square vertical rod P G, so as to vary its distance from the object-glass Y. The observer looking through the eye-piece S, places his right hand upon the screw O, and turns it in the one direction or the other, until he brings the object to the focus, the final adjustment being made by a fine micrometric screw Q, which gives a slower motion to the stage than can be imparted to it by the screw O.

If the object be transparent, it is illuminated by a concave mirror H, which is capable of being adjusted at such an angle with reference to the light as to throw the illumination directly upon the object by means of a horizontal axis K, on which the reflector H turns, and the intensity of the illumination may be varied by moving the reflector

vertically upon the bar P G by means of a screw N, which works in a rack behind the bar and carries with it the frame L, by which the mirror is sustained. In this manner the reflector may be made to approach to or recede from the slider which supports the object, and the light thrown upon it may be accordingly rendered more or less intense.

If the instrument be used in the day-time it will be convenient to place it upon a table near a window, so that the light from the clouds may be received upon the reflector H. If it be used at night, a lamp or candle placed at a convenient height in front of the instrument will form a sufficient illumination.

In order to intercept all light proceeding from the reflector H, except that which falls upon the object, a circular movable stage is placed beneath the stage Z, which supports the object, and pierced by a number of holes of different magnitudes, which, being brought successively under the object, regulate the quantity of light reflected from H which is transmitted to the object.

The focus is determined by the adjustment with the screw O, and still more accurately by the micrometer screw.

When it is required to view opaque objects, they are usually placed upon a blackened glass laid upon the slide Z, and are illuminated by another lens or reflector attached to the side of the slide Z.

When high magnifying powers are used, so that only a part of a minute object can be seen at one time in the field of view, it is desirable to be enabled to move the object slowly under the microscope, so as to bring all its parts successively under examination. To do this by moving the slide with the hand is impracticable, for the motion being magnified in the same proportion as the object, a movement of the hand which is imperceptible will throw the object completely out of the field. To enable, therefore, the observer to move the object so as to bring all its parts successively into view, and to keep any part steadily under examination, two micrometer screws under the stage Z are provided: the first moves the object backwards and forwards to or from the observer, and the second moves it laterally right and left. By the combination of these two motions every possible position can be given to the object.

To keep the eye of the observer undisturbed by extraneous light, a large circular screen S′ S′ is placed between the socket of the eye-piece and the end of the instrument. By this means the observer is not under the necessity of closing that eye which is not directed to the eye-piece.

1209. *Adaptation of the camera lucida to the microscope.* — The adaptation of the camera lucida to this instrument has greatly extended its interest and utility. The instrument is attached to the eye-piece S, so that when the observer looks upon it he sees, by the reflection of the prism, a sheet of paper placed vertically under the

eye-piece on a table before him, and he sees directly the image of the magnified object projected upon the paper. In this manner he is enabled to make a tracing of the object, as already described in the application of the camera lucida.

1210. *Method of determining the magnifying power.* — To determine the magnifying power of the instrument, a slide is provided, of the form represented in *fig.* 385., upon which is engraved a micro-

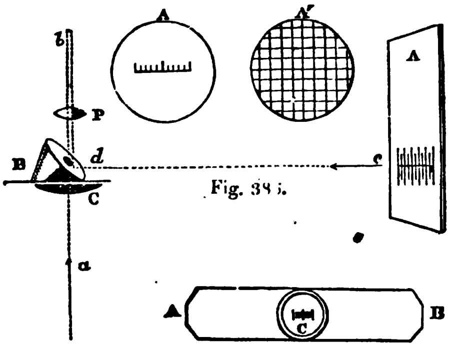

Fig. 385.

Fig. 386.

meter scale, made to the $\frac{1}{100}$th part of a millimetre, which is equivalent to the $\frac{1}{2500}$th part of an inch. There are usually ten of these divisions engraved upon the slider, the entire length of which is therefore the $\frac{1}{250}$th part of an inch. This slide being placed upon the stage, a camera lucida, constructed upon the principle of Amici, is attached to the eye-piece C, *fig.* 385., the effect of which is that the eye sees a magnified image of the scale projected upon a sheet of paper A spread upon the table under the hand of the observer. A divided scale being applied to this magnified image, the length of the magnified divisions may be at once compared with the divisions on the divided scale, which are seen directly without being magnified. The arrangement by which this is accomplished is represented in *fig.* 385., where P represents the eye, B a metallic speculum applied at an angle of 45° with the line of vision, and having a hole pierced in its centre, through which the eye-piece and the magnified image of the scale, *fig.* 386., are seen. At the same time the eye sees by reflection, in the speculum B, the sheet of paper A, upon which a scale is placed at C', parallel to, and coincident with, the image of the scale, *fig.* 386., which is also seen projected on the paper. By comparing the divisions of the image of the scale, *fig.* 386., seen magnified, with the image of the divisions of the scale on the paper A, seen by reflection without being magnified, the magnifying power can be immediately ascertained

The magnifying power being thus ascertained, the real dimensions of any object viewed through the microscope can be easily measured. For this purpose, let a system of parallel lines be described upon the paper A, at right angles to each other as represented at A′, so that the distance between them shall correspond to any fraction of an inch, such as the 2000th part, as ascertained by the method just explained.

When the magnified image of a minute object is seen projected upon the paper upon which this rectangular scale is drawn, the dimensions of the object can be determined by inspection, by merely observing how many squares of the rectangular scale the image occupies.

The details of the microscope represented in *fig.* 384., present many practical conveniences to the general microscopic observer.

When it is desirable to present the main tube R directly upon the object without reflection by the rectangular prism V, the elbow tube which contains this prism can be detached both from the main tube R, and from the tube X, which bears the object-glass, being attached to them by bayonet joints. The tube X is then inserted in the main tube R of the instrument, so as to form a compound microscope, in which the pencils refracted by the object-glass Y will proceed directly to the eye-piece S. The body of the microscope R turns upon a joint at *c*, so that it can be moved through a right angle, so as to be applied in a vertical position, and to enable the observer to look vertically downwards on the stage Z, which supports the object.

The instrument can also be applied at any required angle with the vertical. To accomplish this, it is only necessary to loosen the screw G, by which the straight bar P G is held in the vertical position. When this is done, the bar P G, carrying the reflector H and the stage Z, can be turned at any angle with the pillar C C by means of the pivot E, upon which the entire instrument, including the body of the microscope and the bar P G, turns with a common motion. The pivot works so stiffly, and the weight of the instrument is so equally balanced upon it, that it will rest at any required angle with the vertical.

By means of the two pivots E and *c*, the bar P G, and the body of the instrument R, can be brought into a horizontal direction, so that the stage Z shall be vertical and opposite to the object-glass of the microscope, the axis of which is horizontal. Proper holders are provided on the stage to keep the sliders in their position upon it, at whatever angle it may be applied with the vertical.

The entire instrument is screwed at O upon its own case O A, in which is a drawer, B, properly constructed to receive it and all its accessories.

1211. *Solar microscope.*—This instrument partakes of the character of a magic lantern and a compound microscope. Light proceeding directly from the sun is received upon a plane mirror, and reflected by it into the tube of the instrument, where it is received upon a large convergent lens called the *illuminating lens.* By this

lens the rays are made to converge to a focus, near which they are received upon the minute object which it is desired to exhibit. In this manner, the light thrown upon the object is just so much more intense than that which it would receive directly from the sun, as the area of the illuminating lens is greater than the area of the object. The object being thus illuminated, a magnifying lens is applied before it at a distance greater than the focal length; and an image is accordingly formed of the object at the other side of this lens, which image is greater than the object in the same proportion as its distance from the magnifying lens is greater than the distance of the object from it. A white screen being properly suspended at the necessary distance from the magnifying lens, receives the image, upon which it is seen in the same manner as in the case of the magic lantern.

It is evident that such objects only can be exhibited in this instrument as are naturally transparent, or such as may be rendered so. If opaque objects are exhibited, nothing appears upon the screen except a gigantic *silhouette* or profile of their form.

Such an instrument could only be exhibited in the day-time, and when the sun is unclouded. Its application, however, has recently been rendered more convenient and extensive by adapting it to artificial light called the Drummond light. Instead of the solar rays, a small cylinder of lime rendered incandescent by the oxyhydrogen blow-pipe is applied behind the illuminating lens in such a position that its light is brought to a focus upon the object to be exhibited, and the same effect is thereby produced as with sun-light. Several successful attempts have been still more recently made to apply the electric light to this instrument.

1212. *The telescope.*—What the compound microscope is to minute and near objects, the telescope is to distant objects. The principle in both instruments is the same, the details of its application alone being different. In the telescope, however, as the objects to which it is directed are always at a considerable distance from the object-glass, and generally at a distance which may be considered infinite as compared with any possible magnitude of that lens, it is possible to give the object-glass any desired magnitude without producing such spherical aberration as would render the image indistinct. In fine, the objects to which a telescope is directed being at distances incomparably greater than the diameter of the object-glass, their images will always be formed at a distance from such lens equal to its focal length. The pencils which proceed from the extreme limits of the object passing through the centre of the object-glass, and interesecting there, are continued to the corresponding extreme limits of the image which is formed in an inverted position with respect to the object at a distance from the object-glass equal to its focal length.

This image therefore subtends, at the centre of the object-glass, an angle equal to that which the object subtends at the same point. If

we imagine that an eye be placed at the centre of the object-glass, the apparent magnitude of the image seen from that point would be equal to the apparent magnitude of the object.

The image which is thus formed at the focus of the object-glass is, as in the case of the compound microscope, viewed by the observer with a simple convergent lens, called as in the compound microscope the *eye-glass*. All the observations which have been made in relation to the eye-piece of the compound microscope are equally applicable to the eye-piece of the telescope, which performs precisely the same functions in relation to the image formed at the focus of the object-glass as the eye-piece of a compound microscope with respect to the image formed in the focus of the object-glass of that instrument.

Telescopes differ from each other in the details of their construction, according as the images of the different objects are produced by object-glasses or by concave reflectors. In this respect telescopes, like microscopes, consist of two classes, reflecting telescopes and refracting telescopes. They are also classed in relation to the objects to the vision of which they are directed; those which are used for astronomical purposes being called *astronomical telescopes*, and those which are used for observing objects at less distance on the surface of the earth being called *terrestrial telescopes*.

In this last class it is important that the object should be seen erect, which it would not be if the image formed by the object-glass were the immediate subject of observation by the eye-glass; such image being, as already explained, always inverted. An expedient, however, is adopted in one class of telescopes, as will be presently explained, by which this inconvenience is removed without the introduction of additional lenses.

Having thus explained the general properties upon which telescopes are constructed, we shall briefly explain the different kinds of telescopes.

1213. *The Gregorian reflecting telescope.* — A longitudinal section of this instrument is represented in *fig.* 387. A B is a large

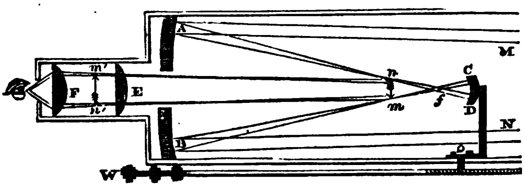

Fig. 387.

concave speculum formed of an alloy of metals adapted to receive a high polish. A circular aperture is made in the centre, so that the reflecting portion of the speculum is that part only which is outside

the circular aperture. A second concave speculum C D is placed with its concavity in the other direction, at a distance from A B greater than the focal length of the great speculum. The eye-glass F is placed in a smaller tube inserted in the greater one opposite the opening of the great speculum.

The extremity of the great tube being open, and presented towards the object of observation, an inverted image of this object is formed at *m n* in the principal focus of the great speculum A B. This image forms an object for the small speculum C D, and another image is formed in the conjugate focus *m' n'*; this latter image being inverted with respect to *m n*, and therefore erect with respect to the object.

The pencils proceeding from C D are sometimes brought to a focus by the interposition of a converging lens E, but this is not necessary.

The image *m' n'* is viewed by the eye-glass F, which, as already explained, may be considered as a simple microscope.

The telescope is mounted with proper apparatus, by which it can be directed to the object, and by which its focus can be regulated.

1214. *Cassegrain's reflecting telescope.* — A longitudinal section

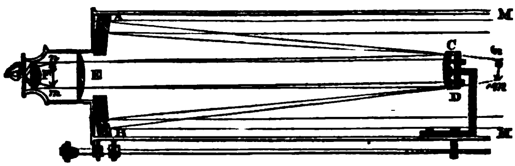

Fig. 388.

of this instrument is given in *fig.* 388. Its details are in all respects similar to the Gregorian reflector, except that the second speculum C D is convex instead of being concave, and receives the pencils proceeding from A B before they come to a focus. It turns them back towards the eye-piece, where an image is formed, as in the former case.

1215. *The Newtonian reflecting telescope.*—A longitudinal section of this instrument is represented in *fig.* 389., where A B is the great

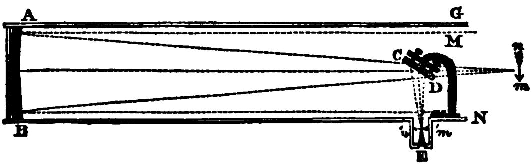

Fig. 389.

speculum which would form an image of the object at *m n* in its principal focus.

But the pencils, before they arrive at that point, being received upon a plane reflector C D placed at an angle of 45° with the axis of the telescope, the image is formed at m' n' in a lateral tube inserted in the great tube, where it is viewed by an eye-piece, as before explained. In this case the open end A of the great tube is directed towards the object, and the observer examines the object by looking in at the side of the telescope in a direction at right angles to its length.

In all these cases, the central rays of the pencils directed upon the great speculum are lost. In the Gregorian and Cassegrain, the central portion of the speculum is removed, and in the Newtonian telescope the central rays are intercepted by the plane reflector C D.

1216. *Herschel's telescope.*—The form of reflecting telescope which has attained by far the greatest celebrity of any that have been hitherto constructed, is that which was erected by Sir W. Herschel, and used by him with such signal success, as to render his name memorable in the history of astronomical science. Herschel, after having constructed a great number of reflecting telescopes on the Newtonian principle, varying from seven to twenty feet in length, aided by the patronage of George III., completed in 1789 his celebrated telescope, forty feet in length, by which, on the very day it was completed, he discovered the sixth satellite of Saturn. The great speculum of this telescope measured nearly fifty inches in diameter, its thickness being three inches and a half, and its weight about a ton. The open end of the telescope being directed to the point of the heavens under observation, and the speculum being fixed at its lower end, the observer is suspended in a chair, so as to be able to look over the lowest part of the edge of the opening. The speculum being a little inclined to the axis of the tube, the image is formed near the lowest point of the edge of the opening, where it is viewed by the observer with proper eyepieces.

The quantity of light obtained by this prodigious speculum enabled Sir W. Herschel to use magnifying powers which greatly exceeded any which before his time had been applied. He was thus enabled, in examining the fixed stars, to apply in some cases a magnifying power of 6450.

1217. *The Galilean telescope.— Opera-glass.*—This telescope, which takes its name from Galileo, by whom it was first used, is a refracting telescope, the principle of which is represented in *fig.* 390.

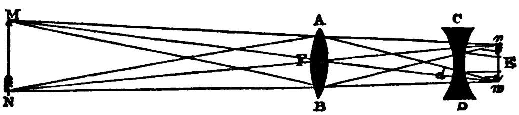

Fig. 390.

A B is the object-glass, in the principal focus of which, E, an inverted image of the object would be formed; but before the pencils arrive at this point, they are received by a divergent lens C D, which, destroying their convergence, causes them to enter the eye parallel, as they would if they proceeded from an object at a considerable distance.

The general direction of the axes of the pencils, however, is not changed, and the eye consequently receives them as if they had proceeded from an object at the same distance from the eye as the image *m n* is from the eye-glass C D. The apparent magnitude, therefore, of the object, as seen with the eye-glass C D, is measured by the angle which the image *m n* subtends at the centre of the lens C D; and the apparent magnitude of the object as seen directly is equal to the angle which the same image subtends at the centre of the object-glass F.

If, therefore, we divide the focal length of the object-glass by the distance of the eye-glass from the image, we shall then obtain the magnifying power.

Let us suppose, for example, that the focal length of the object-glass is fifty inches, that the focal length of the eye-glass is one inch, and that the eye of the observer is adapted to the reception of parallel rays. In this case, the focal length of the object-glass will be fifty times the distance of the eye-glass from the image, and the telescope will magnify accordingly fifty times. But if the eye of the observer be adapted to the reception of diverging rays, then the eye-glass C D must be removed further from the image than its focal length, and, consequently, the magnifying power will be less than it would be for an eye adapted to parallel rays; and if, on the contrary, the eye of the observer be adapted to converging rays, the eye-glass must be moved near to the image, and the magnifying power will be greater.

In all cases, the distance of the eye-glass from the object-glass is equal to the difference between their focal lengths for eyes adapted to parallel rays. It is a little less for short-sighted, and a little more for long-sighted eyes.

This form of telescope has long been disused for all purposes where very distant objects are observed. It is, however, still continued with great convenience where the objects of observation are nearer, as in the case of opera-glasses, which are nothing more than Galilean telescopes.

These instruments have lately been mounted in pairs, so as to enable the spectator to use both his eyes, as with spectacles.

1218. *The astronomical telescope.*—This is the name given to a refracting telescope, consisting of two convergent lenses, one used as an object-lens, to form an image of the object to be observed, and the other as a simple microscope, to examine this image. The principle of this instrument has been already sufficiently explained in the case of the compound microscope, from which it differs in nothing but in

the proportion of its parts. A B, *fig.* 391., is the object-glass; an inverted image *m n* of the object M N is formed at its focus.

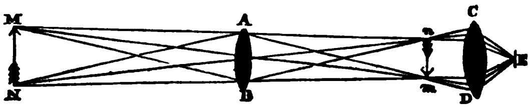

Fig. 391.

This image is viewed by the eye-piece C D, which for eyes adapted to parallel rays is placed at a distance from *m n* equal to its focal length. The image *m n* is seen under an angle equal to that which it subtends at the centre of the eye-glass C D, and its apparent magnitude being equal to the angle which it subtends at the centre of the object-glass A B, it follows that the magnifying power of the instrument is found by dividing the focal length of the object-glass by the focal length of the eye-glass. The image, as seen in this instrument, is always inverted with respect to the object; but as it is used for astronomical purposes, this is unimportant.

1219. *Terrestrial telescope.* — When the telescope described above is applied to terrestrial objects, it exhibits them inverted. This is corrected by interposing between the eye and the image other lenses, by which a second image is formed, inverted with respect to the first, and therefore erect with respect to the object. This arrangement is represented in *fig.* 392., where A B is the object and *m n* the first inverted image. A convergent lens C D is placed before this image, at

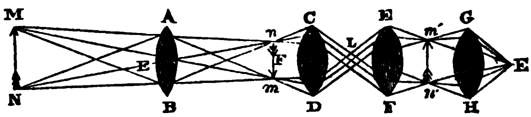

Fig. 392.

a distance equal to its focal length; consequently, the pencils proceeding from *m n*, after passing through C D, will emerge with their rays parallel. These pencils are received by another converging lens of equal focal length E F, by which they are again rendered convergent, and are made to form the image *m' n'*, which is inverted with respect to *m n*, and erect with respect to the object. This image *m' n'* is viewed by the eye-glass G H in the usual manner.

The eye-pieces of telescopes, like those of microscopes, do not necessarily consist of a single lens, but are frequently composed of two.

The object-glass, as well as the other lenses composing refracting telescopes, are usually constructed so as to be achromatic, upon the principle already explained in Chap. XIII.

CHAP. XVI.

THEORIES OF LIGHT.

1220. *Ordinary reflection and refraction explicable independently of theories.* — The optical phenomena attending ordinary reflection and refraction, which have formed the subjects of the preceding chapters, have been explained without reference to any hypothesis or theory. They have been deduced directly from experiments, the results of which are so simple and obvious, that the laws which prevail among them have been rendered evident without reference to theoretical considerations.

Other phenomena, however, will now have to be examined, in which the same simplicity does not prevail, and which do not admit of being explained or reduced to general laws without the occasional use of language derived from one or other of the theories respecting the nature of light which have been imagined by scientific inquirers.

1221. *Two theories of light.* — We shall therefore now explain briefly those theories or hypotheses which have been proposed respecting the nature of light, for the purpose of explaining the phenomena of optics.

It has been already stated that the scientific world for ages has been more or less divided by two theories or hypotheses concerning the nature of light, one of which is known as the corpuscular theory, or the theory of emission, and the other as the undulatory theory, or the theory of undulation.

1222. *Corpuscular theory.* — In the corpuscular theory, which was adopted by Newton as the basis of his optical inquiries, light is considered as a material substance, consisting of infinitely minute molecules which issue from luminous bodies and pass through space with prodigious velocities. Thus, in this hypothesis, the sun is regarded as a source from which such molecules or corpuscles proceed in every direction, with such a velocity that they pass from that luminary to the earth, over a distance of ninety-five millions of miles, in about eight minutes and thirteen seconds.

This immense velocity with which they are endued, amounting to nearly two hundred thousand miles per second, united with the fact established by observation, that they do not impress with the slightest

momentum the lightest objects which they strike, render it necessary to suppose that they are so minute as to be altogether destitute of inertia or gravity. The strongest beam of sunlight acting upon the most delicate substance, upon the fibres of silk or the web of the spider, or upon gold-leaf, does not impress upon them the slightest perceptible motion. Now, in order that a particle of matter endued with a velocity so great should have no perceptible momentum, it is necessary to suppose it to be almost infinitely minute.

But this minuteness requires to be admitted to a still greater ex tent, when it is considered that particle after particle striking upon bodies so light, even after the communication of their forces, impart to them no perceptible motion.

1223. *Difference of colour explained.*—In this system the difference of colour which prevails among the different homogeneous lights, the combination of which constitutes solar light, is ascribed to different velocities.

Thus the sensation of red is produced by luminous molecules animated by one velocity, orange by another, blue by another, and so on.

1224. *Laws of refraction and reflection explained.*—The law which renders the angle of reflection equal to the angle of incidence, is explained by supposing such molecules to have perfect elasticity. The law of refraction is explained by supposing that such molecules are subject to an attraction towards the perpendicular when they enter a denser, and by a repulsion from it when they enter a rarer medium.

1225. *Undulatory theory.*—In the undulatory theory which was adopted by Huygens, and after him by most continental philosophers, light is regarded as in all respects analogous to sound.

The luminous body in this system does not transmit any matter through space any more than a bell transmits matter when it sounds. The luminous body is regarded as a centre of vibration; but in order to explain the transmission of this vibration through space, the existence of a subtle fluid is assumed, which plays, with regard to light, nearly the same part as the atmosphere plays with regard to sound. The sun in this theory, then, is a centre of vibration, and the space which surrounds him being filled with an atmosphere of this subtle fluid, transmits this vibration exactly as the atmosphere transmits the vibration of a sounding body.

1226. *The luminous ether.*—This hypothetical fluid has received the name of *ether*. It is supposed not only to fill all the vacant spaces of the universe which are unoccupied by bodies, but also to fill the interstices which exist between the component parts of bodies. Thus it is not only mingled with the atmosphere which surrounds the earth, but also with the component parts of water, glass, and all transparent substances; and since opaque substances, when rendered suf-

ficiently thin, are penetrable more or less by light, it is necessary to admit that it also fills the pores of such bodies. If this luminous ether did not prevail throughout the whole extent of the atmosphere, the light of the stars could not reach our eyes. If it did not exist in water, glass, precious stones, and all transparent substances, these bodies could not be penetrable by light as they are; in fine, if it did not exist in the humours of the eye, light could not affect this organ, and the undulations could not reach the membrane of the retina.

1227. *Effects ascribed to its varying density.* — But although this luminous ether is thus assumed to be omnipresent, it does not everywhere prevail with the same density. It is probable that its density in the celestial spaces which intervene between planet and planet is the same which it has under the exhausted receiver of an air-pump or above the mercurial column in a barometer.

But its density in transparent media must be different, because to explain the phenomena of light passing through them it is necessary to suppose that the undulations change their magnitude, a supposition which is only compatible with a change in the elasticity of the ether. We shall see further, that in some transparent bodies existing in a crystallized state it is necessary to suppose also that the density of the ether in different directions in the same medium varies.

If this universal ether were for a moment in a state of perfect repose, the universe would be in absolute darkness; but the moment its equilibrium is disturbed, and that an undulation or vibration is imparted to it, that instant light is created, and is propagated indefinitely on all sides, as, in an atmosphere perfectly tranquil, the vibrations of a musical string or the sound of a blow is propagated to a distance in all directions according to determinate laws.

Light itself must not, however, be confounded with the ether which is the medium of its propagation. Light is no more identical with the hypothetical ether than sound is identical with air. The ether, in the one case, and the air in the other, are merely the media by which the systems of undulations which constitute the real sense of light and sound are propagated.

1228. *Analogy of light and sound.* — In considering the analogy between light and sound, however, there is an important distinction which must not escape notice. Sound is propagated, not only by undulations transmitted through the air, but also by undulations transmitted through other fluids as well as solids, as has been already explained. Light, however, according to the undulatory theory, is transmitted only by the undulations of the luminous ether. Light, therefore, does not pass through a transparent body, such as glass, in the same manner as sound is transmitted through the same body. The undulations by which sound is propagated through the air would be imparted to glass itself, which will continue them and transmit them to another portion of air, and thence to the ear; but when the undu-

lations of light are transmitted through glass or any other transparent medium, they must be supposed to be propagated, not by the vibration of the glass itself, but by the vibration of the subtle ether which pervades its pores.

1229. *The undulatory theory affords a more complete explanation of the phenomena.* — These two celebrated theories have, as has been already stated, divided the scientific world for ages; nevertheless, many of the more recent optical discoveries having failed to obtain a satisfactory explanation by means of the corpuscular hypothesis, the other theory has now obtained much more general, if not universal acceptation. We shall therefore, in the succeeding chapters, where it is necessary to use the language of theory, adopt that of the undulatory hypothesis.

All the general principles connected with the theory of undulations, as explained in Book VII., will be applicable to the undulations imparted to the luminous ether in this case.

Thus, the velocity with which such undulations are propagated is the velocity of light, the breadth of the waves determine the colours of the light, and the height of the waves its intensity. Thus the undulations with which red light is propagated are broader than those by which violet light is produced, and the same of the other colours.

CHAP. XVIII.

INTERFERENCE AND INFLECTION.

It has been shown in Book VII., that in all cases where systems of undulation are propagated along the surface of a fluid or through an elastic medium, phenomena are produced by the intersection of systems of waves, by which, at certain points the undulations obliterate each other.

Such effects are called *interference*, one system of waves being said to *interfere* with another when such reciprocal obliterations take place.

An instructive class of interesting optical phenomena are explained upon this principle.

1230. *Fresnel's experiments exhibiting the effects of the interference of light.* — In order to exhibit the phenomena of the interference of light in such a manner as to develop the laws which govern it, and to supply numerical estimates of the data and constants of the undulatory theory, it is necessary to contrive means by which two pencils of light, whether homogeneous or compound, of the same in-

tensity, shall intersect each other at a very oblique angle and at a considerable distance from their foci. Fresnel, to whose experimental researches in this department of physics science is largely indebted, accomplished this object by reflection and refraction in the following manner.

1. *By reflection.*—Let M C, M′ C, *fig.* 393., be two plane reflectors inclined to each other at a very obtuse angle. Let F be a focus of light produced by transmitting the light through a converging lens of short focus, or by reflecting it from a concave speculum. The rays diverging from F are received upon the two plane reflectors M C and M′ C. An image of F will be formed by the reflector M C at P just as far behind the plane of M C as F is before it; and, in like manner, another image of F will be produced by the reflector M′ C at P′ just as far behind the plane of M′ C as F is before it. It follows, therefore, that those rays which proceed from F and are incident upon M C will after reflection diverge as if they had originally proceeded from P, and those rays which are incident upon M′ C will after reflection diverge as if they had originally proceeded from P′. Therefore the pencils after reflection will be optically equivalent to two pencils radiating from P and P′. Thus we shall have a single pencil radiating from the point F converted into two pencils intersecting each other at a very oblique angle, and proceeding from the distant foci P and P′.

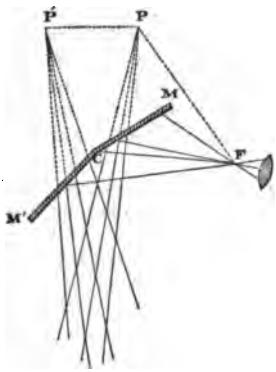

Fig. 393.

2. *By refraction.*—Let A B C, *fig.* 394., be a prism, with a very obtuse angle at B, and let F be a radiant point produced as before by a converging lens or concave reflector. The rays diverging from F, and incident on the surface A B, will be refracted as if they proceeded from F′; and, in like manner, the rays proceeding from F and incident upon B C will be refracted as if they proceeded from F″. Thus we shall have two pencils, as before, the rays of which will intersect each other obliquely at the points I, these pencils consisting of light of the same quality and intensity.

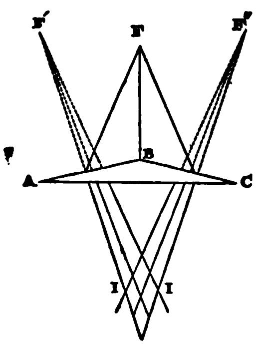

Fig. 394.

1231. *Phenomena of interference exhibited in the case of homogeneous light.*—If

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two pencils of homogeneous light thus obtained be made to diverge from two points F and F′, *fig.* 395., and if the rays of these pencils intersect at very oblique angles below the line A B, which is drawn parallel to the line F F′, which joins the foci of the two pencils, the following effects will ensue: —

If a line C *o* O be drawn from the middle point of F F′ perpendicular to it, any point on this line *o* O will be illuminated; in fact, an illuminated line will be formed from *o* to O, as indicated by the dotted line in the figure. On either side of this illuminated line *o* O will be found a dark curved line 1 1 and 1′ 1′, so that any object held in either of these lines would be deprived of light. Outside these two dark curved lines will be found two other curved lines, 2 2 and 2′ 2′, which will be lines of light, so that any object held at any point of either of them will be illuminated. Beyond this again will be found two other dark curved lines, 3 3 and 3′ 3′, so that any object held in them will be in shadow or darkness; beyond these again will be two curved lines of light, as before, 4 4 and 4′ 4′, so that any object held in either of these will be illuminated. Thus there succeed each other a series of curved lines of light and darkness, the light lines having the colour and qualities of the light of the two pencils. The series of the illuminated curves of light and darkness at each side of the central line *o* O are symmetrically arranged, those on the one side having corresponding forms, positions, and distances to those on the other side.

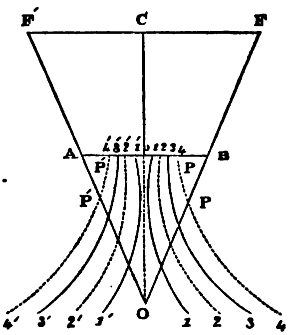

Fig. 395.

The curves formed by these light and dark lines are those known in geometry as the species of conic section called the hyperbola, the points F and F′ being their common foci.

Now, it is a well-known property of this curve that the difference between the distances of every point in it from the two foci is the same. Thus, if lines be drawn from F and F′ to any point in any one of these curves, their difference will be the same as that of lines drawn from F and F′ to any other point in the same curve.

Thus, for example, if P and P be two points upon the curve 4 4, then the differences between the distances of P and P from F and F′ will be equal; and, in like manner, if P′ and P′ be two points on the curve 4′ 4′, the differences between their distances from F and F′ will be equal.

It will presently bo seen that this property gives rise to important consequences.

If an opaque screen be interposed between the line A B and either of the foci, F′ for example, all these curves of bright and dark lines vanish, and there is a uniform illumination produced throughout the space below the line A B. This illumination, however, will be found to have only half the intensity of the bright curves which were previously formed.

Now, since by the interposition of the screen no light has been diffused below the line A B which was not there before, but, on the contrary, all the light proceeding from the focus F′, which was there before, is now excluded, it follows that the effect of the rays which, proceeding from the focus F′, intersect those proceeding from the focus F, is to deprive the spaces marked by the dark curves 1 1, 3 3, 1′ 1′, and 3′ 3′ of light, and to increase in a two-fold proportion the light in the spaces marked *o* O, 2 2, 4 4, 2′ 2′, and 4′ 4′.

Thus it appears, that at the intersections of the rays proceeding from F and F′, which take place upon the dark curves, the one light extinguishes the other; and that at the intersections which take place upon the bright curves, the lights add their mutual intensities, and an intensity is produced equal to their sum; for since they are equal to each other, this intensity is double the intensity of either.

Now it will be evident, by reference to what has been established in Book VII. relating to undulations, that this fact is merely a consequence of the interference of the waves of light. The foci F and F′ may be considered as the centres round which two systems of luminous undulations are propagated. These systems, encountering each other, intersect below the line A B. At those points where the waves meet under corresponding phases, that is to say, where the crest of one wave coincides with the crest of another, or the depression of one with the depression of another, they produce waves of double the height or double the depression of either. But at those points where they meet under contrary phases, that is, where the crest of one wave coincides with the depression of the other, or *vice versâ*, then the waves obliterate each other, and no undulation takes place at such point. In the former case, the light at the point of intersection has double the intensity which it would have if the light from one focus alone was received; in the other case, the lights extinguish each other, and there is darkness.

Now it will be easy to show, that the bright curves indicated by the dotted lines in the figure correspond to points where the systems of waves intersect under the first condition above mentioned, and that the dark curves correspond to those points where they intersect under the second condition.

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The middle line o o, which is a line of light, is at all its points equally distant from F and F′. Thus two lines F O and F′ O drawn from the focus to the same point in it are always equal; consequently the undulations which meet at any point such as O on this line, must necessarily meet under similar phases; for since the waves are of equal lengths, and since the distance F O is equal to the distance F′ O, the same number of waves and parts of a wave must occupy the two distances, and consequently the waves must arrive at O under corresponding phases.

The distance of any point of the first dark curve 1 1 from the focus F′ exceeds its distance from the focus F by half an undulation. If, therefore, the crest of a wave proceeding from F′ arrive at any point on this curve, the depression of a wave proceeding from F must arrive at the same point at the same time; and the same will be true of all points in the dark curve 1 1. The same observation will also be applicable to the curve 1′ 1′, only that in this case the distance of any point from F exceeds its distance from F′ by half an undulation.

Thus it appears that the waves propagated from the centres F and F′ always intersect on the dark curves 1 1 and 1′ 1′ under contrary phases, and consequently obliterate each other's effects and produce darkness.

The distance of any point in the bright curve 2 2 from F′ exceeds the distance of the same point from F by the length of a complete undulation; consequently, if the crest of a wave proceeding from F′ arrive at any point in such line, the crest of the preceding wave proceeding from F must arrive at it at the same time; and the same will be true for every point, so that throughout this bright line 2 2 the intersecting waves increase each other's effect. The same will be true of the line 2′ 2′. Hence the illumination produced along these two bright curves will be equal to the sum of the illuminations proceeding from the two foci.

In the same manner, it appears that the distance of any point on the dark curve 3 3 from F′ exceeds the distance of the same point from F by the length of an undulation and a half, and the same consequences as in the case of the first curve follow; so that the waves intersecting on the dark curves 3 3 and 3′ 3′, meet under opposite phases and obliterate each other.

It is evident, therefore, that the several hyperbolic curves formed by the successive light and dark lines on either side of the central bright line o O derive their character from the multiple of only half a wave's length, which expresses the difference between the distance of their successive points from the two centres of undulation F and F, which are the common foci of all the curves; and this multiple is in such case the length of the transverse axis of the hyperbola, of which the point O is the centre.

The spaces intervening between the bright and dark curves correspond to points where waves intersect under phases which are neither perfectly coincident nor perfectly opposite, and where consequently they only partially efface each other. Hence the light gradually diminishes in these spaces between the bright and the dark curves. The difference between the distances of these intermediate points from the foci F and F′ exceeds a complete number of half undulations by a quantity which is less than half an undulation.

1232. *How the phenomena of interference are affected by the different refrangibilities of different homogeneous lights.* — In what has been here stated, it has been assumed that the light proceeding from the points F and F′ is homogeneous light. Now there are, as has been shown, various species of homogeneous light, differing from each other in refrangibility and colour; and it is necessary to explain in what respects each variety of refrangibility and colour affects the phenomena of the bright and dark curves just explained. We find accordingly, that by causing pencils of homogeneous light of different colours and refrangibilities to intersect as above described, the bright and dark curves formed by their interference retain the character of the hyperbola, and that, although their general disposition on either side of the central line *o* O is the same, they are at different distances from each other; that is to say, the distance of the first bright curve 2 2 from the central line *o* O, as well as the distance of any two corresponding curves from each other, are different for different species of homogeneous light. In general, the more refrangible the light is, the nearer are the bright curves to each other. Thus the distance between one bright curve and another for violet or blue colour is less than the distance between the corresponding bright lines for red or orange colour.

1233. *The lengths of the undulations of the different homogeneous lights computed from the phenomena of interference.* — By an exact measurement of the dark and bright hyperbolic curves produced by each species of homogeneous light, aided by their known geometrical properties, Fresnel was enabled to deduce from these curves the lengths of the undulations of the ether which correspond to each species of homogeneous light. The following are the results of his observations and calculations:—

Colour of homogeneous Light.	Length of Wave in ten-millionth Parts of an Inch.	Number of Undulations to an Inch.
Extreme violet	160	62,500
Mean violet	167	59,880
Violet bordering on dark blue	173	57,803
Dark blue	177	56,497
Dark blue bordering on light blue	180	55,555
Light blue	187	53,422
Light blue bordering on green	194	51,546
Green	205	48,780
Green bordering on yellow	209	47,847
Yellow	217	46,083
Yellow bordering on orange	225	44,444
Orange	230	43,480
Orange bordering on red	235	42,559
Red	244	40,983
Extreme red	254	39,370

1234. *Effects of the interference of compound solar light.*—Since the distances between the bright and dark curves are different for each species of homogeneous light, it follows, that if the light which radiates from F and F′ be white solar light which is composed of all the colours of the spectrum, we shall have all the systems of bright and dark curves which would be separately produced by each of the component parts of the solar lights superposed, and a mixture of colours will consequently ensue which will produce rows of fringes, the colours of which will be determined by the prismatic tints which will be thus mingled together.

A complete analysis of the combination of colour which would produce these fringes in the case of solar light would be extremely complicated. Some idea, however, may be formed of the manner in which the combination of colours is produced from *fig.* 396., in which the relative breadths and distances of the light and dark curves produced by the three homogeneous lights, red, green, and violet, are represented. The series of red fringes with their alternate dark spaces are represented by R R, the series of green stripes are represented by G G, and that of violet stripes by V V. If these be considered, instead of being placed as in the figure one above the other, to be superposed, the effects which would be produced by a

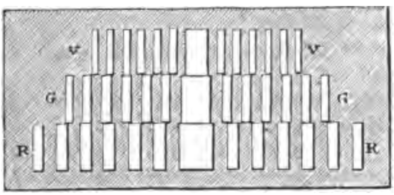

Fig. 396.

light proceeding from the two foci F and F′ composed of these three colours may be inferred.

1235. *Inflection or diffraction of light.*—If the rays of light diverging from a focus F, *fig.* 397., be incident upon an opaque object A B, all those rays of the pencil which are included within the angle A F B will be intercepted, so that a screen held at C D will receive none of those rays.

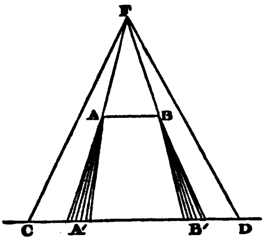

Fig. 397.

If the lines F A and F B be continued to A′ and B′, they will include upon the screen those spaces which would have been illuminated by the rays proceeding from F, which are intercepted by the opaque body A B. All the rays of the pencil included in the angles A F C and B F D will proceed uninterruptedly, and will fall upon the screen. If these rays underwent no change of direction, they would illuminate those portions of the screen included between C and A′ and D and B′. There would thus be an exact and well-defined shadow of the object A B formed upon the screen at A′ B′, and the remainder of the screen would be illuminated in the same manner as it would have been if the opaque body A B had not been present.

It is found, however, by experiment, that no such exact and well-defined shadow of the opaque object would be formed upon the screen.

The outline of the space which would limit an exact and geometrical shadow of A B being determined, it is found that within this space light will enter, and that outside this space the illumination is not the same as it would have been if the object A B had not been interposed.

From this it is inferred that the rays of light which pass the edge A B of the opaque object do not proceed in the same straight lines A A′ and B B′, in which they would have proceeded if the opaque object were not present. In a word, the appearance of the edge of the shadow is not a well-defined line separating the illuminated from the dark part of the screen, but a line of gradually decreasing brilliancy from the illuminated part of the screen to that in which the shadow becomes decided.

This effect produced by the edges of an opaque body upon the light passing in contact with them, by which the rays are bent out of their course either inwards or outwards, is called *inflection* or *diffraction*.

This phenomenon is a consequence of the general property of undulation explained in the Theory of Undulation (see Handbook of Sound, 810, 811, and 812). When the system of waves propagated round F as a centre encounters the obstacles A B, subsidiary systems of undulation will be formed round A and B respectively as centres,

and will be propagated from those points independently of and simultaneously with the original system of waves whose centre is F, and which will also proceed towards C A′ and D B′. In a certain space round the lines A A′ and B B′, along which the rays grazing the edge of the opaque body would have proceeded, the two systems of undulation will intersect each other and produce the phenomena of interference.

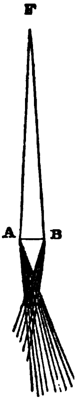

Fig. 398.

1236. *Combined effects of inflection and interference.*—If the opaque body A B be very small, and the distance of the focus F from it be considerable, the two pencils formed by inflection, of which A and B are the foci, will intersect each other as represented in *fig.* 398., and in this case all the phenomena of interference already described in 1231. will ensue. Thus, if the light be homogeneous, a bright line of light will be formed under the centre of the opaque object A B, outside which will be dark lines, and then bright and dark lines alternately. If the arrangment of these lines be examined, they will be found to be hyperbolic, as exhibited in *fig.* 395., and to vary in their relative distance with the quality of the light which radiates from the focus F. If the light radiating from such focus be compound solar light, then a series of coloured fringes will be formed, as already explained.

1237. *Examples of the effects of inflection and interference.*—The variety of optical phenomena produced by light passing the edges of small opaque objects, or small openings made in opaque plates, is infinite. The principles, however, on which all these appearances are explained, are the same.

The following experiments form examples of the variety of which these phenomena are susceptible.

I. If a small sphere formed of any opaque substance be suspended in a dark room, and a pencil of homogenous light be allowed to fall upon it, so that its shadow may be received upon a screen, it will be found that a bright spot will appear in the middle of the shadow, outside which will be a dark circle, beyond which there will be a bright circle, and beyond that a dark circle, and so on, the circles corresponding successively to the interference of the rays, by which their brilliancy is either doubled or extinguished, and the colour of the bright circles corresponding to that of the light.

If the light which falls on the sphere in this case be compound solar light, the central spot on the screen will be white, and will be surrounded by a series of coloured fringes, produced by the superposition of the coloured rings which would be produced separately by each compound of the solar light.

II. If a fine wire or sewing-needle be held close to one eye, the

other being closed, and be looked at so as to be projected upon the light of a window, or a white screen, several needles will be seen.

III. If the eye be directed in a dark room to a narrow slit in the window-shutter by which light is admitted, several slits will be seen separated by dark bands.

IV. If a piece of card having a narrow incision made in it, be held between the eye and a candle, a series of slits will be seen parallel to each other, exhibiting the colours of the spectrum. The same appearance may be produced with increased effect by looking through the slit at the sun-light admitted through an opening in the window-shutter.

1238. *Phenomena of interference of light reflected and refracted by thin transparent laminæ.*—It has been already shown that when light passes from any transparent medium to another of different density, a part of it is reflected from their common surface, and a part only transmitted. Thus, when light passing through air is incident upon the surface of glass, a certain part of it is reflected from such surface, but the greater part enters it. When that portion which penetrates the glass arrives at the second surface, which separates the glass from the air, on the other side a like effect ensues, a portion of the light is reflected from the second surface, the greater part, however, penetrating it, and passing into the air. There are, therefore, two systems of reflected rays, one reflected from the first surface of the glass, and the other by the second surface.

The first system of reflected rays is thrown back immediately into the air; the second system is thrown back into the glass, and must pass through the first surface of the glass before it returns into the air.

If the two surfaces which thus successively reflect a portion of the light which passes through the transparent medium be very close together, and if they be not precisely parallel, the reflected rays will intersect each other, and produce the phenomena of interference.

1239. *Iridescence of fish-scales, soap-bubbles, mother-of-pearl, feathers, &c. explained.*—Hence arise the curious and beautiful appearances of iridescence which are observable whenever transparent substances are exhibited in sufficiently thin plates or laminæ, the prismatic colours observable in the scales of fishes, in spirit of wine spread in thin films on dark surfaces, in oil thinly diffused over the surface of water, and the thin laminæ of crystals and soap-bubbles, and bubbles of glass blown to extreme tenuity, in the laminæ of mother-of-pearl, and in the wings of insects and feathers of birds.

1240. *Newton's experimental illustration of the physical laws of such phenomena.*—In these and similar cases, the forms and thinness of the various laminæ being irregular, the iridescence affords no indications of general laws. Newton, however, by a series of beautiful experiments, reduced these phenomena to a form in which he was

enabled to determine their laws, and by which they have since been shown in a rigorous manner to be consequences of the principle of interference.

Newton placed a flat plate of glass, D E, *fig.* 399., of uniform thinness, upon a convex lens A C B, of a very slight degree of convexity,

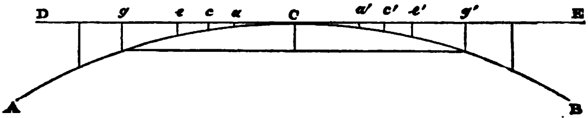

Fig. 399.

so that the surface of the two glasses should be separated by exceedingly minute spaces, even at considerable distances from the point of contact C. By this expedient a plano-concave lens of air of extreme thinness was formed by the two glasses.

Let us now suppose homogeneous light to fall upon the surface D E. The appearance will be that of a dark spot in the centre C, surrounded by a bright ring, outside which is a dark ring, followed by a bright ring, and so on, a series of bright and dark rings being formed round the central black spot.

If homogeneous light of different colours be successively thrown upon the glass, a system of rings, such as here described, will in each case be produced; but their diameters will be different, the rings being closer together for the more refrangible than for the less refrangible lights. If compound solar light be allowed to fall upon the glass, a series of rings will be formed of colours which would be produced by the superposition of all the systems of rings which are separately formed by the various homogeneous lights which form the compound solar light.

1241. *Newton's coloured rings explained by interference.*—These phenomena, which were explained by Newton upon an hypothesis called by him the *theory of fits*, of easy reflection and transmission, are easily explicable upon the principle of interference.

When homogeneous light falls upon the glass, a portion of it is reflected from the under surface of the plate D E, which separates the glass from the thin plano-concave lens of air. Another portion is reflected after passing through the air from the convex surface of the glass. These two systems of rays being reflected from surfaces not precisely parallel, intersect each other, and alternately destroy or increase each other's effect, according as the waves of light meet under the same or different phases. The dark rings comprehend the intersections under different phases, and the light rings the intersections under the same phases.

The thinness of the lens of air at the successive dark and bright rings respectively determine the difference between the lengths of the

intersecting rays measured from their origin to either point of intersection, and thus show where the point of intersection comprehends the waves meeting under the same or under different phases. The measurement of this thinness, accordingly, at the bright and dark rings, is found to be in entire accordance with the calculations already made, and explained in the table of the length of the waves of homogeneous light of different colours.

CHAP. XVIII.

DOUBLE REFRACTION.

1242. *Transparent media resolved into two classes.*—Transparent substances consist of two classes, which present optical phenomena depending on certain physical properties inherent in the constitution of each class of media respectively.

The phenomena, both optical and physical, suggest in the first class the supposition that they consist of molecules which are uniform in their form and reciprocal effects, so that the forces which they exercise one upon the other are the same in every direction. To this class belong every species of æriform fluid, all liquids, and certain transparent solids, such as glass, when properly annealed.

1243. *Single refracting media.*—In all these substances the constituent molecules appear to be so arranged, that we might conceive them to be spherules of matter, from the centres of which forces emanate which are equal in every direction.

1244. *Double refracting media.*—The second class of substances, which includes crystallized minerals, generally exhibits phenomena which lead to the supposition that their constituent molecules are not spherules, or, at least, that they do not exercise like forces in all directions round their centres. The phenomenon of crystallization, explained in the Handbook of Mechanics, sections 60–66., itself suggests this supposition; for when a substance passes from the liquid to the solid state, and undergoes the process of crystallization, the particles affect a particular arrangement with reference to one another, so as to present themselves towards each other in certain directions, as if they had sides which mutually attracted or repelled each other.

1245. *Effects of an uncrystallized medium on light.*—To render more clearly intelligible the effects produced by crystallized substances on light transmitted through them, we shall first briefly recapitulate the effects produced on rays of light by an ordinary transparent uncrystallized medium, such as air, water, or glass.

Let us suppose such a substance reduced to the form of a sphere, which, if it be gas or liquid, may be done by enclosing it in a thin

globe of glass; and if it be a solid, it may be reduced to the spherical form in the lathe.

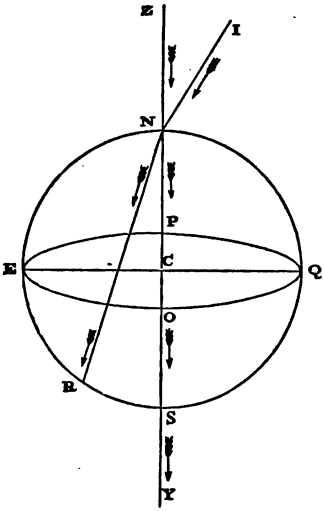

Fig. 400.

Let E N Q S, *fig.* 400., represent a section of this transparent sphere, and E P Q O another section at right angles to it.

Let Z N and I N represent two rays of homogeneous light incident at N, one in a direction which, being continued, would pass through the centre C of the sphere, and the other, I N, in a direction oblique to the former.

If the sphere be composed of non-crystallized transparent matter, the ray Z N will pass through it, pursuing the original direction, and consequently, after passing the centre C, will emerge from the lowest point S in the direction S Y, so that its course shall be in no wise changed by the transparent medium through which it has passed; but the ray I N, which falls obliquely at the point N, will, according to the law of refraction already explained, be deflected from its course towards the diameter N C S, and will follow a direction such as N R, which makes an angle with N S less than that which I N makes, with N Z.

The laws which govern in this case the refracted ray are as follows:—

1. If the incident ray be perpendicular to the surface at the point of incidence, its direction will not be changed in passing through the transparent medium.

2. If the incident ray form an angle, such as I N Z, with the perpendicular N Z at the point of incidence, then the refracted ray N R will form an angle with the same perpendicular N Z, or with its production N S, the plane of which will coincide with the plane of the angle of incidence Z N I.

3. If the angle of incidence I N Z be varied, the angle of refraction R N S will be also varied, but in such a manner that the ratio of the sine of the angle of incidence I N Z to that of the angle of refraction R N S shall always be the same, so long as the transparent medium into which the ray passes is the same.

4. If while the incident rays Z N and I N preserve their position, the sphere be turned round its centre C, so as to bring successively every part of its surface to coincide with the point of incidence N, the

refracted ray N R will still maintain the same direction and position, and the ray Z N will still pass through the centre of the sphere C, no matter what position may be given to the sphere, so long as the position of its centre C remain unchanged.

Thus the direction and position of the incident rays I N and Z N, and of the refracted rays N R and N S, will remain fixed, although the transparent sphere which they penetrate may be changed in an infinite variety of ways, so as to bring all its points in succession to coincide with the point of incidence N of the rays.

Such are the phenomena which are produced when the rays I N and Z N are incident upon a sphere composed of uncrystallized transparent substance. The same phenomena will always prevail in the case even of certain crystallized substances; but in the case of other crystallized media, different and far more complicated phenomena are developed, which we shall now proceed to explain.

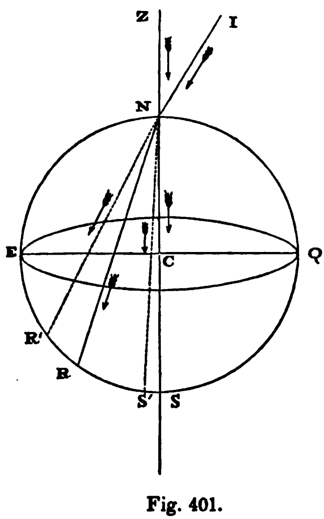

Fig. 401.

1246. *Effects of certain media on light.*—Let a sphere be formed of one of the class of crystals of which Iceland spar or the crystallized carbonate of lime is a specimen, and let this sphere be submitted to the same experiments as have been described in the former case. When the rays I N and Z N, *fig.* 401., penetrate the sphere at N, they will each of them be resolved into two rays, one of which, in the figure, is indicated by the uniform line, and the other by the dotted line. The rays indicated by the uniform lines N S and N R, will conform to the laws of refraction which prevail in uncrystallized media; that is to say, the ray N S will pass through the centre of the sphere C, preserving the direction of the incident ray Z N, which strikes the surface of the sphere at N in a perpendicular direction, and the ray N R will be in the plane of the angle of incidence I N Z. Also, if the ray I N be made to fall at N, so as to form any other angle of incidence, the ray N R will vary its inclination to the perpendicular N S, in conformity with the law of refraction, which establishes a constant ratio between the sines of the angles of incidence and refraction.

But none of these characters are found to attend the other rays N S′ and N R′, into which the original incident rays are resolved by the crystal.

The ray N S′, although proceeding from the ray Z N, which is inci-

French Theatre.

dent perpendicularly at the point N, does not penetrate the medium in the same direction, but makes a certain angle S′ N S with the perpendicular. Thus, in the case of this ray there is an acute angle of refraction corresponding to perpendicular incidence. In the case of the ray N R′ it is found that it deviates on the one side or the other of the plane of the angle of incidence I N Z, and thus this ray violates that general law of common refraction which declares that the plane of the angle of refraction coincides with the plane of the angle of incidence.

If the angle formed by the incident ray I N with the perpendicular Z N be varied, the angle which the refracted ray N R′ makes with the perpendicular N S will be also varied, but not according to the law of sines which prevails in the case of ordinary refraction.

1247. *The ordinary and extraordinary rays.*—Thus it appears that in such crystallized media the incident ray is resolved into two rays, one of which conforms to the laws of common refraction, and the other violates them, and is regulated by other and different conditions. The two rays into which the incident ray is thus resolved are called the *ordinary* and *extraordinary* rays; that which conforms to the laws of common refraction being called the *ordinary*, and that which violates them the *extraordinary* ray.

If the sphere be now supposed to be moved, as before, round its centre C, so as to bring successively all the points of its surface to coincide with the point of incidence N, it will be found that the ordinary rays N S and N R will preserve their direction and position fixed in all positions which the sphere shall assume; but that the direction and position of the extraordinary rays N S′ and N R′ will vary with every change of position of the sphere. They will sometimes approach to, and sometimes recede from the ordinary rays; and they will sometimes deviate on one side, and sometimes on the other, of the plane of the angle of incidence; but in all cases there will be a maximum deviation from the ordinary ray, which will not be exceeded.

1248. *The axis of double refraction.*—By varying the position of the sphere so as to bring the various points of its surface to coincide with the point of incidence N, a point will be found upon it at which the extraordinary ray N S′ will coincide with the ordinary ray N S. As this point approaches the point N, the angle S′ N S under the ordinary and extraordinary ray will be observed continually to diminish; an effect which will indicate the change of position necessary to bring the desired point to coincide with the point of incidence N.

This point of the sphere then possesses a distinctive character, in virtue of which the incident ray Z N is not, as at all other points, resolved into two rays, but passes through the sphere in the direction N C S, exactly as it would pass through a sphere composed of an uncrystallized substance.

The diameter of the sphere which possesses this property is called its *optical axis*, or the *axis of double refraction*, being the only line in the sphere along which a ray of ordinary light can pass without being decomposed into two.

1249. *Laws of double refraction.* — Having thus determined this optical axis of the sphere, let us next examine the conditions which affect a ray of light, such as I N, which falls obliquely at the extremity of such optical axis.

Let N O S, *fig.* 402., be the optical axis of the sphere. The ray Z N will then, as has just been explained, pass through the centre O to the point S, without double refraction, as it would through an ordinary medium. The ray I N, which falls obliquely at N, will, however, be doubly refracted, and will be resolved into the ordinary ray N R, and the extraordinary ray N R′. But this extraordinary ray N R′ will, in this case, conform to one of the laws of ordinary refraction, for it will invariably lie in the plane of the angle of incidence I N Z; and so long as the angle of incidence shall not be varied, the direction of this extraordinary ray will remain the same. This may be proved by causing the sphere to revolve round the axis N S. While it so revolves, the extraordinary ray N R′ will remain fixed in its direction, being always in the plane of the angle of incidence, and forming always the same angle of refraction with the axis N S.

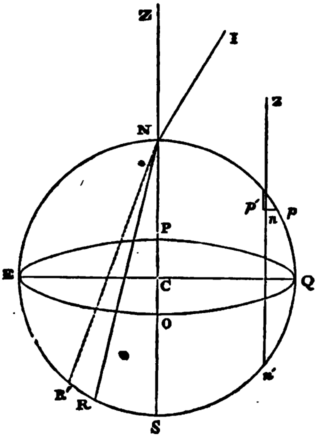

Fig. 402.

If the incident ray I N be varied in its inclination, so as to form, as before, a greater angle with Z N, the extraordinary ray N R′ will also vary its inclination to the axis N S and to the ordinary ray N R. But, although it will remain during such variation always in the plane of the angle of incidence, it will not conform to the invariable ratio of sines which constitutes the law of ordinary refraction.

If we suppose the incident ray I N gradually to approach Z N, so that the angle of incidence continually diminishes, then the two rays N R and N R′ will at the same time approach the axis N S and each other; and when the incident ray coincides with Z N, the ordinary and extraordinary rays N R and N R′ will coalesce with the axis N S.

As, on the other hand, the inclination of the ray I N to Z N is gradually increased, the ordinary and extraordinary rays N R and N R′ will also gradually recede from the axis N S, so that their angles of

refraction will continually increase, and they will also recede from each other.

1250. *Positive and negative crystals.*—In the case represented in the figure, the angle of refraction of the extraordinary ray N R′ is greater than that of the ordinary ray N R, so that the latter is more deflected by the refraction of the crystal than the former. This, however, is not always the case.

In some crystals the angle of refraction of the extraordinary rays is less than that of the ordinary ray, and, consequently, the former is more deflected towards the perpendicular than the latter.

Crystals are accordingly resolved into two classes, based upon this distinction; those in which the extraordinary ray is less deflected than the ordinary ray being called *negative crystals*, and those in which it is more deflected *positive crystals*.

It is evident that in the former case the index of ordinary refraction is greater, and in the latter less than the index of extraordinary refraction.

It must be observed, that while the incident ray varies its obliquity to Z N, increasing gradually from 0 to 90°, and while the index of ordinary refraction throughout this variation remains constant, the index of extraordinary refraction varies with every change of obliquity. In the case of positive crystals this index increases, in the case of negative crystals it diminishes, with the angle of incidence; while, in all, it is equal to the index of ordinary refraction when the ray of I N coincides with Z N. It increases and becomes a maximum when I N is at right angles to Z N in positive crystals, it diminishes and becomes a minimum when I N is at right angles to Z N in negative crystals.

1251. *All lines parallel to the axis of double refraction are themselves axes of double refraction.*—It is easy to show that all lines passing through the crystal which are parallel to the line N S possess also the property which characterizes such axis; that is to say, a ray which is incident perpendicularly in the direction of such lines will penetrate the crystal without double refraction. This we may prove by cutting a portion of the crystal in a direction perpendicular to the line N S.

Thus, at the point *p*, let a surface *p p′* be formed, which shall be perpendicular to N S. Then a ray *z n*, falling perpendicularly on such surface *p p′* will penetrate the crystal in the direction *n n′* without double refraction.

1252. *Axis of double refraction coincides with crystallographic axis.*—Thus it appears that the lines passing through the crystal parallel to N S are axes of double refraction as well as the line N S. On comparing the direction of the line N S with the direction of the planes of cleavage of the crystal, it is found that this line has a direction which is symmetrical with respect to all these planes, and that it is

in fact the direction of the crystallographic axis; that is to say, a line the direction of which bears the same relation to all the faces of the crystal.

1253. *Case of Iceland spar.*—Thus in the case of Iceland spar, the primitive form of whose molecules is that of such a rhomboid as is represented in *fig.* 403., the crystallographic axis is the diagonal A X joining the obtuse angles of the rhomb. The rhomb itself is a solid bounded by six equal and similar parallelograms, whose obtuse angles B A C and C D B are each 101° 55′, and whose acute angles A B C and A C D are accordingly each 78° 5′.

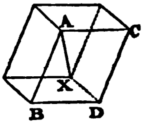

Fig. 403.

The inclination of the faces of the rhomb, which meet at A, to each other is 105° 5′, consequently the inclination of those which meet at B is 74° 55′. The crystallographic axis A X is equally inclined, not only to the three faces of the rhomb, which meet at A and X respectively, but also to its three edges. The angles which this axis makes with the three edges of the rhomb forming the angle A are equal to each other, their common magnitude being 66° 44′ 46″.

It is evident from this measurement, that the line A X is symmetrically placed with respect to all the elements which determine the primitive form of the crystal, and we thus find accordingly a distinct relation established between the optical and mineralogical characters of this substance, so that whenever the direction of its crystallographic axis is required to be ascertained, it can be done without any mechanical experiment or measurement, by merely determining that direction in which a ray of light incident perpendicularly on a surface of the crystal will pass through it without double refraction.

What has been here stated with regard to Iceland spar will, *mutatis mutandis,* be applicable to a numerous class of crystallized substances, which are distinguished by the denomination of crystals having a single axis of double refraction.

In all such crystals the crystallographic axis coincides with the optical axis.

1254. *General description of the phenomena of double refraction in uni-axial crystals.*—In attempting to explain the complicated phenomena of double refraction and other effects related to them, much convenience and clearness will be obtained by the adoption of a nomenclature indicating the position of the axis of double refraction in certain sections of the crystal analogous to the well-known circles used in geography and astronomy for expressing the relative position of points on the earth and in the heavens. We shall therefore call the extremities of the axis N and S the *poles* of the crystal, and a section of the crystal E P Q O, *fig.* 402., intersecting this axis at right angles the *equator*. We shall also call all sections of the crystal made by planes passing through the axis *meridians.*

These terms being understood, it will follow that whenever the plane of the angle of incidence coincides with the plane of a meridian, the angles of refraction, both of the extraordinary and ordinary rays, will be in the plane of the same meridian; but the ratio of the sine of the angle of incidence to the sine of the angle of extraordinary refraction will not in this case be constant.

If the plane of the angle of incidence intersect the crystal at right angles to the optical axis N S, and be consequently parallel to the line coincident with the plane of the equator, the angle of extraordinary refraction will have its plane coincident with that of the angle of incidence, thus fulfilling one of the laws of ordinary refraction, as is the case when the plane of the angle of incidence coincides with the plane of a meridian. But in this case the second law of refraction, which establishes a constant ratio between the sines of the angles of incidence and refraction, is also fulfilled by the extraordinary ray, so that when the angle of incidence coincides with, or is parallel to, the plane of the equator, the extraordinary refraction fulfils all the conditions of ordinary refraction, although the extraordinary ray does not coincide with the ordinary ray; the constant index of refraction of the one being greater or less than the constant index of refraction of the other, according as the crystal is positive or negative.

There are therefore two systems of planes which intersect crystals, one system having the axis of the crystal as their common line of intersection, and the other having directions parallel to each other and perpendicular to this axis. In the former, one of the laws of ordinary refraction is fulfilled, and in the latter both of them. In the former, the plane of the angle of extraordinary refraction coincides with the plane of the angle of incidence, but the ratio of the sines is not constant; in the latter, the planes also coincide, and the ratio of the sines is constant, but not the same as that of the ordinary ray.

1255. *Table of uni-axial crystals.* — The following is a table, according to Sir David Brewster, of the crystals which have a single axis of double refraction, arranged under their respective primitive forms; the sign + being prefixed to those which have a positive axis of double refraction, and — to those which have a negative axis of double refraction.

Rhomb with obtuse summit.

— Carbonate of lime (Iceland spar).
— Carbonate of lime and iron.
— Carbonate of lime and magnesia.
— Phosphato-arseniate of lead.
— Carbonate of zinc.
— Nitrate of soda.
— Phosphate of lead.
— Ruby silver.
— Levyne.
— Tourmaline.
— Rubellite.
— Alum stone.
+ Dioptase.
+ Quartz.

Rhomb with acute summit.

— Corundum.
— Sapphire.
— Ruby.
— Cinnabar.
— Arseniate of copper.

Regular hexahedral prism.

— Emerald.
— Beryl.
— Phosphate of lime (apatite).
— Nepheline.
— Arseniate of lead.
+ Hydrate of magnesia.

Octohedron with a square base.

+ Zircon.
+ Oxide of tin.
+ Tungstate of lime.
— Mellite.
— Molybdate of lead.
— Octohedrite.
— Prussiate of potassa.
— Cyanuret of mercury.

Right prism with a square base.

— Idocrase.
— Wernerite.
— Paranthine.
— Meionite.
— Somervillite.
— Edingtonite.
— Arseniate of potassa.
— Subphosphate of potassa.
— Phosphate of ammonia and magnesia.
— Sulphate of nickel and copper.
— Hydrate of strontia.
+ Apophyllite of uton.
+ Oxahverite.
+ Superacetate of copper and lime.
+ Titanite.
+ Ice (certain crystals).

M. Pouillet, "Élémens de Physique," tome ii., Paris, 1847, gives also the following: —

— Hydrochlorate of lime.
— Hydrochlorate of strontia.
+ Mica de kariat.
+ Oxide of iron.
+ Tungstate of zinc.
+ Stannite.
+ Boracite.
+ Sulphate of potassa and iron.
+ Hydrosulphate of lime.
+ Red silver.

1256. *Crystals having two axes of double refraction.* — There is another class of crystals which present optical phenomena still more complicated. Let us suppose, as before, one of these formed into a sphere, and let its various points, as before, be brought to coincide with the point of incidence N of two rays, one of which, Z N, *fig.* 402., is directed to the centre of the sphere, and the other I N forming any angle with the latter. By bringing the various points of the spherical surface to coincide with the point N, it will be found that two points, and two only, upon it possess the property of transmitting the ray Z N, which falls perpendicularly upon the surface, through the object without double refraction. The diameters passing through these two points have each of them the character of an axis of double refraction; and the crystals characterized by this property are accordingly called crystals with two axes of double refraction.

In this class of crystals it is found that neither of the rays into which the incident ray is resolved conforms to the laws of ordinary refraction; that both deviate from the plane of the angle of incidence, and that neither of them fulfils the second law, which determines the constant ratio between the sines of incidence and refraction.

Both rays, therefore, are extraordinary rays.

There are, however, two planes in which the angle of incidence may be placed, in one of which one of the two rays and in the other

the other will conform to both the laws of ordinary refraction, so that in these planes one or other of the two extraordinary rays becomes an ordinary ray. The position of these planes is determined by the following conditions.

Let N S and N′ S′, *fig.* 404., be the two axes of double refraction. Let P P′ be a line which divides into equal parts the angle N C N′ formed by these two angles, and let Q Q′ be a line which divides into equal parts the other angle N′ C S formed by the same axis.

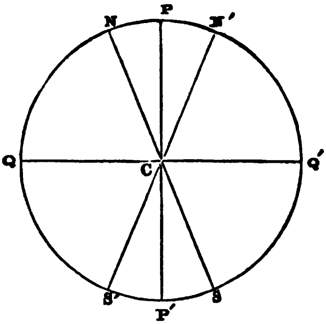

Fig. 404.

If a plane pass through C perpendicular to P C, any ray incident upon the crystal in that plane will be resolved into two rays, one of which will conform to the laws of ordinary refraction; and if a plane be drawn perpendicular to the line Q Q′, any ray incident upon the crystal in that plane will be resolved into two, one of which will also conform to the laws of ordinary refraction, and the ray which thus becomes an ordinary ray in the one plane will be different from that which becomes an ordinary ray in the other plane.

1257. *Table of bi-axial crystals.* — The following list of crystals having two axes of double refraction, with the magnitude of the angle included between such axes, is given by M. Pouillet in the work already cited.

TABLE OF CRYSTALS WITH TWO AXES.

Names of Substances.	Angles of Axis.	
	°	′
Sulphate of nickel (certain samples)	3	0
Sulpho-carbonate of lead	"	"
Carbonate of strontia	6	56
Carbonate of baryta	"	"
Nitrate of potassa	5	20
Mica (certain samples)	6	0
Talc	7	24
Pearl	11	28
Hydrate of baryta	13	18
Mica (certain samples)	14	0
Arragonite	18	18
Prussiate of potassa	19	24
Mica (certain samples)	25	0
Cymophane	27	51
Anhydrite	28	7
Borax	28	42

Names of Substances.	Angles of Axes.	
	°	′
Mica (several samples examined by M. Biot)	30	0
	31	0
	32	0
	34	0
	37	0
Apcphyllite	35	8
Sulphate of magnesia	37	24
Sulphate of baryta	37	40
Spermaceti (about)	37	42
Borax (native)	38	48
Nitrate of zinc	40	0
Stilbite	41	42
Sulphate of nickel	42	4
Carbonate of ammonia	43	24
Sulphate of zinc	44	28
Anhydrite (examined by M. Biot)	44	21
Mica	45	0
Lepidolite	45	0
Benzoate of ammonia	45	8
Sulphate of soda and magnesia	46	49
Sulphate of ammonia	49	42
Brazilian topaz	49 to 50	0
Sugar	50	0
Sulphate of strontia	50	0
Sulpho-hydrochlorate of magnesia and iron	51	16
Sulphate of magnesia and ammonia	51	22
Phosphate of soda	55	20
Comptonite	56	6
Sulphate of lime	60	0
Oxynitrate of silver	62	16
Iolite	62	50
Feldspar	63	0
Aberdeen topaz	65	0
Sulphate of potassa	67	0
Carbonate of soda	70	1
Acetate of lead	70	25
Citric acid	70	29
Tartrate of potassa and soda	80	0
Carbonate of potassa	80	30
Cyanite	81	48
Chlorate of potassa	82	0
Epidote	84	19
Hydrochlorate of copper	84	30
Peridot	87	56
Succinic acid	90	0
Sulphate of iron	90	0

1258. *Images formed by double refracting crystals.* — If a visible object be placed behind a double refracting crystal, the pencil of rays proceeding from each point in it will be resolved into two pencils, and will emerge from the crystal as if they had proceeded from two different objects in directions corresponding to the respective directions of the two pencils.

An eye, therefore, placed before the crystal, so as to receive these emerging pencils will see two different images of the object, corresponding to the two systems of pencils. If the crystal be one having a single axis of double refraction, then one of these images will be that produced by the pencils consisting of ordinary rays, and the other will be that produced by pencils consisting of extraordinary rays.

1259. *Ordinary and extraordinary image.*—The one is called th ordinary, the other the extraordinary image.

Thus, if P, *fig.* 405., be such an object, and A B C D be a doubly refracting crystal, such as Iceland spar, the pencils which proceed from P and are incident upon the surface B C will be divided into two systems of pencils, the axis of the ordinary system passing perpendicularly through the crystal in the direction I O, and emerging on the other side in the same direction, so as to meet the eye at Y. The extraordinary pencils will follow the direction I E through the crystal, and will emerge parallel to the ordinary pencil in the direction E Y′, so as to reach the eye at Y′. An eye placed therefore at any point, in looking towards the crystal, will perceive two images of the point P in juxtaposition in the direction of the rays Y′ E and Y O.

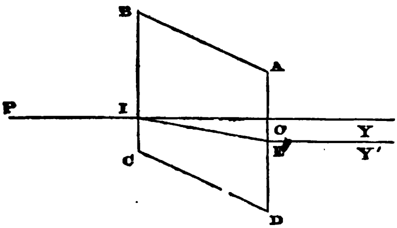

Fig. 405.

1260. *The separation of the images dependent on the thickness of the crystal.*—It is evident that the thicker the crystal is, the more widely separated will be these two images.

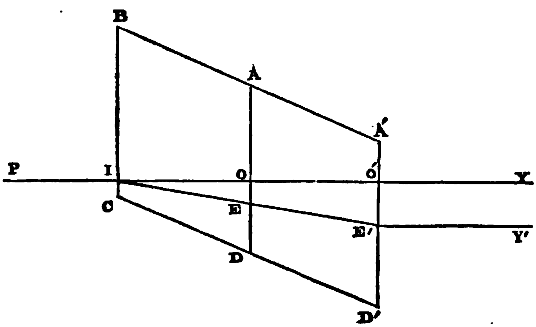

Fig. 406.

A crystal of Iceland spar three inches thick will be sufficient to produce a distinct separation of the two images of a spherical object having a diameter of one-third of an inch.

If while the object and the eye remain fixed, the crystal be turned round the line P Y, joining the eye and the object as an axis, the extraordinary image will appear to revolve round the ordinary image, showing that in this case the extraordinary pencil I E revolves round the ordinary pencil I O, so as to move in the surface of a cone. This effect is in conformity with what has been already explained.

If, after passing through a crystal A B C D, *fig.* 406., the rays be received by another crystal A A′ D′ D, whose sides and axes have a position similar to those of the first, the two crystals being in contact at the surface A D, the ordinary and extraordinary rays will pass through the second crystal, following the same direction as those which they followed in the first crystal, the lines O O′ and E E′ being the continuation of the lines I O and I E.

1261. *Case in which two similar crystals neutralize each other.*— If the two crystals in this case have the same thickness, then the effect will be that the rays E′ Y′ and O′ Y emerging from the second will be separated by a space twice as great as that by which they were separated in passing through the first crystal.

If the second crystal, instead of having been placed upon the first crystal so that its corresponding sides shall have the same direction, be placed upon it so that they shall have contrary directions, as represented in *fig.* 407., then the second crystal will have the effect of causing the reunion of the two pencils separated by the first crystal,

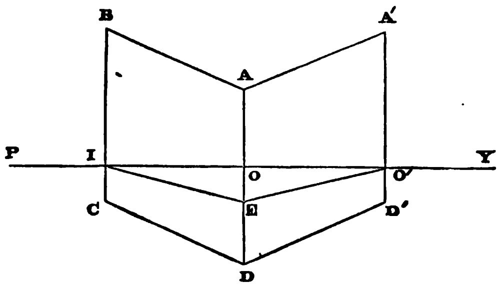

Fig. 407.

and the ordinary and extraordinary rays will accordingly emerge from the same point O′ of the second crystal in the same direction, so that an eye placed at Y will see but one image of the object P. In this case the ordinary ray follows the direction P I O O′ Y, and the extraordinary ray follows the direction P I E O′ Y. Thus, the separation of

the rays takes place only in passing through the crystals, the reunion being established at the point of emergence O′ from the second crystal.

1262. *Cases in which four images are formed by the combination of two similar crystals.*—If we suppose the second crystal, A A′ D′ D, *fig.* 406., to be turned round the line P I O Y as an axis, the moment it moves from the position represented in *fig.* 406., the ordinary and extraordinary rays I O and I E incident upon it from the first crystal will be each doubly refracted, so as to be resolved into four rays, and thus an eye placed at Y would see four images of the point P. As the second crystal is gradually turned round, these four images assume a series of different positions with relation to each other, and also have different degrees of brilliancy. After the crystal has made one half a revolution and assumed the position represented in *fig.* 407., all these four images unite in one. In the position intermediate between these two, that is to say, when the second crystal has made a quarter of a revolution round the line P I O Y, then the four images will be reduced to two, which, however, will have a different position relative to the line A D from that which the images produced in the position represented in *fig.* 406. have.

1263. *Their successive positions.*—The successive positions assumed by the four images during the half revolution of the second crystal between the position represented in *fig.* 406., and that represented in *fig.* 407., are given in *fig.* 408., where B represents the

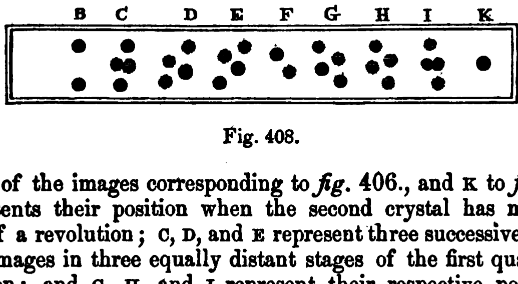

Fig. 408.

position of the images corresponding to *fig.* 406., and K to *fig.* 407.; F represents their position when the second crystal has made one-fourth of a revolution; C, D, and E represent three successive positions of the images in three equally distant stages of the first quarter of a revolution; and G, H, and I represent their respective positions in three equally distant stages of the second quarter of a revolution. The relative brilliancies of the images are indicated by the shading of the dots, the dark dots being understood to represent greater brilliancy than the shaded ones.

1264. *Position of axes different for different coloured lights in bi-axial crystals.*—In uni-axial crystals the axis has the same position, whatever be the colour of the light, but in bi-axial crystals the position of the axes is different for different coloured lights. Sir John Herschel found that in tartrate of potassa and soda (Rochelle salts), their inclination for violet light was 56°, and for red light 76°. In

other crystals, such as nitre, their inclination for violet was greater than for red, but in all cases the axes for all coloured light in the same crystal are in the same plane. Sir David Brewster found that glauberite had two axes, for red light, inclined at an angle of 50°, and only one for violet light. The same eminent philosopher found that in the case of analcine there were several planes along which there was no double refraction, however various the angle of incidence might be, so that that substance might be considered as having an infinite number of axes of double refraction.

1265. *Doubly refracting structure produced by artificial processes.* — The property of double refraction may in some cases be imparted by artificial processes to substances which do not naturally possess it. If a cylinder of glass be brought to a red heat, and held upon a plate of metal until it becomes cold, it will acquire the doubly refracting property, the axis of the cylinder being a single positive axis of double refraction. This axis differs, however, from the positive axis of crystallization, because in this case it is a single line, while in the crystal the lines parallel to it are equally axes of double refraction. Sir David Brewster says, that if instead of heating the cylinder it had been immersed in a vessel of boiling water, it would have acquired the same doubly refracting virtue when the heat had reached its axis, but that the property would not be permanent, disappearing when the cylinder should become uniformly heated. Also if the cylinder were uniformly heated in boiling oil, or at a fire so as not to soften the glass, and had been placed in a cold fluid, it would acquire a temporary doubly refracting virtue when the cooling had reached the axis; but in this case the axis would be a negative one, instead of a positive, as in the former case.

According to him some other analogous structures may be produced by pressure, and by the induration of soft solids, such as animal jellies, isinglass, &c.

If the cylinder in the preceding explanations is not a regular one, but have its section perpendicular to the axis, every where an ellipse in place of a circle, it will have two axes of double refraction.

In like manner, if we use rectangular plates of glass instead of cylinders, as in the preceding experiment, we shall have plates with two planes of double refraction, a positive structure being on one side of each plane, and a negative one on the other.

If we use perfect spheres there will be axes of double refraction along every diameter, and consequently an infinite number of them.

The crystalline lenses of almost all animals, whether they are lenses, spheres, or spheroids, have one or more axes of double refraction.

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CHAP. XIX.

POLARIZATION OF LIGHT.

1266. *Characteristic property of polarized light.* — When a ray of light, whether natural or artificial, has been submitted, under peculiar conditions, to reflection or refraction, it is then in a state in which it acquires new properties, and is denominated polarized light; and the process by which this modification in the ray is effected is called polarization.

To render the properties by which polarized is distinguished from unpolarized or common light clearly intelligible, let us imagine a ray of light admitted into a dark room through a hole in the window-shutter, so as to pass in a horizontal direction. Supposing such a ray to be cylindrical, let its section, made by a vertical plane, be represented by the circle A C B D, *fig.* 409.

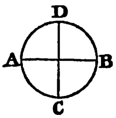

Fig. 409.

This ray, if it were common or unpolarized light, would be reflected or refracted in exactly the same manner, and according to the common laws of reflection and refraction already explained, on whatever side of it, and at whatever angle with it, the reflecting or refracting surface might be presented.

If, however, the ray be polarized, the effects will be different.

Let a plate of glass be blackened on one side so that when used as a reflector no light will be reflected from its posterior surface. Such a plate will therefore reflect light only from one surface, which will be its anterior surface. This precaution is necessary in the cases now to be examined, in order to prevent the effects which would ensue from the combination of the rays, which would otherwise be reflected from both the anterior and posterior surfaces of the glass.

Let such a plate, so prepared, be presented to the polarized ray at an angle of incidence of 54° 35′, so that the plate shall make with the ray an angle of 35° 25′; and let it be turned round the ray, so as to be presented on every side of it, still forming, however, the same angle with it. During this process, it will be observed that there is a certain direction of the plane of the angle of incidence at which no reflection will take place; the ray will be absorbed or extinguished, so to speak, by the reflecting surface. The plane of incidence will have this direction in two opposite positions of the reflector.

Let the line D C, *fig.* 409., represent this position of the plane of incidence: then D and C will be the two opposite sides of the ray, at which the reflector being presented will cause the extinction of the light. Now as the reflector is carried round from either of these positions respectively, so that the plane of the angle of incidence shall turn round the axis of the ray, reflection will begin to take

place, and will increase in intensity until the plane of the angle of incidence take a position, such as A B, at right angles to D C, when the intensity of the reflection will be a maximum. After passing this position, the intensity of the reflection will again diminish, and will continue to decrease until the plane of the angle of incidence shall again coincide with the diameter D C.

It is evident, therefore, that different sides of such a ray have different properties. Thus, the sides A and B have a susceptibility of being reflected, of which the sides D and C are deprived; and the susceptibility of reflection diminishes gradually in going round the ray from either A or B towards C or D, when it altogether ceases.

A plane passing through the axis of the ray and coinciding with the diameter A B is called the plane of polarization.

It is evident, therefore, from what has been explained, that when the reflector is so presented to the ray that the plane of the angle of incidence shall coincide with the plane of polarization, reflection will take place with the greatest intensity, and that when the plane of the angle of incidence is at right angles to the plane of polarization, no reflection takes place, and the ray is extinguished.

1267. *Angle of polarization.* — If, instead of glass, any other reflecting surface be used, like effects would be produced; only that the angle at which it would be necessary to present the reflecting surface to the ray would be different, each species of reflector having its own particular angle.

This angle is, for reasons which will be hereafter explained, called the angle of polarization.

1268. *Polariscopes.* — Instruments called *Polariscopes* adapted for the experimental illustration of the phenomena of polarization, have been constructed in various forms.

One of the most convenient for the purposes of elementary explanation consists of several detached pieces, which are represented in *fig.* 410. A B is a brass tube like that of a telescope, along the axis of which the polarized pencil to be submitted to examination is transmitted. C is a short tube capable of being inserted, after the manner of telescopic tubes, in the main tube at A. This tube C carries a plane reflector D, of the blackened glass already described, which is capable of being turned on pivots, and is supplied with a double scale and index, by which the angle it makes with the axis of the tube can be regulated at pleasure. By turning the tube C round its axis, the plane of the reflector D may be presented successively on every side of the axis of the main tube.

A diaphragm is fixed in the tube at *d*, having a circular hole in its centre, to limit the magnitude of the transmitted pencil. The pieces E, F, G, and H, are severally capable of being inserted in the ends of the tube, and of being turned round in the same manner as already described with relation to the piece C inserted at the end A.

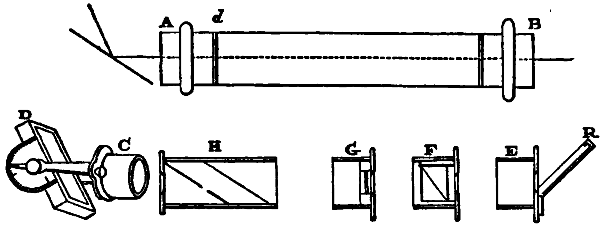

Fig. 410.

The short tube E carries a plane reflector R, similar to that already described, which is capable of being adjusted at any desired angle with the axis of the tube. The tube F contains a doubly refracting prism, the tube G contains a thin disk of tourmaline with parallel faces, so cut that the optic axis is parallel to these faces. In fine, the tube H contains a bundle of plates of glass, with parallel surfaces placed in contact with each other, and inclined obliquely to the axis of the tube.

All these pieces being severally inserted in the tube A B can be turned round its axis, so that the reflector R, or the prism, or the tourmaline G, or the included plates H, may be severally presented in succession on all sides of the ray transmitted along the axis of the tube A B.

1269. *Polarization by reflection.* — Let the tube C, *fig.* 410., carrying the reflector D, be inserted in the main tube A, and let a plate of blackened glass be inserted in the frame D, as already described. Let the apparatus be so adjusted that when a ray of light falling upon the plate D at an angle of incidence equal to 54° 35′ is reflected, the reflected ray will pass along the axis of the tube A B. Such a ray will be polarized, and the plane of its polarization will coincide with the plane of the angle of incidence upon the plate D.

To prove this, let the tube E carrying the reflector R be inserted in the end B of the main tube, and let the reflector R be adjusted so that the ray which passes along the axis of the tube shall fall upon it at the same angle of incidence, 54° 35′, as represented in *fig.* 411.

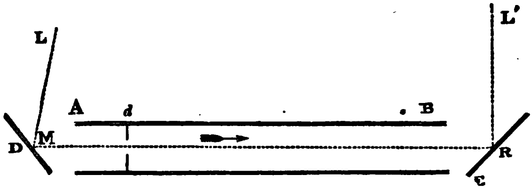

Fig. 411.

If the tube E be so placed that the plane of the angle of incidence upon the reflector R shall coincide with the plane of the angle of incidence on the reflector D, then the ray coming along the axis of the tube will be reflected from R with the greatest possible intensity. If the tube E be then turned round within the tube B, so as to present the reflector R successively on different sides of the ray which passes along the axis of the tube, it will be found that when the reflector R assumes such a position that the plane of the angle of incidence upon it is at right angles to the plane of the angle of incidence upon the reflector D, no reflection will take place, and the ray will be extinguished.

It follows, therefore, from this, first, that the ray passing along the axis of the tube is polarized; and, secondly, that its plane of polarization coincides with the plane of the angle of incidence of the original ray upon the reflector D.

If, instead of a blackened glass, any other reflecting surface were placed in the frame D, the same effects would ensue; but the angle of incidence upon such surface which would produce polarization, would be different for different surfaces.

1270. ***Method of determining the polarizing angle for different reflecting surfaces.*** — It was discovered by Sir David Brewster by observation, and afterwards confirmed by theory, that the polarizing angle for any reflecting surface is that angle of incidence which, being added to the corresponding angle of refraction, supposing the ray to enter the medium, would make up the sum of 90°. Thus, if A B C D, *fig.* 412., be a transparent medium bounded by parallel surfaces A B and C D, and if P I be a ray of light incident upon it at such an angle of incidence P I F that the angle of refraction R I F′ corresponding to it shall, when added to F I P, make 90°, then the angle P I F will be the polarizing angle, and a ray incident at such angle and reflected from I in the direction I P′ will be polarized.

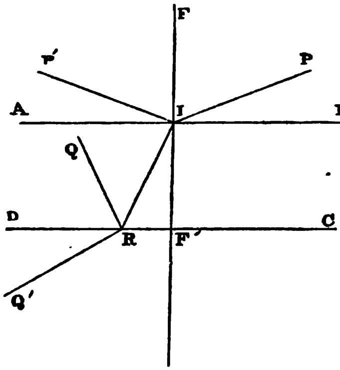

Fig. 412.

It is easy to show that in this case the directions of the reflected ray I P′ and the refracted ray I R are at right angles; for we have

$$\text{FIP} + \text{PIB} = 90°.$$

And since P I B is equal to P′ I A, we shall have

$$\text{FIP} + \text{P}'\text{IA} = 90°.$$

But since F I P + R I F′ = 90°, it follows that

P′ I A = R I F′.

If to both of these we add the angle A I R, we shall have the angle P′ I R equal to the angle A I F′; but since A I F′ is 90°, the angle P′ I R will be also 90°.

The angle of polarization is therefore determined by the condition that the reflected ray I P′ shall be at right angles to the direction it would have pursued, had it been reflected instead of refracted at I.

It is easy to show that when the ray I R emerges from the lower surface in the direction R Q′, parallel to P I, it will be at right angles to the direction it would have taken, if, instead of passing through the surface at R, it were reflected from it in the direction R Q; for since R Q′ and R D are respectively parallel to P I and B I, the angle D R Q′ is equal to the angle P I B, or, what is the same, the angle P′ I A, or, in fine, to the angle R I F′.

But the angle I R F′ is equal to the angle Q R D, therefore the angles R I F′ and I R F′ taken together, are equal to the angle Q R Q′; and since the former are equal to 90°, Q R Q′ is a right angle. Hence it follows that the ray I R also falls upon the surface D C at R at the angle of polarization, since its directions reflected and refracted are at right angles.

It follows from what precedes, that the polarizing angle corresponding to any surface separating two media is that angle whose trigonometrical tangent is equal to the index of refraction; for since the angle R I F′ is the complement of the angle F I P, the sine of F I P divided by the sine of R I F′ will be equal to the tangent of the angle F I P.

Thus, whenever the index of refraction for any medium is known, the polarizing angle for the surface of such medium can be determined; and whenever the polarizing angle can be found by observation, the index of refraction may be inferred.

Since the indices of refraction for the different component parts of solar light are different, it follows that the polarizing angle for each species of homogeneous light will also be different.

1271. *Table showing the polarizing angle of certain media.*—Sir David Brewster gives the following table, showing the polarizing angles corresponding to the mean and extreme rays for the undermentioned transparent media:—

		Index of Refraction.	Maximum polarizing Angle.	Difference between the greatest and least polarizing Angles.
			° ′	° ′
Water	Red rays	1·330	53 4	
	Mean rays	1·336	53 11	15
	Violet rays	1·342	53 19	
Plate glass	Red rays	1·515	56 34	
	Mean rays	1·525	56 45	21
	Violet rays	1·535	56 55	
Oil of cassia	Red rays	1·597	57 57	
	Mean rays	1·642	58 40	1 24
	Violet rays	1·687	59 21	

1272. *Effects of reflection on polarized light.*—If a ray of polarized light be incident upon any plane reflecting surface, the position of the plane of its polarization will in general be changed after reflection, and will be turned more or less towards the plane of the angle of incidence. If the angle at which the ray is incident be equal to the polarizing angle, then the plane of polarization, whatever may be its position in the incident ray, will coincide with the common plane of incidence and reflection in the reflected ray, so that the effect of reflection will be to turn this plane round the axis of the ray through the angle formed by it with the plane of incidence.

If, however, the angle at which the ray is incident be not equal to the polarizing angle, then the plane of polarization will not be turned entirely round to coincide with the plane of the angle of incidence, but will be turned towards that plane, so that the angle formed by the plane of polarization of the reflected ray with the plane of incidence will be less than the angle formed by the plane of the angle of polarization of the incident ray with the same plane.

The angle through which the plane of polarization is thus turned will depend upon the relation which the angle of incidence bears to the polarizing angle.

If the ray be incident perpendicularly upon the surface, no change will take place in the position of the plane of polarization, that of the reflected ray coinciding with that of the incident ray. If the angle of incidence be very small, then the plane of polarization of the reflected ray will be slightly turned towards the plane of incidence, and it will be more and more turned towards it as the angle of incidence approaches to equality with the polarizing angle. When they are equal, the plane of polarization will coincide with the plane of the angle of incidence. When the angle of incidence exceeds the angle of polarization, the plane of polarization of the reflected ray will be turned from the plane of the angle of incidence, and on the other side of it; and it will continue to be turned from it more and more as the angle

of incidence is increased, until it becomes a right angle. All these phenomena can be illustrated experimentally by means of the polariscopic apparatus already described, the plane of polarization being always capable of being determined by the means already explained.

1273. *Effects of ordinary refraction on polarized light.*—When a ray of polarized light enters any transparent medium, the plane of its polarization is changed after refraction, and is turned from th plane of the angle of incidence more or less, according as the angle o incidence differs more or less from the polarizing angle. The effect, therefore, of refraction on the plane of polarization is contrary to that produced by reflection. The more nearly the angle of incidence approaches to equality with the polarizing angle, the more nearly will the plane of polarization in the refracted ray be turned to a direction at right angles to the plane of incidence; and if the angle of incidence be absolutely equal to the polarizing angle, then the plane of polarization of the refracted ray will be at right angles to the plane of incidence, whatever may have been its position in the incident ray.

It follows, therefore, that if the plane of polarization of the incident ray be at right angles to the plane of incidence, it will suffer no change by refraction; but the further it departs from this direction the greater will be the change produced upon it by refraction.

1274. *Composition of unpolarized light.*—It was first suggested by Sir D. Brewster, and since confirmed by theory, that a ray of ordinary or unpolarized light consists of two rays polarized in planes at right angles to each other, the absolute direction of these planes being arbitrary. When such a ray is perfectly polarized, these planes of polarization are made to coincide, either or both being turned round the axis of the ray.

Polarized rays, may, however, also be obtained from a ray of natural light, either by resolving the ray into the two pencils of which it consists, and exhibiting them separately polarized in planes at right angles to each other, or by extinguishing one of the two rays, and not the other.

1275. *Polarization by double refraction.*—A doubly refracting crystal supplies the means of obtaining polarized rays by the first method.

When a ray of common light is incident upon such a crystal in a plane passing through its axis, it will be divided, as has been already explained, into two rays, the ordinary and extraordinary, both of which will be found to be polarized if examined by the test already explained. The plane of polarization of the ordinary ray will coincide with the plane of the angle of incidence, and the plane of polarization of the extraordinary ray will be at right angles to it. Thus the doubly refracting crystal resolves the ray of common light into its two component polarized rays, exactly as a common prism resolves a ray of solar light into its component rays of different refrangibility.

1276. *Partial polarization.*—As a ray of light is completely polarized when the two planes of polarization naturally at right angles are brought to absolute coincidence, and as it is completely unpolarized when these planes are at right angles, it is partially polarized when they are in any intermediate position; and it approaches more and more to the state of complete polarization as the obliquity of the two planes of polarization increases. Thus when they form an angle of 45° the ray may be considered as half polarized.

It was long contended that a pencil partially polarized consisted of rays completely polarized mixed with rays completely unpolarized in various proportions, according to the degree of partial polarization of the pencil; but Sir David Brewster suggested, what has been since confirmed by theory, that partial polarization must be otherwise understood, and that a pencil partially polarized contains in it no ray, either perfectly polarized or perfectly unpolarized, but consists of rays, each of which is imperfectly polarized, as just explained.

1277. *Polarization by successive refractions.*—It has been already shown that a ray of polarized light when it enters a transparent medium, and is refracted by it, has its plane of polarization turned from the plane of the angle of incidence through an angle greater or less in magnitude according to the relation which the angle of incidence bears to the polarizing angle. Now, since a ray of natural light consists of two rays of light polarized in planes at right angles to each other, such a ray when it enters a refracting medium will have both planes of polarization of its component rays turned towards a right angle with the plane of the angle of incidence.

If such a ray then be successively refracted by a series of media bounded by parallel planes, the planes of polarization of its component rays will undergo a series of changes of direction, each having a tendency to turn them into a direction at right angles to the common plane of incidence and emergence.

Sir David Brewster found that the light of a wax candle placed at the distance of ten or twelve feet from a series of parallel plates of ground glass was polarized at angles of incidence which depended on the number of plates as exhibited in the following table:—

No. of Plates of Crown Glass.	Observed Angle at which the Pencil is polarized. °	′
8	79	11
12	74	0
16	69	4
21	63	21
24	60	8
27	57	10
31	53	28
35	50	5
41	45	35
47	41	41

He inferred from these experiments that if we divide the number 41·84 by any number of crown glass plates, we shall obtain the tangent of the angle at which a pencil of light may be polarized by this number. He also inferred that the power of polarizing the refracted light increased with the angle of incidence between 0, or a minimum, at a perpendicular incidence, and the greatest possible, or a maximum, as the incidence approached 90°.

The apparatus represented at H, *fig.* 410., is adapted for the experimental demonstration of this. In the tube H is placed a series of five or more plates of glass, resting with their surfaces one upon the other, and capable of being adjusted in the tube, so as to form any desired angle with its axis.

If this piece H be inserted in the end A of the tube, and if the plates of glass be applied at the proper angle, it will be found that the light, after passing through them, is nearly polarized, and that its plane of polarization is perpendicular to the common plane of the angles of incidence and refraction. In this case, the more brilliant the pencil of light transmitted through the plates, the more numerous the plates must be in order to effect complete polarization.

Strictly speaking, no number of plates can bring the planes of polarization to absolute coincidence; but they may be said to approach so near to it, that the pencil will be to all appearance completely polarized with lights of ordinary intensity

A pencil thus polarized by refraction will exhibit the same properties when submitted to reflection, or when incident upon a plate of tourmaline, as have been already described with respect to light polarized by reflection.

1278. *Effect of tourmaline on polarized light.* — Let a plate of tourmaline be cut with surfaces parallel to each other and to its optic axis. Such a plate being fixed in the piece G, *fig.* 410., may be inserted in the end of the tube B, so as to receive the polarized ray transmitted along the axis of the tube perpendicular to its surface. When thus arranged, the tube G being turned within the tube B, so as to bring the optic axis of the tourmaline to coincide with the plane of polarization of the ray, the ray will be totally intercepted. If the tube be then turned, so that the axis of the tourmaline shall form an increasing angle with the plane of polarization, light will begin to be transmitted, and the intensity of the light so transmitted will gradually increase, until the axis of the tourmaline is at right angles to the plane of polarization, when its intensity will be a maximum. After it passes that, the tube G being slowly turned, the intensity will again diminish until the axis of the tourmaline again coincides with the plane of polarization, when the light will be completely intercepted. The tourmaline supplies, therefore, a test of polarization, and a means of ascertaining the position of the plane of

polarization more convenient still than that which has been already explained by means of the reflecting surface R.

1279. *Polarization by absorption.* — Sir David Brewster showed that agate and some other crystals had the effect of intercepting one of the two polarized rays which constitute common light and transmitting the other, and suggested this as a means of obtaining polarized light. Thus, if a ray of common light be transmitted through a plate of agate, one of the oppositely polarized beams will be converted into nebulous light in one position of the crystal, and the other in another position, so that one of the polarized beams will in each case be transmitted. The same effect may be produced by Iceland spar, Aragonite, or artificial salts, prepared in a peculiar manner, so as to produce a dispersion of one of the two polarized rays forming common light.

If common light be transmitted through a thin plate of tourmaline, one of the polarized rays which constitute it will in like manner be absorbed by the tourmaline, and the other transmitted; and when the tourmaline is applied in a position at right angles to this, the ray which was before transmitted is absorbed, and *vice versâ*.

1280. *Polarization by irregular reflection.* — When a pencil of light is directed obliquely on any imperfectly polished surface so as to be irregularly reflected from it, the rays thus reflected will be partially polarized, as may be ascertained by looking at the reflecting surface through the plate of tourmaline G, *fig.* 410. On turning round the plate of tourmaline, it will be found that the brightness of the surface will vary according to the direction of the axis of the tourmaline, the positions of the axis which render its brilliancy greatest and least being at right angles to each other. That the polarization in this case is imperfect is demonstrated by the fact that the tourmaline in no position produces a complete extinction of the light.

Since light is more or less polarized by successive refractions and by successive reflections, whether regular or irregular, it follows that light is almost never found without being more or less polarized. Thus the light of day proceeding from the solar rays reflected and refracted by the atmosphere and the clouds must always be more or less polarized, — an effect of which may be verified by examining this light by one or other of the tests of polarization, but more especially by the tourmaline already described.

1281. *The interference of polarized pencils.* — If two pencils of light have their planes of polarization parallel, they will exhibit the same phenomena of interference as have been already described for ordinary light. The production of bright and dark fringes, when the pencils are homogeneous, and the production of coloured fringes, when the pencils consist of compound light, will occur as in the case of unpolarized light

But if the two pencils be polarized in planes at right angles to each other, none of the phenomena of interference will be exhibited. No

matter under what circumstances the rays shall intersect, it can never happen that either ray will extinguish the other, or that the phenomena of dark and light or coloured fringes are produced.

When two pencils are polarized in planes forming with each other an oblique angle, they will produce fringes, but of inferior brilliancy to those exhibited when their planes of polarization are parallel.

If two pencils are first polarized in planes at right angles to each other, and afterwards have their planes of polarization rendered parallel, which may always be accomplished either by refraction or reflection, they will not recover the property of forming fringes of interference, of which they were deprived by rectangular polarization.

But if a pencil of common light be first completely polarized, and then be divided into two pencils polarized in rectangular planes, these two pencils, if their planes of polarization be again rendered parallel, will acquire the property of interference, and will exhibit fringes.

All these phenomena admit of verification by the polariscopic apparatus already described.

1282. *Compound solar light cannot be completely polarized by reflection, but may be nearly so.* — Since the polarizing angle varies with the index of refraction, and since white solar light is a compound of rays having different indices of refraction, it follows that a pencil of solar light can never be completely polarized by a reflecting surface, for the angle which would polarize completely one of its constituents would be different from the angle which would polarize completely another. But since the difference between the polarizing angles for the extreme rays in the case of glass is only 21′, and in the case of water still less, it follows that if the polarizing reflector be adjusted at the polarizing angle of the rays of mean refrangibility, the rays of extreme refrangibility will fall upon it at an angle differing very little from their polarizing angle, and, consequently, although they will not be completely, they will still be very nearly polarized.

1283. *Absence of complete polarization rendered evident by tourmaline.* — Nevertheless, the absence of complete polarization in this case is rendered extremely evident by the the test of the plate of tourmaline already described.

If the reflector D, *fig.* 410., be adjusted to the polarizing angle of the rays of mean refrangibility, and the plate of tourmaline G be applied to the end B of the tube, the rays corresponding to the middle of the spectrum only will be completely intercepted when the axis of the tourmaline is brought into the plane of polarization. A portion of the extreme rays at both ends of the spectrum will be transmitted through the tourmaline, and will be perceivable as bright purple light proceeding from the mixture of the red and violet rays which are transmitted. If the plate D be then adjusted to the polarizing angle of the violet rays, the red rays will be transmitted in considerable quantity, and the yellow less, so that the light transmitted will be a

reddish-orange; and if, on the other hand, the polarizing plate D be adapted to the polarizing angle of the red rays, the light transmitted will be a bluish-green. If the polarizing plate D be composed of any highly dispersive substance, such as cassia, diamond, chromate of lead, realgar, or specular iron, the colour of the unpolarized light transmitted from the tourmaline will be found to be extremely bright and beautiful.

1284. *Effect of a doubly refracting crystal on polarized light.*— Let us suppose a pencil of polarized light R P, *fig.* 413., to be incident

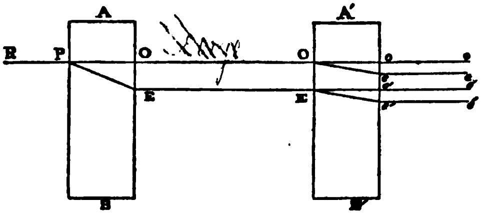

Fig. 413.

perpendicularly upon a plate A B, cut from a doubly refracting crystal, in such a manner that its surfaces are parallel to each other and to the optic axis of the crystal. The pencil R P, in passing through this plate, will be doubly refracted, the ordinary pencil proceeding in the direction P O O of the original pencil R P, and the extraordinary pencil taking another direction P E through the crystal, and emerging in the direction E E, parallel to that of the incident ray R P. These two pencils will be polarized in rectangular planes, the plane of polarization of the ordinary pencil O O coinciding with the optical axis of the crystal, and the plane of polarization of the extraordinary pencil E E being perpendicular to it.

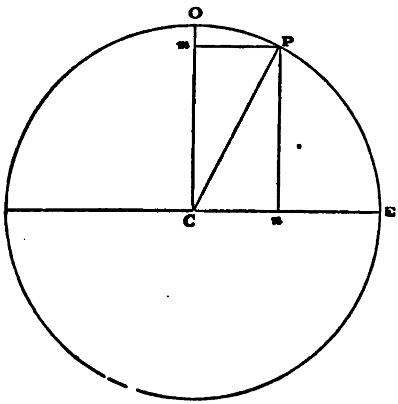

Fig. 414.

To render this more clear, let the circle, *fig.* 414., represent a section of the incident ray R P, and let C P be the direction of the plane of primitive polarization of the ray R P. Let C O be parallel to the optic axis of the crystal A B, and C E be perpendicular to it. It follows, therefore, that C O, *fig.* 414., will be

the direction of the plane of polarization of the ordinary pencil O O, *fig.* 413., and C E will be the direction of the plane of polarization of the extraordinary pencil E E, *fig.* 413.

It follows from the principles of the undulatory theory (and this consequence is confirmed by observation) that the proportion in which the light of the original pencil R P is shared by the ordinary and extraordinary pencils O O and E E will be expressed by the squares of the cosines of the angles which the plane of primitive polarization C P, *fig.* 414., makes with the planes of polarization of the two pencils O O and E E, *fig.* 413., respectively.

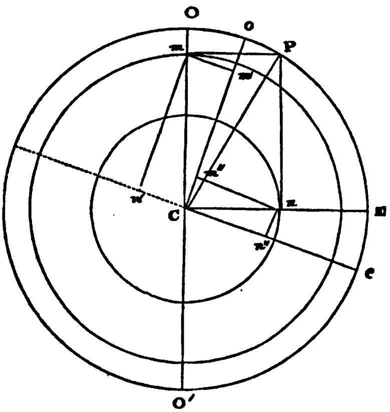

Fig. 415.

If, therefore, the number of rays in the original pencil R P be expressed by the square of the radius, *fig.* 415., the number of rays in the ordinary pencil O O will be expressed by the square of C *m*, and the number of rays in the extraordinary pencil E E will be expressed by the square of C *n*. The changes incident to the relative intensities of the ordinary and extraordinary pencils produced by the plate A B, may then be easily inferred from the diagram, *fig.* 414.

If the plane of polarization of the original ray R P coincide with the axis of the crystal A B, then C P, *fig.* 414., will coincide with C O, and the number of rays in the pencil O O, *fig.* 413., will be expressed by the square of the radius C O, while the pencil E E will vanish; for, in this case, C *m* will become equal to C O, and C *n* will vanish.

As the plane of primitive polarization C P makes an increasing angle with C O, C *m*, whose square represents the number of rays in the pencil O O, will decrease, and C *n*, whose square represents the number of rays in the pencil E E, will increase. The one pencil, therefore, will diminish, and the other increase in intensity. When the plane of primitive polarization C P makes an angle of 45° with the axis C O of the crystal, the line C P will bisect the angle O C E, and and C *m* will become equal to C *n*. In this position, therefore, the ordinary and extraordinary pencils O O and E E, *fig.* 413., will become equally intense, or contain the same number of rays.

When the plane of primitive polarization C P makes with the axis C O of the crystal A B a greater angle than 45°, C *m* becomes less than C *n*, and consequently the ordinary pencil O O, *fig.* 413., contains less

rays than the extraordinary pencil E E; and as the angle included between C P and C O increases, the extraordinary pencil will become relatively more intense, and the ordinary pencil less so, until the plane of primitive polarization C P makes a right angle with the axis C O of the crystal; in which case C P will coincide with C E, C *n* will become equal to C E, and C *m* will vanish.

Thus the ordinary pencil O O, *fig.* 413., will disappear, and all the rays of the incident pencil R P will pass into the emergent extraordinary pencil E E. A like succession of changes of intensity will take place if we suppose the axis of primitive polarization C P to revolve through another quadrant; the rays of the extraordinary pencil gradually passing into the ordinary one, and the extraordinary one vanishing, and the ordinary pencil acquiring the same intensity as the incident pencil, when the plane of polarization again coincides with the direction of the axis of the crystal.

It thus appears, that in a complete revolution of the plane of primitive polarization, or, what is the same, if that plane be fixed, in a complete revolution of the plate A B in its own plane, there will be two positions, 180° asunder, in which all the rays of the primitive pencil will pass into the ordinary pencil, and consequently, in which the primitive pencil will undergo no change either in its intensity or its polarization. Therefore, there will be two positions at right angles to these in which the primitive pencil again undergoes no change in intensity, but in which it is converted into the extraordinary pencil E E, its plane of polarization being turned through 90°, and receiving a direction at right angles to that of the plane of primitive polarization. In the intermediate positions between these four directions, the relative intensities of the ordinary and extraordinary pencils undergo constant change; that of the ordinary pencil being greater or less than that of the extraordinary pencil, according as the plane of primitive polarization makes a less or greater angle than 45° with the axis of the crystal A B, and the intensities of the two pencils are equal in the four positions in which the axis of primitive polarization is inclined at 45° to the axis of the crystal.

1285. *Effects produced by a second doubly refracting crystal.* — If we now suppose the ordinary and extraordinary pencils O O and E E, *fig.* 413., to be incident perpendicularly upon another doubly refracting plate A′ B′, cut with surfaces parallel to each other and to its optic axis, as before, they will each be again doubly refracted. The ordinary pencil O O will be divided into another ordinary pencil *o o* and an extraordinary pencil *e e*, while the extraordinary pencil E E will also be doubly refracted and resolved into two, an ordinary pencil *o′ o′*, and an extraordinary pencil *e′ e′*, all these four pencils emerging parallel to the primitive pencil R P.

To determine the proportion in which the rays of the original pencil R P are distributed among these four pencils, let C O, *fig.* 415.,

represent, as before, the direction of the optical axis of the plate A B and therefore the plane of polarization of the ordinary pencil O O; and consequently C E, perpendicular to C O, will represent the plane of polarization of the extraordinary pencil E E. Let C *o* represent the direction of the optical axis of the plate A′ B′, and let C *e* be a line perpendicular to it.

According to what has been already explained, the planes of polarization of the ordinary pencils *o o* and *o′ o′* will coincide with C *o*, the optical axis of the plate A′ B′, while the planes of polarization of the extraordinary pencils *e e* and *e′ e′* will coincide with the line C *e* perpendicularly to C *o*.

If the square of the radius C P, *fig.* 415., express the number of rays in the original pencil R P, the square of C *m*, as already explained, will express the number of rays in the pencil O O, and the square of C *n* the number of rays in the pencil E E

To obtain expressions for the intensities of the pencils into which these latter are resolved by the second crystal A′ B′, let circles be described with C as a centre, and C *m* and C *n* respectively as radii. From *m* draw *m n′* perpendicular to C *e*, and *m m′* perpendicular to C *o*. Since, then, the square of C *m* expresses the number of rays in the pencil O O, the square of C *m′* will express the number of rays in the pencil *o o*, and the square of C *n′* will express the number of rays in the pencil *e e*.

In like manner, if from *n* we draw *n m″* and *n n″* at right angles respectively to C *o* and C *e*, the number of rays in the pencil *o′ o′* will be expressed by the square of C *m″*, and the number of rays in the pencil *e′ e′* will be expressed by the square of C *n″*. We shall therefore have the following analysis of the intensities of the emergent pencils of the ordinary and extraordinary rays produced by the first plate A B, and of the four pencils, ordinary and extraordinary, produced by the second plate A′ B′.

Intensity of	original pencil R P	is expressed by	C P^2.
"	ordinary pencil O O	"	C m^2.
"	extraordinary pencil E E	"	C n^2.
"	ordinary pencil *o o*	"	C m'^2.
"	extraordinary pencil *e e*	"	C n'^2.
"	ordinary pencil *o′ o′*	"	C m''^2.
"	extraordinary pencil *e′ e′*	"	C n''^2.

If we suppose the plate A′ B′ to be turned round its centre, so as o make its optical axis C *o*, *fig.* 415., revolve, making varying angles with the planes of polarization of the rays O O and E E, a succession of changes will take place in the two pairs of ordinary and extraordinary pencils emerging from the plate A′ B′, in all respects analogous to those which have been already described as having taken place in the pencils O O and E E emerging from the first plate A B.

This change can be easily inferred from *fig*. 415., where C *o* represents the direction of the optical axis of the crystal A′ B′, and C O and C E the planes of polarization of the pencils O O and E E.

Thus, if we suppose the crystal A B turned into such a position that its optical axis C *o* shall coincide with C O, then C *m*′ will become equal to C *m*, and C *n*′ will vanish; therefore the pencil *o o* will contain all the rays of the incident pencil O O, and will have the same plane of polarization, while the pencil *e e* will vanish. At the same time that this takes place, C *e* will coincide with C E, and consequently C *n*″ will become equal to C *n*, and C *m*″ will vanish. Therefore the pencil *e*′ *e*′ will contain all the rays of the incident pencil E E. Thus it appears that in this case the second plate A′ B′ will make no change whatever, either on the intensities or the planes of polarization of the two rays O O and E E that emerge from the first crystal A B. If the axis of the second crystal C *o* be turned round so as to make a gradually increasing angle with the axis C O of A B, then the lines C *n*′ and C *m*″ will gradually increase, and the lines C *m*′ and C *n*″ will gradually diminish. Therefore the intensities of the ordinary pencil *o o* will gradually diminish, and that of the extraordinary pencil *e e* will gradually increase; and, at the same time, the intensity of the extraordinary pencil *e*′ *e*′ will gradually diminish, and that of the ordinary pencil *o*′ *o*′ will gradually increase.

When the axis C *o* of the crystal A′ B′ makes an angle of 45° with the axis C O of the crystal A B, then the four pencils will have equal intensities, for in such case C *o* will bisect the angle O C E, and the line C *e* will bisect the angle O′ C E; and in this case it is evident that all the four lines C *m*′, C *n*′, C *m*″, and C *n*″ will be equal; and since their squares express the intensities of the four pencils, these intensities will be equal. When the angle formed by the axis C *o* of the plate A′ B′, still increasing, forms an angle greater than 45° with the axis C O of the plate A B, then the line C *n*′ becomes greater than C *m*′, and consequently the pencil *e e* becomes more intense than the pencil *o o*. At the same time, the line C *n*″ will become less than C *m*″, and consequently the pencil *e*′ *e*′ will become less intense than the pencil *o*′ *o*′. These inequalities between the respective pencils will gradually increase with the gradually increasing angle formed by the axis of the plate A′ B′ with the axis of the plate A B, until these axes form a right angle with each other, in which case the pencils *o o* and *e*′ *e*′ wil vanish, and the pencil *e e* will contain all the rays of the pencil O O and the pencil *o*′ *o*′ will contain all the rays of the pencil E E. Thus when the axis of the crystal A′ B′ is applied at right angles to the axis of the crystal A B, no change is made in the intensities of the two pencils incident upon this second crystal; but the planes of polarization are respectively moved through a right angle, the ordinary pencil being converted into an extraordinary one, and the extraordi-

nary pencil being converted into an ordinary pencil. It is clear that the same succession of changes will take place throughout each successive quadrant through which the optical axes of the plates are turned.

CHAP. XX.

CHROMATIC PHENOMENA OF POLARIZED LIGHT.

1286. *Chromatic phenomena explicable by undulatory hypothesis.* — The splendid prismatic colours arranged in the form of concentric rings, intersected by dark and bright rectangular crosses, and occasionally by hyperbolic curves, are among the most remarkable and beautiful phenomena developed by modern experimental researches in optics. No triumph of theory can be more complete than the solution of these complicated appearances afforded by the undulatory hypothesis.

Any description, however, of these multitudinous and various appearances, much more any exposition of the mathematical solution of them supplied by the undulatory theory of light, would be incompatible with the objects and the necessary limits of this volume. While, however, we cannot enter into these details, we must not, on the other hand, pass over in absolute silence such phenomena.

1287. *Effect produced by the transmission of polarized light through thin doubly refracting plates.* — To convey some idea of the principles on which these phenomena are based, let us suppose the plates A B and A′ B′ to be so thin that the separation of the pencils into which the primitive pencil R P is resolved will be inconsiderable In such case, although the changes described in the last chapter will still be made in their planes of polarization, the pencils will more or less overlay each other, so that the rays composing one will fall within the limits of and be mixed with, the rays of the other.

It might therefore be inferred that the intensity or brilliancy of the pencils formed by each combination would be found by adding together the measures of their separate intensities. Thus, the two pencils $o\,o$ and $o'\,o'$, whose separate intensities are expressed by $\mathrm{C}\,m'^2$, and $\mathrm{C}\,m''^2$, would have thire combined intensity expressed by

$$\mathrm{C}\,m'^2 + \mathrm{C}\,m''^2.$$

But it must be considered that polarized light is subject to interference when its planes of polarization are parallel, which they are in the two cases here supposed, the planes of polarization of the pencils

$o\,o$ and $o'\,o'$ being both parallel to the axis of the crystal A′ B′, and the planes of polarization of the pencils $e\,e$ and $e'\,e'$ being both perpendicular to it. If, therefore, the other conditions of interference be fulfilled, it will follow that the rays of these two pairs of pencils would alternately extinguish one another, or produce a brilliancy equal to the sum of their intensities, according to the phases under which the luminous undulations meet.

But it is easy to show, that, provided one or both of the crystals A B and A′ B′ have a certain degree of thinness, the rays of the two pencils would fulfil the conditions which determine interference.

To prove this, it must be considered that the indices of ordinary and extraordinary refraction are different; therefore the velocities of the undulations in passing through the crystals will be different, if one be ordinarily and the other extraordinarily refracted; and if this difference be such as to produce by the undulation of the emergent pencils that relation which determines interference, that phenomenon must ensue. Now, on considering the refraction which the pencils $o\,o$ and $o'\,o'$ have suffered, it will appear that the former has undergone ordinary refraction by both crystals, while the latter has suffered extraordinary refraction, by the crystal A B, and ordinary refraction by the crystal A′ B′. Their velocities, therefore, through the crystal A B will be different; and if the thinness of the crystal be such that the undulations of the original rays are so related as to fulfil the conditions of interference, interference will ensue.

The same observations will be applicable to the pencils $e\,e$ and $e'\,e'$, the latter of which has suffered extraordinary refraction by both crystals, and the former ordinary refraction by A B, and extraordinary refraction by A′ B′.

1288. *Coloured rings and crosses explained.*—If, therefore, the plates be reduced to such a degree of thinness as to produce the phenomena of interference, a series of bright and dark rings will be produced; but as such rings will depend on the indices of refraction, and as these indices differ for each species of homogeneous light, it will follow that a different system of rings would be produced by each species of homogeneous light, of which the primitive pencil R P might be composed; and if such pencil be composed of compound solar light, then the resulting appearances are those which will be produced by the superposition of all the systems of rings which would be separately produced by each species of homogeneous light. The effect of the optical axes of the crystals, and of the revolution of either of them round its centre in its own plane, will be to produce dark or bright rectangular crosses corresponding to the planes of polarization of the emergent pencils, these crosses intersecting the systems of coloured rings.

We have here adopted for illustration, for the sake of simplicity, the case of crystals having a single axis of double refraction. The

appearances produced by crystals with two axes are analogous to these, though somewhat more complicated.

In these, two systems of rings, which sometimes assume the form of the curves called lemniscates, which have the form of the figure of 8, are produced, and the cross is often converted into hyperbolic curves, which in certain positions assume the form of a cross, the hyperbola passing into its asymptos.

To give a complete analysis of these complicated and beautiful chromatic phenomena would be impossible within the necessary limits of this volume; enough, however, has been explained of the principles of polarization to render their general theory intelligible; and we shall therefore now confine ourselves to a general description of some of the most interesting of the phenomena produced by transmitting polarized light through doubly refracting media.

1289. *Method of observing and analyzing these phenomena.— Apparatus of Noremberg.*—The polariscopic apparatus of Noremberg, represented in *fig.* 416., supplies convenient means of observing and analyzing the chromatic phenomena of polarized light.

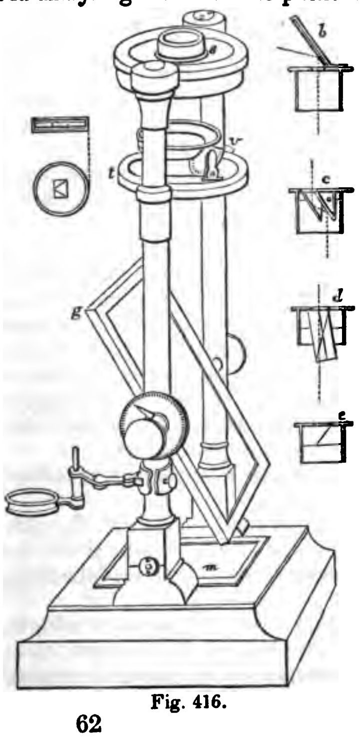

Fig. 416.

The polarizing apparatus is mounted in the lower part of the instrument, and consists of the frame *g* containing the polarizing plate, the horizontal reflector *m*, and other accessories. By means of these a pencil of light polarized in any required plane can be transmitted vertically upwards, so as to pass through the centre of the rings *v* and *s*.

The rings *v* and *s* are graduated, and a tube is inserted in each of them, having an index which plays on the divided scale as the tube is turned round its centre within the ring. Plane reflectors inclined at variable angles, plates of doubly refracting crystals, doubly refracting prisms, bundles of parallel plates of glass and other polariscopic tests, are set in short tubes capable of being fixed in one or other of the rings *v* and *s*. So the polarized pencil transmitted upwards along the axis of the

apparatus may first be made to pass through the plate inserted in *v*, and may then be examined by an inclined reflector or tourmaline plate, a doubly refracting prism, or by any other polariscopic test which may be fixed in *s*. The position of the indices which move on the divided circles of *v* and *s* will indicate the position and changes of position of the planes of polarization.

1290. *Effect of rock crystal.*—Let a plate of rock crystal, with surfaces cut parallel to its optic axis, the thickness of which does not exceed the 50th of an inch, be placed on the ring *v*; and let a doubly refracting prism, with a single axis of double refraction, be placed in *s*.

Let us first suppose that the axis of this prism coincides with the plane of polarization of the pencil incident on the plate *v*, and let the axis of this plate be first placed in the plane of polarization. In that case the incident ray will pass through both crystals without change, and an eye placed above the prism at *s* will see only the ordinary image of the object from which the pencil issues. If the axis of *v* be turned at right angles to the plane of polarization, a single image only will be seen; but in this case it will be the extraordinary image, and the plane of its polarization will be perpendicular to the plane of primitive polarization. The images will in both cases be white.

In all intermediate positions of the axis of the plate *v*, two images will be seen, which will partly overlay each other, as represented in *fig.* 417. Those parts which are not superposed will have colours exactly complementary, and the superposed parts on which these colours are combined will be white.

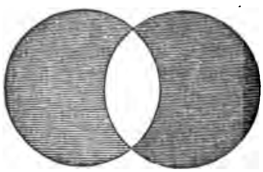

Fig. 417.

As the plate *v* is turned round its centre through 90°, from the position in which its axis coincides with the plane of primitive polarization to the position in which it is at right angles to that plane, the two images pass through a series of tints of colour (always, however, complementary), and through various degrees of relative brightness, their most vivid colours being exhibited when the axis is at 45° with the plane of primitive polarization.

The same changes take place in each successive quadrant through which the axis of *v* revolves.

If the axis of the prism *s* be placed at right angles with the plane of primitive polarization, a like succession of appearances will be exhibited, the ordinary and extraordinary images, however, interchanging places.

If the axis of the prism *s* be placed at any oblique angle with the plane of primitive polarization, a like succession of effects will be observed; but, in this case, the single images will be exhibited when the axis of the prism *s* coincides with, and is at right angles with that of

the plate *v*; and the double coloured images appear in the intermediate positions, the images having the greatest splendour when the two axes intersect at an angle of 45°.

There is therefore, in all cases, a single image in four positions in each revolution, these four positions being at right angles to each other; and intermediate between these, there are four other positions, also at right angles to each other, at which the complementary images attain their greatest brightness.

Plates of rock crystal more than the 50th of an inch in thickness produce like effects, but with less brilliant colours. In general, the colours vary with the thickness of the plate, the more brilliant tints being produced by the thinnest plates.

Different crystals exhibit striking differences in these chromatic phenomena. Thus Biot found that carbonate of lime cut parallel to the axis, required to be eighteen times thinner than rock crystal to produce the same tint. This circumstance renders it difficult to observe these phenomena with carbonate of lime.

1291. *Effect of Iceland spar inclosed between two plates of tourmaline.* — Let a plate of Iceland spar less than an inch thick be cut with parallel surfaces at right angles to its optic axis. If this be placed between two plates of tourmaline cut parallel to their axes, a series of beautiful chromatic phenomena will be observed by looking through it at the clouds. If the axes of the tourmalines are placed at right angles, the crystal will exhibit a system of concentric rings of the most vivid colours, intersected by a dark cross, as represented in *fig.* 418.

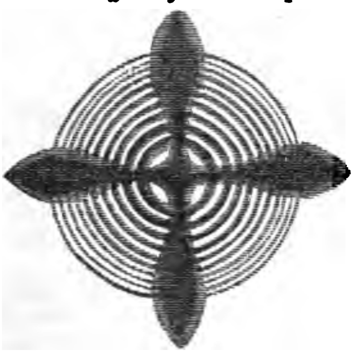

Fig. 418.

If the axis of one of the tourmalines be turned gradually round, making a decreasing angle with the axis of the other, the tints of the rings will undergo a series of changes, and the dark cross will show a space in the midst of each of its arms faintly luminous, as represented in *fig.* 419. These changes will proceed until the axis of the one tourmaline becomes parallel to the other, when the cross will become white, and all the tints of the rings will become complementary to those which they had in the first position, as represented in *fig.* 420.

If, instead of presenting the crystal to the white light of the heavens, a pencil of homogeneous light be transmitted through it, the rings, instead of showing various tints will be alternately dark

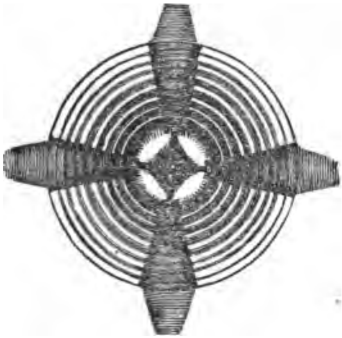

Fig. 419.

Fig. 420.

and of the colour of the homogeneous light; and the cross, in like manner, will be either dark or of the colour of the same light. The diameters of the successive rings will be different for each coloured light, being greater for the more refrangible colours; and the diameters of rings for the same colour will increase as the thickness of the crystal is diminished.

It is evident that the system of rays produced by white light results from the superposition of the several systems produced separately by the homogeneous coloured lights.

The white cross produced by white light, when the axes of the tourmalines are parallel, is in like manner produced by the superposition of all the coloured crosses produced by the homogeneous lights severally.

1292. *Effects produced by other uni-axial crystals.*—Phenomena analogous to these are produced by all crystals having a single axis of double refraction, such as rock crystal, tourmaline, zircon, nitrate of soda, mica, hyposulphate of lime, apophyllite, &c. In some cases, however, the effects are modified by conditions peculiar to the species of crystal under examination. Thus, in the case of rock crystal, the cross disappears, in consequence of the effect of circular polarization, which we shall presently notice. In other crystals there appear to be different optic axes for lights of different refrangibilities, which produce modifications in the appearance of the rings and crosses.

Of all crystals, the most convenient for the exhibition of these phenomena is Iceland spar.

1293. *Effect of bi-axial crystals; nitrate of potassa.*—If a plate of nitrate of potassa (a crystal having two axes), with parallel surfaces cut at right angles to its optic axis, be placed in like manner between two plates of tourmaline cut parallel to their axes, a series of chromatic appearances will be observed, which are represented in *figs.* 421., 422., and 423.

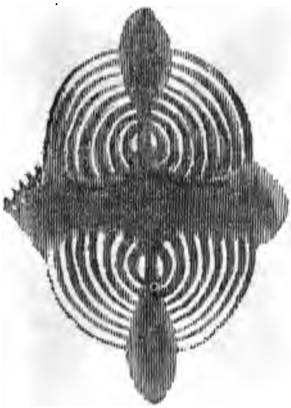

Fig. 421.

Fig. 422.

If the axes of the tourmalines are placed at right angles, the crystal itself being properly placed between them, a dark cross, *fig.* 421., will be seen intersecting a double system of coloured rings, the common centres of which correspond to the position of the two axes of the intermediate crystal.

If the crystal be turned gradually round its centre between the tourmaline plates without deranging the position of the latter, the cross will gradually assume the form of two hyperbolic curves, and the rings will change their position and tints as represented in *fig.* 422. When the crystal has been turned through half a quadrant, the appearance will be that represented in *fig.* 423., and after which it will assume a form like that of *fig.* 422., but more inclined to the horizontal position; and, in fine, when the crystal has been turned through a quadrant, the appearance will be that represented in *fig.* 421., the vertical arms of the cross, and the line joining the centres of the systems of concentric rings, being, however, horizontal.

1294. *Effect of the carbonate of lead.* — The carbonate of lead, another crystal with two axes, gives appearances analogous to those of nitrate of potassa. These are represented in *fig.* 424.

1295. *Coloured bands produced by an acute prism of rock crystal.* —If a piece of rock crystal be cut in the form of a prism, with a very acute angle, one surface forming the angle being parallel to the optic axis, and the other therefore slightly inclined to it, a pencil of polarized light transmitted through it will exhibit to the naked eye a series of alternated red and green fringes, provided the eye is placed at some distance from the crystal, and the thickness through which the light passes does not exceed the 50th of an inch. These coloured bands

Fig. 423.

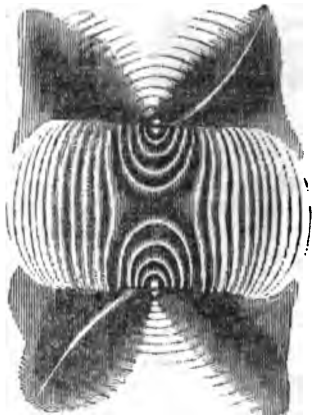

Fig. 424.

are more vivid when viewed through a plate of tourmaline, and it is easy to observe that they attain their greatest brightness when the axis of the prism is inclined at 45° to the plane of primitive polarization.

1296. *Polarizing structure artificially produced in glass and other media.* — A doubly refracting and polarizing structure may be produced in glass and other transparent bodies by molecular changes in their structure consequent on sudden changes of temperature, and sometimes by mere mechanical pressure.

If a circular plate of glass, about an inch in diameter and half an inch thick, be exposed to a high temperature by contact with a heated body which is a good conductor, so that its temperature near the edges shall be higher than at the centre; or if, on the contrary, it be raised to a higher temperature at the centre than near the edges, it will exhibit the phenomena of rectangular crosses and coloured rings, like those produced by doubly refracting crystals.

If, in this case, the plate be oval, it will exhibit appearances indicating two axes of double refraction.

When a plate is reduced to a uniform temperature, these appearances cease.

These phenomena are susceptible of infinite variation, according to the shape of the plate, which may be square, oblong, or of any other form. The disposition and form of the fringes and rings will vary with the form of the plate.

A permanent doubly refracting and polarizing structure may be imparted to glass by raising it to a high temperature, and then cooling it rapidly, by placing it in contact with the cold surfaces of metals.

The metallic surfaces, in this case, may be formed into an infinite variety of fancy patterns, which will have the effect of producing corresponding optical effects of great beauty.

CHAP. XXI.

CIRCULAR POLARIZATION.

1297. *Cases in which the change of the plane of polarization varies with the thickness of the crystal.*—In all the cases noticed in the preceding chapter in which a ray of polarized light passes through a plate of doubly refracting crystal, the change produced upon its plane of polarization is quite independent of the thickness of the crystal; this change depending solely upon the relative position of the plane of primitive polarization and the axis of the crystal.

We have now, however, to notice another class of polarizing influences, in which the change produced in the plane of polarization of the ray transmitted will vary with the thickness of the crystal.

If a plate of rock crystal be cut with parallel surfaces perpendicular to its optical axis, a ray of polarized light transmitted through it will have its plane of polarization changed, and turned through a certain angle. If the thickness of the plate be doubled or halved, then the angle through which the plane of polarization of the ray is turned will also be doubled or halved. In a word, the angle through which the plane of polarization would revolve when the ray passes through the crystal, will, for the same crystal, be proportional to the thickness of the plate.

The direction in which the plane of polarization is thus made to turn is different in different specimens. Thus, two different plates of rock crystal, having the same thickness, will turn the plane of polarization, one to the left and the other to the right.

1298. *The angle through which that plane is turned varies with the refrangibility of the light.*—The angle through which the plane of polarization is turned depends also upon the refrangibility of the light transmitted through the crystal, the angle increasing with the refrangibility. Thus, if a polarized ray of red light be transmitted through such a plate, the angle through which its plane of polarization will be turned will be less than that through which the plane of polarization of an orange ray would be turned, and this latter less than that through which the plane of polarization of a yellow, green or any other more refrangible ray would be turned.

It follows from this, that if a polarized pencil of white light be in-

cident upon such a plate, the emergent pencil will have different planes of polarization for light of each degree of refrangibility.

1299. *The plane may make a complete revolution if the thickness of the crystal be sufficient.*—It follows from these phenomena, that in its progress through the thickness of such a crystal the plane of polarization of a ray of homogeneous light is gradually turned round its centre, so that a thickness may be assigned which will cause this plane to make a complete revolution, so that the emergent ray will in this case appear as if it had suffered no change, although in reality, in its progress through the crystal, the plane of its polarization had in succession formed all angles with its original direction from 0° to 360°.

This effect on the plane of polarization may be illustrated by the motion of the thread of a screw in penetrating any substance.

To understand this distinction, as Sir John Herschel has observed, it is only necessary to take a common corkscrew, and holding it with the head towards him, let the observer turn it in the usual manner, as if to penetrate a cork. The head will then turn the same way as the plane of polarization of a ray in its progress from the spectator through a right-handed crystal may be conceived to do. If the thread of the corkscrew be reversed, as in a left-handed screw, then the motion of the head, as the instrument advances, would represent that of the plane of polarization in a left-handed crystal.

1300. *Angles through which the plane is turned by a crystal of quartz.*—The angles through which the plane of polarization of each of the component rays of the spectrum is made to turn by a plate of quartz cut perpendicular to the axis of the twenty-fifth of an inch thick is given in the following table:—

Homogeneous Ray.	Arcs of Rotation. °	′
Extreme red	17	30
Mean red	19	00
Limit of red and orange	20	29
Mean orange	21	24
Limit of orange and yellow	22	19
Mean yellow	24	00
Limit of yellow and green	25	40
Mean green	27	51
Limit of green and blue	30	03
Mean blue	32	19
Limit of blue and indigo	34	34
Mean indigo	36	07
Limit of indigo and violet	37	41
Mean violet	40	53
Extreme violet	44	05

1301. *Effect of amethyst.*—Sir David Brewster says, that in examining the phenomena of circular polarization produced in amethyst, he found that it possessed the same power in the same specimen of

turning the plane of polarization, both from left to right and from right to left, and that it actually consisted of alternate strata of right and left-handed quartz, whose planes were parallel to the axis of double refraction. These strata are not united together like the parts of certain composite crystals, whose dissimilar faces are brought into mechanical contact; for the right and left-handed strata destroy each other at the middle line between each stratum, and each stratum has its maximum polarizing force in its middle line, the force diminishing gradually to the line of junction.

1302. *Circular polarization in liquids and gases.*—Rock crystal is the only solid substance in which circular polarization has been observed. This phenomenon, however, has been discovered in several fluids. Thus, right-handed circular polarization exists in turpentine, essence of laurel (?), gum arabic and inuline; and left-handed polarization is observed in essence of citron (?), syrup of sugar, alcoholic solution of camphor, dextrine, and tartaric acid.

1303. *Magnetic circular polarization.*—It has lately been shown by Dr. Faraday, that several transparent solids and liquids acquire the property of circular polarization, when they have been submitted in a certain manner to magnetic and electric action.

These bodies appear to acquire a photogyric virtue, or a property by which they are enabled to cause the planes of undulation of the liquid which traverse them to revolve.

Thus, a link of connection is indicated between two physical influences which seemed hitherto distinct and independent; the force which produces the undulation of the luminous ether, and those of the electric and magnetic fluids.

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PRACTICAL QUESTIONS FOR THE STUDENT.

1. What must be the length of a plane mirror, in order that an observer may see his whole length therein, the mirror being placed parallel to the observer?

2. The radius of a concave reflector is 3 inches, and the distance of the focus of incident rays from the vertex 9 inches: what is the position of the focus of reflected rays? (959.)

NOTE.—The formula (A), given in 959., may be reduced to a more convenient form for use.

Thus, by transposing the term $\frac{1}{f}$, we have

$$\frac{1}{f'}=\frac{2}{r}-\frac{1}{f}=\frac{2f-r}{rf}.$$

Hence

$$f'=\frac{rf}{2f-r};$$

which gives us the following Rule for obtaining the position of the focus of reflected rays:—*Multiply the radius of the mirror by the distance of the focus of incident rays from the vertex, and divide the product by twice that distance minus the radius.*

By a proper attention to the signs of r and f, the formula and rule may be applied to all cases of reflection from spherical mirrors, whether concave or convex, and whether the rays be diverging or converging.

For concave mirrors, r is positive; for convex, negative. For diverging rays, f is positive; for converging, negative.

We will discuss briefly the various cases which may occur. For *concave* mirrors, there are three cases.

1st. *The rays may be diverging and the focus beyond the principal focus.* In this case $2f$ being greater than r, it will be seen from the formula that f' will always be positive; that is, the reflected rays will converge to a *real* focus in front of the mirror.

For the question proposed above, we have

$$f' = \frac{3 \times 9}{18 - 3} = 1\tfrac{4}{5} \text{ inches.}$$

2d. *The rays may be diverging and the focus between the principal focus and the vertex.* In this case $2f$ being less than r, f' will be negative, that is, the reflected rays will diverge from an *imaginary* focus behind the mirror.

3d. *The rays may be converging.* In this case, f is negative, and the formula becomes

$$f' = \frac{r \times -f}{-2f - r} = \frac{rf}{2f + r};$$

which is always positive. Hence such rays are always brought to a real focus.

For *convex* mirrors, there are also three cases.

1st. *The rays may be diverging.* Here, f is positive and r negative; and the formula becomes

$$f' = -\frac{rf}{2f + r};$$

which makes f' always negative. Hence, in this case, the reflected rays always diverge from an imaginary focus behind the mirror.

2d. *The rays may converge, and their focus be between the principal focus and the vertex.* In this case, r and f are both negative; $2f$ being less than r Hence the formula is

$$f' = \frac{-r \times -f}{-2f + r} = \frac{rf}{-2f + r};$$

which is positive so long as $2f$ is less than r. Consequently, in this case, the reflected rays converge to a real focus in front of the mirror.

3d. *The rays may converge, and their focus be beyond the principal focus.* In this case, $2f$ being greater than r, the value of f', in the preceding formula, will be negative. Hence the reflected rays will diverge.

It will be perceived from this discussion, that rays, incident upon a concave mirror, are always reflected converging, unless their focus be between the principal focus and the vertex.

On the contrary, rays, incident on a convex mirror, are always reflected diverging, unless their focus be between the principal focus and the vertex

3. A candle is placed 16 feet from the vertex of a convex mirror whose radius is 2 inches: what is the position of the focus of reflected rays?

4. The focus of converging rays incident upon a convex mirror is 2 inches behind the vertex, the radius of the mirror being 5 inches; the vertex of a concave mirror having the same radius is placed at a distance of 8 feet from the vertex of the first mirror: determine the position of the focus of the rays reflected from the second mirror.

5. Show that, in all cases of reflection from spherical surfaces, the conjugate foci lie on the same side of the principal focus; that they move in opposite directions; and that they meet at the centre of the reflector.

6. A plane mirror, moveable about an axis in its own plane parallel to the axis of the earth, revolves from east to west with half the sun's appa-

rent diurnal motion. Show that the direction of the reflected rays of sunlight will not be sensibly altered during the day.

7. The distance of Venus from the sun is about 69 millions of miles, and that of the earth about 95 millions. How does the brightness of the earth, as seen from Venus, compare with the brightness of Venus as seen from the earth, supposing the sizes and reflecting powers of the two bodies equal? (907.)

8. What is the focal length of a double-convex lens, the radius of each surface being 3 inches? (1038.)

9. What is the focal length of a plano-concave lens, the radius of the concave surface being 5 inches?

10. Why do objects appear further off and smaller, when viewed through the wrong end of the telescope?

11. A person can see distinctly at the distance of four inches: what is the focal length and nature of a lens which will enable him to see distinctly at the distance of sixteen inches?

12. A person can see distinctly at the distance of 12 feet: what is the focal length and nature of a lens which will enable him to see distinctly at the distance of 12 inches?

13. Place an object before a double-convex lens, so that the image may be double of the object, and erect.

14. An object is placed before a double-concave lens of glass, at the distance of 5 feet, and has the linear magnitude of its image 7 times less than its own: what is the focal length of the lens?

15. An object placed 4 inches before a double-convex glass lens, has its image formed 9 inches from the lens on the same side: what is the focal length of the lens?

16. A person, who can see distinctly at the distance of 3 feet, wishes to see an object at 12 feet distance; what sort of glass must he use, and what must be its focal length?

17. An object placed 4 inches before a double-convex lens has its image erect with respect to itself, and of three times its linear magnitude: what is the focal length of the lens?

18. Show that if a plane mirror recede from a fixed object, the image will recede twice as fast.

19. If an object be placed between two plane reflectors inclined to each other at any angle, and the eyes of a spectator be in any point between the planes, show that the distance of the eye from any of the images seen by him, is equal to the length of the path described by the rays which form that image.

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☞ The numbers refer to the foot of the pages.

THE END.

J. FAGAN, STEREOTYPER.

COLLINS, PRINTERS.

A Tandem Team

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A.

[illegible] is a gentleman

I am right up on "light" you know:

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As beautiful as a butterfly an

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Was.

Pretty little Polly Perkins

of Abington Green.

She was a nice Gal

Fol de riddle ri do

She was a lucky gal

&

Fol de riddle eh.

~~George Bolight~~

Frank Bowen Frank Bowen

Frank Bowen. Frank Bowen

Francis Bowen ~~Frank~~ Frank ~~Exercise~~